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Backward Regularity for some Infinite Dimensional Hypoelliptic Semi-groups

Hélène Airault and Paul Malliavin

CONTENTS

We dedicate this work to Kiyosi Itô, the "Newton of continuous Stochastic Dynamic", one of the most influential scholars of the last century; the topic of this paper underlines in itsef the deep influence that the 1976 Kyoto Symposium [12] has had on the whole subsequent carrier of the second author who is also deeply indebted to Kiyosi Itô for fifty years of warm personnal relations; his attentive support from the beginning to some of our scientific enterprises has been a key step towards their international recognition.

In classical Stochastic Analysis regularity properties are time independent : the Brownian motion is for all time Hölderian of order $(\frac{1}{2} - \epsilon)$ regular, the tangent space to the Wiener space (i.e. the Cameron-Martin space) is also time independent. The Stochastic Analysis on Loop groups have recently confirmed the paradigm that regularity properties are time independent.

It has been a surprise that regularity exponents for highly non linear infinite dimensionnal diffusion as the canonic diffusion above Virasoro algebra are time dependent [2],[9]. We shall discuss in this paper the status of tangent space to Virasoro diffusion; we shall exhibit a minimal tangent space which is time independent; it is conceivable that the maximal tangent space is time dependent, fact which will be established on a toy model. The finite dimensional root of of this phenomen lies in the fact that hypoelliptic diffusion on R^d does not satisfy simple scaling relation when the time goes to zero [4], [11].

Stability of interest models in Mathematical Finance are deeply affected by these infinite dimensional effects.

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1. Regularity of the canonical diffusion above Virasoro algebra.

The group of C^∞ diffeomorphism of the circle S^1, Diff(S^1), has for Lie algebra diff(S^1) the C^∞ vector fields on S^1; we identify a function $u(\theta)$ to the vector field $u(\theta)\frac{d}{d\theta}$; with this identication the bracket of vector fields becomes $[u,v] = \dot{v}u - \dot{u}v$. Complexifying the underlying real vector space we get the following expression for this bracket in the complex trigonometric basis :

$$[e^{in\theta}, e^{im\theta}] = i(m-n)e^{i(m+n)\theta}$$

Given a positive constant $c > 0$, define the bilinear antisymmetric form

$$\omega_c(f,g) := -\frac{c}{12}\int_{S^1} (f' + f^{(3)})g \, d\theta;$$

then

$$\omega_c([f_1,f_2],f_3) + \omega_c([f_2,f_3],f_1) + \omega_c([f_3,f_1],f_2) = 0,$$

$$\omega_c(e^{in\theta}, e^{-im\theta}) = i\delta_n^m \frac{c}{6}(n^3 - n), \quad n > 0.$$

Virasoro algebra is defined as $\mathcal{V}_c := R \oplus \text{diff}(S^1)$ with the following bracket :

$$[\xi\kappa + f, \eta\kappa + g] := \omega_c(f,g)\kappa + [f,g].$$

Brownian motion on Diff $(\mathbf{S^1})$.

Define the Hilbertian metric $\frac{3}{2}$ by :

$$\|\phi\|^2_{\mathcal{H}^{\frac{3}{2}}} = \sum_{n>1}(n^3-n)\,(a_n^2+b_n^2), \quad \phi(\theta) = \frac{1}{2}a_0 + \sum_{n=1}^{+\infty}(a_n \cos n\theta + b_n \sin n\theta);$$

define

$$e_n : R^2 \mapsto \text{diff}(S^1), \quad e_n(\xi) = \frac{1}{\sqrt{n^3-n}}(\xi^1 \cos n\theta + \xi^2 \sin n\theta), \quad n > 1.$$

Let X_k be independent copies of Wiener space of the R^2-valued Brownian motion; define $X = \bigotimes X_k$ and consider the Stratonovich SDE :

$$dg_x(t) = \left(\sum_{k>1} e_k(dx_k(t))\right) o \; g_x(t), \quad g_x(0) = \text{Identity}$$

$$dg_x^r(t) = \left(\sum_{k>1} r^k \; e_k(dx_k(t))\right) o \; g_x^r(t), \quad g_x^r(0) = \text{Identity};$$

then, $g_x^r(t) \in$Diff (S^1) $\forall r < 1$.
Theorem.[2],[9].

Denote $\mathcal{H}^\beta(S^1)$ the group of homemorphism of S^1, with an Hölderian modulus of continuity β, then

$$\lim_{r\to 1} g_x^r(t) := g_x(t) \in \mathcal{H}^{\beta(t)}(S^1), \;\; a.s.,$$

$$\beta(t) = \frac{1-\sqrt{1-e^{-\frac{t}{2}}}}{1+\sqrt{1-e^{-\frac{t}{2}}}}.$$

The laws ν_t of $g_x(t)$ satisfy $\nu_t * \nu_{t'} = \nu_{t+t'}$.
Remark. The composition of two homemorphisms of Hölderian exponents γ, γ' can have an Hölderian exponent as worst as $\gamma\gamma'$: this fact explains the exponential decrease of $\beta(t)$ when $\to +\infty$.

It is obvious that the metric used to construct the Brownian motion degenerates on the vector fields $\cos\theta$, $\sin\theta$, 1. The Lie subagebra generated by these three vector fields is isomorphic to sl$(2, R)$; the corresponding subgroup Γ of Diff(S^1) is the restriction to the circle of the group of Möbius transformations of the unit disk.

It had be shown [1] that $\mathcal{M}_1$:=Diff$(S^1)/\Gamma$ is an homogeneous Riemannian manifold, that the Hilbert transform on the circle pass to the quotient and defines an integrable almost complex structure for which $\mathcal{M}_1$ becomes an homogeneous Kähler manifold. Denote π : Diff$(S^1) \to \mathcal{M}_1$, then $\pi(g_x^{-1}(t))$ is the Brownian motion on $\mathcal{M}_1$ and defines the heat semi-group on function on $\mathcal{M}_1$. This section will prove the backward regularity of this heat semi-group.

Background of finite dimensional Stochastic Riemannian Geometry.

Denote by M a Riemannian manifold of dimension d; a *frame* r is a Euclidean isomorphism of R^d onto the tangent plane $T_{\pi(r)}(M)$; the collection of all frames on M is a smooth manifold $O(M)$ on which the orthogonal group operates on the right : this is the *bundle of orthonormal frames.* The Levi-Civita connection defines on $O(M)$ a *parallelism* that is a canonical differential form of degree 1, with values in $R^d \oplus R^d \otimes_a R^d$ let $\omega = (\dot\omega, \ddot\omega)$. Riemannian geometry is encompassed in the Darboux-Cartan structural equations :

$$< A\wedge B \;,\; d\dot\omega >= \ddot\omega(A)\dot\omega(B) - \ddot\omega(B)\dot\omega(A),$$

$$< A \wedge B \; , \; d\ddot{\omega} >= \ddot{\omega}(A)\ddot{\omega}(B) - \ddot{\omega}(B)\ddot{\omega}(A) + \Omega(\dot{\omega}(A), \dot{\omega}(B)),$$

where Ω is the Riemann curvature tensor.

Given an R^d valued brownian motion $x(\tau)$ the *horizontal diffusion* is defined by the Stratonovitch SDE

$$< dr_x, \dot{\omega} >= dx, \qquad < dr_x, \ddot{\omega} >= 0, \;\; r_x(0) = r_0,$$

where $r_0 \in O(M)$ is fixed. The *Itô parallel transport* is the isometry

$t^x_{0\leftarrow\tau} : \; T_{\pi(r_x(\tau))}(M) \mapsto T_{\pi(r_0)}(M)$ defined by $t^x_{0\leftarrow\tau} = r_x(0) \; o \; (r_x(\tau))^{-1}$.

A variation induces $x \mapsto x + \epsilon\tilde{\zeta}$ induces a variation of the path (ζ, ρ) defined by

$$\zeta(\tau) :=< \frac{dr^\epsilon(\tau)}{d\epsilon_{=0}} \; , \; \dot{\omega} >, \;\; \rho(\tau) :=< \frac{dr^\epsilon(\tau)}{d\epsilon_{=0}} \; , \; \ddot{\omega} >, \;\; r^\epsilon(\tau) := r_{x+\epsilon}(\tau).$$

These two variations are linked by the two following key SDE [6], [10], [7], [14], the first being an Itô SDE, the second a Stratonovitch SDE :

$$\text{(1.1)} \qquad d\tilde{\zeta} = d\zeta - \frac{1}{2}\text{Ricci}(\zeta) \; d\tau - \rho \; dx, \quad d\rho = \Omega(\zeta, \; o \; dx).$$

Two parallel transports on $\mathcal{M}_1$.

We follow Bowick-Lahiri [5]. We have on $\mathcal{M}_1$ two connections : the Levi-Civita connection ∇_X and the connection $\mathcal{L}_X$ induced by the left invariant Maurer-Cartan form on Diff(S^1); we introduce a tensorial operator on $T_0(\mathcal{M}_1)$ defined by

$$\phi_X = \mathcal{L}_X - \nabla_X$$

The operator ϕ, extended to the complexification, has the following expression in the complex trigonometric basis :

$$\text{(1.2)} \qquad \phi_{e^{ir\theta}}(e^{iq\theta}) = i(r-q)\Theta(-q-r), \;\; r > 1,$$

where $\Theta(t) := 1_{[0,+\infty[}$ is the Heaviside function. For $s < -1$ we prolongate ϕ_* by requiring hermitian symmetry : $\phi_{e^{is\theta}} := (\phi_{e^{-is\theta}})^*$.

Then the Riemannian curvature of $\mathcal{M}_1$ can be expressed in terms of the operator ϕ_* by

$$\Omega(X,Y) = [\nabla_X, \nabla_Y] - \nabla_{[X,Y]} = [\phi_X, \phi_Y] - \phi_{[X,Y]},$$

the last identity results from $[\mathcal{L}_X, \mathcal{L}_Y] - \mathcal{L}_{[X,Y]} = 0$ together with $[\mathcal{L}_X, \nabla_Y] = \nabla_{[X,Y]}$ identity coming from the invariance of the Kählerian metric under the left action of Diff (S^1). The curvature tensor is of trace class [5], and its trace is

$$\text{Ricci} = -\frac{13}{6} \times \text{Identity} \tag{1.3}$$

Lemma.

Denote V_q the space generated by $\cos k\theta$, $\sin k\theta$, $k \in [2,q]$ *then the operators ϕ_* preserve V_q and are nilpotent on V_q.*

Denote $\eta_n(\xi) = \tilde{\phi}_{e_n(\xi)}$, $\xi \in R^2$, where $\tilde{\phi}_X$ is the matrix associated to ϕ_X in the real trignometric basis $(n^3-n)^{-\frac{1}{2}} \cos n\theta$, $(n^3-n)^{-\frac{1}{2}} \sin n\theta$.

Theorem

The matrix Stratonovich SDE

$$d\mathcal{U}_t = \mathcal{U}_t \; o \left(- \sum_{k>1} \eta_k(dx_k(t)) \right), \quad \mathcal{U}_0 = \text{Identity} \tag{1.4}$$

has a unique solution and $\mathcal{U}_t$ is a unitary matrix.

Proof.

The restriction to V_q of this SDE is equivalent to an SDE which is driven only by $2q$ Brownian motion; this SDE which is solvable by the finite dimensional theory •

Backward regularity. (Minimal tangent space)

Theorem.

Given z such that $\|z\|_{H^{\frac{3}{2}}} < \infty$ then, for a generic test function Φ defined on $\mathcal{M}_1$,

$$\begin{aligned} &\left| \frac{d}{d\epsilon_{=0}} E((\pi^*\Phi)(\exp(\epsilon z) g_x(t))) \right|^2 \\ &\leq \frac{13}{6(1-\exp(-\frac{13}{6}t))} \|z\|^2_{H^{\frac{3}{2}}} \; E(|\pi^*\Phi(g_x(t))|^2). \end{aligned} \tag{1.5}$$

Proof.

We follow the strategy that Driver [8] developped in the case of Loop groups making the change of variables

$$y_t = \int_0^t \mathcal{U}_s \; dx(s);$$

then y_t is a new brownian motion to which we can apply the finite dimensional Riemannian geometry because the curvature operator preserves the V_q •

2. Infinite dimensional non autonomous Riemannian metrics.

Consider a group G of dimension finite or infinite; for instance G could be the group of diffeomorphism of a compact manifold, case which includes the theory of Stochastic Flows.

We consider a left invariant diffusion on G; denote by $\Delta = \frac{1}{2}\sum_{k\geq 1}\partial^2_{A_k} + \partial_{A_0}$ its infinitesimal operator where the A_k are left invariant vector field on G; denote by ∇ the corresponding gradient : $\nabla\phi * \nabla\psi := \Delta(\phi\psi) - \phi\Delta\psi - \psi\Delta\phi$.

We denote by $p_T(dg)$ the law of the process starting from the identity. Given a tangent vector at the identity z define the "logarithmic derivative" of p_T by the identity

$$\frac{d}{d\epsilon_{=0}} E(\Phi(\exp(\epsilon z)g_x(T)) = E(K_{z,T}(g_x(T))\Phi(g_x(T))), \tag{2.1}$$

where Φ is a generic test function.

For all $T > 0$ define a Hilbertian norm by

$$\|z\|_T^2 := E(|K_{z,T}(g_x(T))|^2)). \tag{2.2}$$

Theorem.

If $T < T'$ then

$$\|z\|_{T'} \leq \|z\|_T. \tag{2.3}$$

Proof.

For $\eta > 0$ define $\Psi(g) := E_{g_x(T)=g}(\Phi(g_x(T+\eta))$, then

$$\begin{aligned} E(\Phi(\exp(\epsilon z)g_x(T+\eta)) &= E(E^{\mathcal{N}_T}(\Phi(\exp(\epsilon z)g_x(T+\eta)) \\ &= E(\Psi(\exp(\epsilon z)g_x(T))); \end{aligned}$$

differentiating relatively to ϵ we obtain

$$E(K_{z,T+\eta}(g_x(T+\eta))\ \Phi(g_x(T+\eta))) = E(K_{z,T}(g_x(T))\Psi(g_x(T))),$$

letting $\eta \to 0$ we write $\simeq$ equalities modulo $o(\epsilon)$; then by Itô calculus :

$$K_{z,T+\eta}(g_x(T+\eta) - K_{z,T}(g_x(T) \simeq \eta(\frac{\partial K}{\partial T} + \Delta K) + \nabla K * (x(T+\eta) - x(T))$$

$$\Psi(g) - \Phi(g) \simeq \eta\Delta\Phi(g),$$
$$\Phi(g_x(T+\eta)) \simeq \Phi(g_x(T)) + \eta(\Delta\Phi(g_x(T))) + \nabla\Phi * (x(T+\eta) - x(T))$$

$$\frac{1}{\eta}E^{\mathcal{N}_T}(K_{z,T+\eta}(g_x(T+\eta))\ \Phi(g_x(T+\eta))) - (K_{z,T}(g_x(T))\Phi(g_x(T)))$$

$$\simeq \Phi(\frac{\partial K}{\partial T} + \Delta K) + K\Delta\Phi + \nabla\Phi * \nabla K;$$

$$\frac{1}{\eta}E\Big(K_{z,T+\eta}(g_x(T+\eta))\ \Phi(g_x(T+\eta))) - (K_{z,T}(g_x(T))\Phi(g_x(T))\Big)$$

$$\simeq \Phi(\frac{\partial K}{\partial T} + \Delta K) + K\Delta\Phi + \nabla\Phi * \nabla K) - K\Delta\Phi$$

$$E\Big(\Phi(\frac{\partial K}{\partial T} + \Delta(K)) + \nabla K * \nabla\Phi\Big) = 0. \tag{2.4}$$

From the other hand

$$\frac{\partial}{\partial T}E[(K_T(g))^2)] = E[\Delta(K_T^2) + \frac{\partial K_T^2}{\partial T}]$$

$$= E[2K_T(\frac{\partial K}{\partial T} + \Delta(K_T)) + \nabla K_T * \nabla K_T)] = -E[\nabla K_T * \nabla K_T] < 0,$$

the last equality is obtained by applying (2.4) with $\Phi = K_T$ •

Consider now the free Lie algebra $\mathcal{G}$ generated by d vector fields $A_1, \dots A_d$; denote G the infinite dimensional group associated. Denote x a d-dimensional Brownian motion and define on G the process by the following Stratanovitch SDE

$$dg_x(t) = g_x(t)\ o \sum_{k=1}^{d} A_k\ dx^k(t), \quad g_x(0) = \text{Identity}$$

denote $\mathcal{H}_T$ the completion of $\mathcal{G}$ for the norm $\|z\|_T$.
Theorem. For $T \neq T'$, we have
(2.5)
$\mathcal{H}_T \neq \mathcal{H}_{T'}$, *which means the inequivalence of the corresponding norms.*

Proof.
We shall use the Ben-Arous expansion [3] (see Theorem 15)

$$g_x(t) = \exp\Big(\sum_{m=1}^{\infty}\sum_{J\in\sigma_m} M_J(t)U^J\Big)$$

where $A^J := [A_{j_1}, [A_{j_2}, \dots, [A_{j_{n-1}}, A_{j_n}]$, where σ_m denotes a maximal subset of $[1, d]^m$ such that the A^J are linearly independent in $\mathcal{G}$ and where iterated integrals M_J have been constructed by Meyer and are mutually orthogonal in L^2. We decompose

$$z = \sum_{m=1}^{\infty} z_m, \quad z_m = \sum_{J \in \sigma_m} c_J A^J.$$

Lemma.

$$\|z\|_T^2 = \sum_{m=1}^{\infty} \|z_m\|_T^2 \tag{2.6}$$

By the rescaling of Meyer integrals we have

$$\|z_m\|_{T'}^2 = \left[\frac{T}{T'}\right]^m \|z_m\|_T^2$$

relation which shows the inequivalence of the two norms •

3. Instability of Heath-Jarrow-Merton model of interest rates.

All long terms loans (States bounds, mortgages, companies bounds) are appearing on a single market, the " zero coupon default free bonds market". Every day it is possible to buy bonds at any maturity between 1 up to 360 months; for each maturity the market gives a price; all these prices can be summarized by a single positive function $r_t(x)$ *the instantaneous forward rate* such that the discount price today of a 1 dollar bound paid in five years is equal to

$$\exp(-\int_0^{60} r_t(x)\, dx).$$

The associated configuration space $\mathcal{C}$ is $(R^+)^{360}$.

The HJM model replace the $\mathcal{C}$ by the space of continuous positive functions $r_t(x), \ x \in [0, 360]$ and propose that "for the risk free measure" the interest rate curve dynamic can be described by the following Itô SDE, driven by q independent Brownian motion $W^*(t)$,

$$dr_t(x) = \left(\frac{\partial r_t(x)}{\partial x} + Z_t(x)\right) dt + \sum_{k=1}^{q} \phi_{k,t}(x)\, dW^k(t),$$

$$Z_t(x) = \sum_{k=1}^{q} \phi_{k,t}(x) \int_0^x \phi_{k,t}(s)\, ds. \tag{3.1}$$

This HJM modell can be mathematically established under the two general assumptions : market where an agent cannot increase his wealth without risk (*arbitrage free)* and market variations free from jumps.

A practical fact is that the variance injected in the equation is very low : $q \leq 4$. This means that the operator associated with the SDE (3.1) is an *hypoellitic operator driven by at most four vectors fields in a Euclidean space of large dimension.*

Consider the Stochastic flow $U^{W}_{t\leftarrow t_0}$ defined as $U^{W}_{t\leftarrow t_0}(r_0)$ being the solution of (3.1) for $r_W(t_0) = r_0$. Denote by $J^{W}_{t\leftarrow t_0}$ the Jacobian of the flow $U^{W}_{t\leftarrow t_0}$ which is defined by solving the linearized SDE.

Greeks means the reaction of the market at an infinitesimal pertubation δ_0 of r_0 appearing at time t_0, $W^*(s) - W^*(t_0)$, $s \geq t_0$ being fixed :

$$\frac{d}{d\epsilon_{\epsilon=0}} U^{W}_{t\leftarrow t_0}(r_0 + \epsilon\delta_0) = J^{W}_{t\leftarrow t_0}(\delta_0) := \delta^{W}(t),$$

is called the *Greek propagation.*

Every trader can buy or sell *european options* which is a contract by which the seller obliges himself to pay at maturity T an amount of money equal to $F(r_T)$. The option is called *digital* if the function F is discontinuous.

Sensitivities at the option F is defined

$$\frac{d}{d\epsilon_{\epsilon=0}} E(F(U^{W}_{T\leftarrow t_0}(r_0 + \epsilon\delta_0)) = E(< dF \ , \ J^{W}_{T\leftarrow t_0}(\delta_0) >).$$

Sensitivities regularization for digital european options

Denote $\mathcal{C}$ the vector space of all possible infinitesimal pertubation δ_0 of the market at time t_0; consider the Hilbertian norm $\|\delta\|_{T,t_0}$ defined in (2.2) and denote $\mathcal{C}_{t_0,T}$ the corresponding Hilbert space then

$$|\frac{d}{d\epsilon_{\epsilon=0}} E(F(U^{W}_{T\leftarrow t_0}(r_0 + \epsilon\delta))| \leq \|\delta\|_{T,t_0}(E(|F(r_W(T)|^2))^{\frac{1}{2}}$$

Compartimentage Principle.

"Generically" the sequence of Hilbert spaces $\mathcal{C}_{T,t_0}$ is strictly increasing relatively the parameter T and strictly decreasing relatively to the parameter t_0.

Hedging

The Clark-Ocone-Karatzas formula

(3.2)

$$F(r_W(T)) - E(F(r_W(t_0))) = \sum_{k=1}^{q} \int_{t_0}^{T} E^{\mathcal{F}_s}(D_{s,k}(F(r_W(T))) \, dW^k(s)$$

gives a realization of the option along each trajectory. The corresponding strategy of *replication,* consist for the trader to balance at each time t his portfolio according the infinitesimal observed variation of the driving Brownian $W^k(t+\epsilon) - W^k(t)$, multiply by $E^{\mathcal{F}_s}(D_{s,k}(F(r_W(T)))$.

The formula (3.2) is a specialization of the general Itô theorem saying that any random variable of zero expectation is representable by a Stochastic integral; at this level of generality the integrand is only in $L^2([t_0, T])$ on each trajectory. As the *financial replication* of the option is given by this integrand, it is impossible to realize this replication if this integrand is not at least continuous; otherwise instabilities appear.

3.3. Theorem [13].
Denote Θ the stopping time such that

$$J^W_{t_0 \leftarrow t}(\Phi_k(r_W(t))) \in \mathcal{C}_{T,t} \quad \forall\, t \leq \Theta, \quad \forall k \in [1, q];$$

then $E^{\mathcal{F}_\Theta}(F(r_W(T)))$ is replicable by a stable Clark-Ocone-Karatzas formula.

Proof.

$$E^{\mathcal{F}_s}(D_{s,k}(F(r_W(T))) = E(< dF\ ,\ J^W_{T \leftarrow s}(\Phi_k(r_W(s)) >) \qquad \bullet$$

Consequence : Traders must try to sale digital options before the stopping time Θ.

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H.A.
INSSET, Université de Picardie, 48 rue Raspail, 02100 Saint-Quentin, France;
hairault@insset.u-picardie.fr

P.M.
10 rue Saint Louis en l'Isle, 75004 Paris, France;
sli@ccr.jussieu.fr