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On certain one-dimensional elliptic systems under different growth conditions at respective infinities

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Abstract.

A one-dimensional semi-linear elliptic system with constraint to the growth of its solutions at respective infinities is discussed. This system appears in a rescaling limit of a competition-diffusion system which describes very strong inter-specific competition between two biological species. A necessary and sufficient condition for the existence of solutions is characterized in terms of the constraint at respective infinities. Also the uniqueness of a solution and several asymptotic estimates of its derivatives at infinities are stated. Moreover similar problems are discussed for an associated inhomogeneous linear elliptic system with constraint to the growth of its solutions at infinities.

§1. Introduction

In this article we prepare fundamental facts which are necessary to describe asymptotic profiles of positive solutions to a Lotka-Volterra competition-diffusion system

$$(1)\qquad \begin{cases} u_t = \Delta u + (a-u)u - \epsilon^{-3}buw, & x \in \mathbf{R}^N,\ t>0, \\ w_t = D\Delta w + (d-w)w - \epsilon^{-3}cuw, & x \in \mathbf{R}^N,\ t>0 \end{cases}$$

as a positive parameter ϵ is nearly equal to 0. Here a, b, c, d and D are positive constants; $u = u(x,t)$ and $w = w(x,t)$ denote the population densities of competing two species at position x and time t. When ϵ is very small, the habitats of the two species become soon separated in a short time period because of the "strong inter-specific competition" (see [3]). In the limit as $\epsilon \to +0$, (1) is reduced to a free boundary problem which describes such "spatial segregation" phenomena between the competing species: $\mathbf{R}^N$ is divided into the habitat $\Omega_u(t)$ for u and the habitat $\Omega_w(t)$ for w by a moving interface $\Gamma(t)$; in particular, $(\nabla_x u,\ \nabla_x w)$ has

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a gap across $\Gamma(t)$ while (u, w) is continuous (cf. [1]). Thus, when ϵ is very small, we expect that $u(\cdot, t)$ (resp. $w(\cdot, t)$) almost vanishes in $\Omega_w(t)$ (resp. $\Omega_u(t)$) and that $(\nabla_x u, \nabla_x w)$ becomes almost singular along $\Gamma(t)$; namely $(u(\cdot, t), w(\cdot, t))$ has a "corner layer" along $\Gamma(t)$.

We would like to investigate the profile of (u, w) near such a corner layer. Assuming that $\Gamma(t)$ is a smooth hypersurface, formally expand $u(x, t)$ and $w(x, t)$ in a narrow region along $\Gamma(t)$ as

$$(2)\qquad \begin{cases} u(x,t) = \epsilon U_1\left(\frac{r}{\epsilon};\, \sigma, t\right) + \epsilon^2 U_2\left(\frac{r}{\epsilon};\, \sigma, t\right) + o(\epsilon^2), \\ w(x,t) = \epsilon W_1\left(\frac{r}{\epsilon};\, \sigma, t\right) + \epsilon^2 W_2\left(\frac{r}{\epsilon};\, \sigma, t\right) + o(\epsilon^2) \end{cases}$$

for ϵ small enough. Here $r = r(x, t)$ is the signed distance from x to $\Gamma(t)$ with $r(x, t) > 0$ in $\Omega_w(t)$ and $\sigma = (\sigma_1(x, t), \ldots, \sigma_{N-1}(x, t))$ is a local coordinate of the point on $\Gamma(t)$ closest to x; the functions $(U_i, W_i) = (U_i(\rho; \sigma, t), W_i(\rho; \sigma, t))$ $(i = 1, 2)$ are independent of ϵ. A method of matched asymptotic expansion formally tells us that (U_i, W_i) $(i = 1, 2)$ satisfy

$$(3)\qquad \begin{cases} U_{1,\rho\rho} = bU_1W_1, & \rho \in (-\infty, +\infty), \\ DW_{1,\rho\rho} = cU_1W_1, & \rho \in (-\infty, +\infty), \\ U_1 > 0, \quad W_1 > 0, & \rho \in (-\infty, +\infty), \\ (U_1,\, W_1) \approx (\alpha_0^- + \alpha_1^- \rho,\ 0) & \text{as } \rho \to -\infty, \\ (U_1,\, W_1) \approx (0,\ \alpha_0^+ + \alpha_1^+ \rho) & \text{as } \rho \to +\infty \end{cases}$$

and

$$(4)\qquad \begin{cases} U_{2,\rho\rho} - b(W_1U_2 + U_1W_2) = \gamma^- U_{1,\rho}, & \rho \in (-\infty, +\infty), \\ DW_{2,\rho\rho} - c(W_1U_2 + U_1W_2) = \gamma^+ W_{1,\rho}, & \rho \in (-\infty, +\infty), \\ (U_2,\, W_2) \approx \left(\beta_0^- + \beta_1^- \rho + \beta_2^- \frac{\rho^2}{2},\ 0\right) & \text{as } \rho \to -\infty, \\ (U_2,\, W_2) \approx \left(0,\ \beta_0^+ + \beta_1^+ \rho + \beta_2^+ \frac{\rho^2}{2}\right) & \text{as } \rho \to +\infty \end{cases}$$

for certain parameters $\alpha_i^\pm = \alpha_i^\pm(\sigma, t)$ $(i = 0, 1)$, $\beta_j^\pm = \beta_j^\pm(\sigma, t)$ $(j = 0, 1, 2)$ and $\gamma^\pm = \gamma^\pm(\sigma, t)$. Here and hereafter we use the notation "$f \approx g$ as $\rho \to +\infty$ (etc.)" which means that $f(\rho) - g(\rho) \to 0$ as $\rho \to +\infty$ (etc.) for functions f and g of $\rho \in (-\infty, +\infty)$. The parameters α_i^- and β_j^- (resp. α_i^+ and β_j^+) $(i = 0, 1;\ j = 0, 1, 2)$ are actually expressed with use of some asymptotic values of $u(\cdot, t)|_{\overline{\Omega_u(t)}}$ (resp. $w(\cdot, t)|_{\overline{\Omega_w(t)}}$) on $\Gamma(t)$ as $\epsilon \to +0$; $\gamma^\pm$ are expressed with use of the normal velocity and the mean curvature of $\Gamma(t)$; in particular

$$(5)\qquad \alpha_1^-(\sigma, t) < 0, \quad \alpha_1^+(\sigma, t) > 0,$$

which suggests that $(\nabla_x u, \nabla_x w)$ has a gap across $\Gamma(t)$ in the limit $\epsilon \to +0$. Hereafter we assume (5), even if it is not maintained. Thus (U_1, W_1) is formally determined by the semilinear elliptic boundary value problem (3) on $(-\infty, +\infty)$ and (U_2, W_2) is formally determined by the inhomogeneous linear elliptic boundary value problem (4) on $(-\infty, +\infty)$, where σ and t are regarded as parameters.

Our concern is the following two:

(i) the necessary and sufficient conditions on the parameters $\alpha_i^{\pm}$ $(i = 0, 1)$, $\beta_j^{\pm}$ $(j = 0, 1, 2)$ and $\gamma^{\pm}$ for the solvability of (3) and (4);

(ii) smooth dependence of (U_i, W_i) upon (σ, t) $(i = 1, 2)$.

We give an answer to (i) and (ii) in the following section. As its application we will rigorously formulate the matched asymptotic expansion (2) of positive solutions (u, w) to (1) as $\epsilon \to +0$ in the forthcoming paper.

§2. Results

The fundamental properties of (U_i, W_i) $(i = 1, 2)$ can be stated with use of a solution $\psi = \psi(\xi)$ of the elliptic boundary value problem on the whole line $(-\infty, +\infty)$:

$$\begin{cases} \psi'' = \psi^2 - \xi^2, & \xi \in (-\infty, +\infty), \\ \psi > |\xi|, & \xi \in (-\infty, +\infty), \\ \psi \approx |\xi| & \text{as } |\xi| \to \infty. \end{cases} \tag{6}$$

Here we give two propositions on ψ.

Proposition 2.1. *A solution $\psi = \psi(\xi)$ of (6) exists and is unique. Moreover it satisfies*

$$\psi(-\xi) \equiv \psi(\xi) \quad \text{on } (-\infty, +\infty), \tag{7}$$

$$0 < \psi'(\xi) < 1 \quad \text{on } (0, +\infty), \qquad \lim_{\xi \to +\infty} \psi'(\xi) = 1. \tag{8}$$

Proposition 2.2. *For any positive number λ there exists a positive constant C such that the solution $\psi(\xi)$ of (6) satisfies*

$$\begin{cases} 0 < \psi(\xi) - \xi \leq Ce^{-\lambda\xi}, \\ 0 > \psi'(\xi) - 1 \geq -Ce^{-\lambda\xi} \end{cases} \tag{9}$$

on $[0, +\infty)$.

We can state the solvability of (3) as follows:

Theorem 2.3. *The elliptic boundary value problem* (3) *possesses a solution* $(U_1, W_1) = (U_1(\rho;\sigma,t), W_1(\rho;\sigma,t))$ *if and only if*

$$c\alpha_0^- = -bD\alpha_0^+, \quad c\alpha_1^- = -bD\alpha_1^+. \tag{10}$$

Fix a (σ, t) *satisfing* (10). *Then the solution* $(U_1(\cdot;\sigma,t), W_1(\cdot;\sigma,t))$ *is unique and satisfies*

$$\begin{cases} U_1(\rho;\sigma,t) > \max\{\alpha_0^-(\sigma,t) + \alpha_1^-(\sigma,t)\rho,\ 0\}, \\ W_1(\rho;\sigma,t) > \max\{0,\ \alpha_0^+(\sigma,t) + \alpha_1^+(\sigma,t)\rho\}, \\ \alpha_1^-(\sigma,t) < U_{1,\rho}(\rho;\sigma,t) < 0, \\ 0 < W_{1,\rho}(\rho;\sigma,t) < \alpha_1^+(\sigma,t) \end{cases}$$

for $\rho \in (-\infty, +\infty)$.

We can state the solvability of (4) as follows:

Theorem 2.4. *Assume* (10) *and let* (U_1, W_1) *be the solution of* (3). *The elliptic boundary value problem* (4) *possesses a solution* $(U_2, W_2) = (U_2(\rho;\sigma,t), W_2(\rho;\sigma,t))$ *if and only if*

$$\begin{cases} c\beta_0^- + bD\beta_0^+ \\ \quad = (\gamma^+ - D\gamma^-)\left\{\dfrac{\alpha_0^+}{\alpha_1^+}\dfrac{b\alpha_0^+}{2} + \left(\dfrac{b\alpha_1^+}{2}\right)^{\frac{1}{3}} \displaystyle\int_{-\infty}^{+\infty} (\psi(\xi) - |\xi|)\, d\xi\right\}, \\ c\beta_1^- + bD\beta_1^+ = b\alpha_0^+(\gamma^+ - D\gamma^-), \\ \beta_2^- = \alpha_1^-\gamma^-, \qquad D\beta_2^+ = \alpha_1^+\gamma^+, \end{cases} \tag{11}$$

where ψ *is the solution of* (6). *For a fixed* (σ, t) *satisfing* (10) *and* (11) *the solution* $(U_2(\cdot;\sigma,t), W_2(\cdot;\sigma,t))$ *is unique.*

Under the following assumption on $\alpha_i^\pm$, $\beta_j^\pm$ and $\gamma^\pm$ we can deduce several asymptotic estimates for (U_i, W_i) $(i = 1, 2)$.

(A): Let T be a positive number and $\overline{\Gamma}$ a smooth hypersurface in $\mathbf{R}^N$. As smooth functions of $(\sigma, t) \in \overline{\Gamma} \times [0, T]$, $\alpha_i^\pm = \alpha_i^\pm(\sigma, t)$ $(i = 0, 1)$, $\beta_j^\pm = \beta_j^\pm(\sigma, t)$ $(j = 0, 1, 2)$, $\gamma^\pm = \gamma^\pm(\sigma, t)$ and all their derivatives with respect to t and local coordinates $(\sigma_1, \sigma_2, \ldots, \sigma_{N-1})$ of σ are bounded on $\overline{\Gamma} \times [0, T]$. Moreover (10) and (11) hold true on $\overline{\Gamma} \times [0, T]$, and $\alpha_1^\pm$ satisfy

$$\sup_{(\sigma,t)\in\overline{\Gamma}\times[0,T]} \alpha_1^-(\sigma,t) < 0, \qquad \inf_{(\sigma,t)\in\overline{\Gamma}\times[0,T]} \alpha_1^+(\sigma,t) > 0.$$

Theorem 2.5. *Assume* (A) *and take an arbitrary positive number* λ. *Then there exists a positive constant* C *such that the solution* $(U_1, W_1) = (U_1(\rho;\, \sigma, t),\, W_1(\rho;\, \sigma, t))$ *of* (3) *satisfies*

$$\left\{ \begin{array}{l} 0 < U_1 - \max\{\alpha_0^- + \alpha_1^- \rho,\, 0\} \leq Ce^{-\lambda|\rho|}, \\ 0 < W_1 - \max\{0,\, \alpha_0^+ + \alpha_1^+ \rho\} \leq Ce^{-\lambda|\rho|} \end{array} \right. \tag{12}$$

for $(\rho, \sigma, t) \in (-\infty, +\infty) \times \overline{\Gamma} \times [0, T]$;

$$\left\{ \begin{array}{l} \left|\dfrac{\partial U_1}{\partial \rho} - \alpha_1^-\right| + \left|\dfrac{\partial W_1}{\partial \rho}\right| \leq Ce^{\lambda\rho}, \\ \left|\dfrac{\partial U_1}{\partial t} - \left(\dfrac{\partial \alpha_0^-}{\partial t} + \dfrac{\partial \alpha_1^-}{\partial t}\rho\right)\right| + \left|\dfrac{\partial W_1}{\partial t}\right| \leq Ce^{\lambda\rho}, \\ \left|\dfrac{\partial U_1}{\partial \sigma_i} - \left(\dfrac{\partial \alpha_0^-}{\partial \sigma_i} + \dfrac{\partial \alpha_1^-}{\partial \sigma_i}\rho\right)\right| + \left|\dfrac{\partial W_1}{\partial \sigma_i}\right| \leq Ce^{\lambda\rho}, \\ \left|\dfrac{\partial^2 U_1}{\partial \sigma_i \partial \sigma_j} - \left(\dfrac{\partial^2 \alpha_0^-}{\partial \sigma_i \partial \sigma_j} + \dfrac{\partial^2 \alpha_1^-}{\partial \sigma_i \partial \sigma_j}\rho\right)\right| + \left|\dfrac{\partial^2 W_1}{\partial \sigma_i \partial \sigma_j}\right| \leq Ce^{\lambda\rho} \\ \qquad\qquad\qquad\qquad\qquad\qquad (i, j = 1, 2, \ldots, N-1) \end{array} \right. \tag{13}$$

for $(\rho, \sigma, t) \in (-\infty, 0] \times \overline{\Gamma} \times [0, T]$;

$$\left\{ \begin{array}{l} \left|\dfrac{\partial U_1}{\partial \rho}\right| + \left|\dfrac{\partial W_1}{\partial \rho} - \alpha_1^+\right| \leq Ce^{-\lambda\rho}, \\ \left|\dfrac{\partial U_1}{\partial t}\right| + \left|\dfrac{\partial W_1}{\partial t} - \left(\dfrac{\partial \alpha_0^+}{\partial t} + \dfrac{\partial \alpha_1^+}{\partial t}\rho\right)\right| \leq Ce^{-\lambda\rho}, \\ \left|\dfrac{\partial U_1}{\partial \sigma_i}\right| + \left|\dfrac{\partial W_1}{\partial \sigma_i} - \left(\dfrac{\partial \alpha_0^+}{\partial \sigma_i} + \dfrac{\partial \alpha_1^+}{\partial \sigma_i}\rho\right)\right| \leq Ce^{-\lambda\rho}, \\ \left|\dfrac{\partial^2 U_1}{\partial \sigma_i \partial \sigma_j}\right| + \left|\dfrac{\partial^2 W_1}{\partial \sigma_i \partial \sigma_j} - \left(\dfrac{\partial^2 \alpha_0^+}{\partial \sigma_i \partial \sigma_j} + \dfrac{\partial^2 \alpha_1^+}{\partial \sigma_i \partial \sigma_j}\rho\right)\right| \leq Ce^{-\lambda\rho} \\ \qquad\qquad\qquad\qquad\qquad\qquad (i, j = 1, 2, \ldots, N-1) \end{array} \right. \tag{14}$$

for $(\rho, \sigma, t) \in [0, +\infty) \times \overline{\Gamma} \times [0, T]$.

Theorem 2.6. *Assume* (A) *and take an arbitrary positive number* λ. *Then there exists a positive constant* C *such that the solution*

$(U_2, W_2) = (U_2(\rho;\, \sigma, t),\, W_2(\rho;\, \sigma, t))$ *of* (4) *satisfies*

(15)

$$\begin{cases} \left| U_2 - \left(\beta_0^- + \beta_1^- \rho + \beta_2^- \frac{\rho^2}{2} \right) \right| + |W_2| \le Ce^{\lambda\rho}, \\ \left| \frac{\partial U_2}{\partial \rho} - (\beta_1^- + \beta_2^- \rho) \right| + \left| \frac{\partial W_2}{\partial \rho} \right| \le Ce^{\lambda\rho}, \\ \left| \frac{\partial U_2}{\partial t} - \left(\frac{\partial \beta_0^-}{\partial t} + \frac{\partial \beta_1^-}{\partial t}\rho + \frac{\partial \beta_2^-}{\partial t}\frac{\rho^2}{2} \right) \right| + \left| \frac{\partial W_2}{\partial t} \right| \le Ce^{\lambda\rho}, \\ \left| \frac{\partial U_2}{\partial \sigma_i} - \left(\frac{\partial \beta_0^-}{\partial \sigma_i} + \frac{\partial \beta_1^-}{\partial \sigma_i}\rho + \frac{\partial \beta_2^-}{\partial \sigma_i}\frac{\rho^2}{2} \right) \right| + \left| \frac{\partial W_2}{\partial \sigma_i} \right| \le Ce^{\lambda\rho}, \\ \left| \frac{\partial^2 U_2}{\partial \sigma_i \partial \sigma_j} - \left(\frac{\partial^2 \beta_0^-}{\partial \sigma_i \partial \sigma_j} + \frac{\partial^2 \beta_1^-}{\partial \sigma_i \partial \sigma_j}\rho + \frac{\partial^2 \beta_2^-}{\partial \sigma_i \partial \sigma_j}\frac{\rho^2}{2} \right) \right| + \left| \frac{\partial^2 W_2}{\partial \sigma_i \partial \sigma_j} \right| \le Ce^{\lambda\rho} \\ \hfill (i, j = 1, 2, \ldots, N-1) \end{cases}$$

for $(\rho, \sigma, t) \in (-\infty, 0] \times \overline{\Gamma} \times [0, T]$;

(16)

$$\begin{cases} |U_2| + \left| W_2 - \left(\beta_0^+ + \beta_1^+ \rho + \beta_2^+ \frac{\rho^2}{2} \right) \right| \le Ce^{-\lambda\rho}, \\ \left| \frac{\partial U_2}{\partial \rho} \right| + \left| \frac{\partial W_2}{\partial \rho} - (\beta_1^+ + \beta_2^+ \rho) \right| \le Ce^{-\lambda\rho}, \\ \left| \frac{\partial U_2}{\partial t} \right| + \left| \frac{\partial W_2}{\partial t} - \left(\frac{\partial \beta_0^+}{\partial t} + \frac{\partial \beta_1^+}{\partial t}\rho + \frac{\partial \beta_1^+}{\partial t}\frac{\rho^2}{2} \right) \right| \le Ce^{-\lambda\rho}, \\ \left| \frac{\partial U_2}{\partial \sigma_i} \right| + \left| \frac{\partial W_2}{\partial \sigma_i} - \left(\frac{\partial \beta_0^+}{\partial \sigma_i} + \frac{\partial \beta_1^+}{\partial \sigma_i}\rho + \frac{\partial \beta_2^+}{\partial \sigma_i}\frac{\rho^2}{2} \right) \right| \le Ce^{-\lambda\rho}, \\ \left| \frac{\partial^2 U_2}{\partial \sigma_i \partial \sigma_j} \right| + \left| \frac{\partial^2 W_2}{\partial \sigma_i \partial \sigma_j} - \left(\frac{\partial^2 \beta_0^+}{\partial \sigma_i \partial \sigma_j} + \frac{\partial^2 \beta_1^+}{\partial \sigma_i \partial \sigma_j}\rho + \frac{\partial^2 \beta_2^+}{\partial \sigma_i \partial \sigma_j}\frac{\rho^2}{2} \right) \right| \le Ce^{-\lambda\rho} \\ \hfill (i, j = 1, 2, \ldots, N-1) \end{cases}$$

for $(\rho, \sigma, t) \in [0, +\infty) \times \overline{\Gamma} \times [0, T]$.

Under the condition (10) we can show that (U_1, W_1) is represented by

$$\text{(17)} \qquad \begin{cases} U_1(\rho; \sigma, t) = \left(\frac{(\alpha_1^-)^2 D}{4c} \right)^{\frac{1}{3}} \{\psi(\xi_-) - \xi_-\}, \\ W_1(\rho; \sigma, t) = \left(\frac{(\alpha_1^+)^2}{4b} \right)^{\frac{1}{3}} \{\psi(\xi_+) + \xi_+\}, \end{cases}$$

where ψ is the solution of (6) and $\xi_\pm = \xi_\pm(\rho;\sigma,t)$ is defined by

$$(18)\qquad \begin{cases} \xi_- := -\left(\dfrac{c}{2(\alpha_1^-)^2 D}\right)^{\frac{1}{3}} (\alpha_0^- + \alpha_1^- \rho), \\ \xi_+ := \left(\dfrac{b}{2(\alpha_1^+)^2}\right)^{\frac{1}{3}} (\alpha_0^+ + \alpha_1^+ \rho). \end{cases}$$

Note that

$$(19)\qquad \begin{cases} \xi_-(\rho;\sigma,t) = \xi_+(\rho;\sigma,t), \\ \xi_{-,\rho} = \xi_{+,\rho} = \left(\dfrac{b\alpha_1^+}{2}\right)^{\frac{1}{3}} > 0 \end{cases}$$

holds true for $\rho \in (-\infty, +\infty)$, as long as the parameter (σ, t) satisfies (10). Essential properties of the boundary value problems (3) and (4) become clear when they are expressed with use of $\xi_\pm$ instead of ρ. Owing to some properties of the solution ψ of (6), we can construct (U_2, W_2) as

(20)

$$\begin{cases} U_2(\rho;\sigma,t) = \dfrac{1}{c}\left(-\dfrac{1}{2}\left\{1 - \dfrac{\xi_-}{\psi(\xi_-)}\right\} Q(\xi_-;\sigma,t) + \omega(\xi_-;\sigma,t)\right), \\ W_2(\rho;\sigma,t) = \dfrac{1}{bD}\left(\dfrac{1}{2}\left\{1 + \dfrac{\xi_+}{\psi(\xi_+)}\right\} Q(\xi_+;\sigma,t) + \omega(\xi_+;\sigma,t)\right), \end{cases}$$

as long as (10) and (11) hold. Here $\omega(\xi;\sigma,t)$ is a smooth function satisfying

$$(21)\qquad \begin{cases} \omega = O(e^{-\lambda|\xi|}), \\ -\dfrac{\partial^2 \omega}{\partial \xi^2} + 2\psi(\xi)\omega = O(e^{-\lambda|\xi|}) \end{cases}$$

as $|\xi| \to \infty$ for an arbitrary positive number λ, and $Q(\xi;\sigma,t)$ is given by

(22)

$$\begin{aligned} Q(\xi;\sigma,t) \;&:=\; \left(\frac{b\alpha_1^+}{2}\right)^{\frac{1}{3}} \left((\gamma^+ - D\gamma^-)\int_0^\xi \psi(\eta)\,d\eta + (\gamma^+ + D\gamma^-)\frac{\xi^2}{2}\right) \\ &\quad + B_0 + B_1\xi \end{aligned}$$

with use of appropriate B_0 and B_1 which are independent of ξ. Thus Propositions 2.1 and 2.2 play an essential role in the proof of Theorems 2.3-2.6. We can prove Propositions 2.1 and 2.2 as an application of the maximum principle. For the detail of the above arguments, see [2].

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