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Memories of Professor Teiji Takagi

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I am particularly sensible of the honor of being given this chance of speaking today at this international meeting on the memories of Professor Takagi, Japanese mathematician who founded the class field theory, as well known to all of you. Personally, he was an unforgettable teacher for me to whom I owe, so to speak, my whole existence as a mathematician. Not only myself, the Japanese community of mathematicians as a whole, owes much to him, I believe. May I remind you that he was born in 1875 and deceased in 1960? I was born in 1906 so that I have a privilege of having lived 54 years on this earth with him, but more precisely I met him for the first time at a class-room at the University of Tokyo when I had 20 years of age. Thus I could share with him more than 30 years in mutual acquaintance, which have left me numerous personal memories, on which I shall touch in the present talk, together with the role of Takagi in the development of mathematics in Japan, a more important subject. But the main theme of my talk of today should be, I believe, to trace how he came to found the class field theory. With this in mind, I have made the following short chronological table of his life.
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This table contains only five titles of Takagi's work: three in German, one in French and one in Japanese, the last one is of his talk on Reminiscences and Perspectives, particularly important for the history of the class field theory. Takagi published many other papers and books, many of which only in Japanese, which contributed greatly to the progress of Japanese mathematics. His Collected Papers, gathering all his papers published in European languages, ran into the second edition; the first edition being sold out, the second augmented edition is now on sale at bookstores. It has an appendix containing Iwasawa's "On papers on Takagi in number theory" and if you permit me to mention, also my

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Year	Age	Event
1875	0	Born on April 21 in Gifu prefecture
1891-94	16-19	Studied in the Third High School in Kyoto
1894-98	19-23	Studied in Todai (the University of Tokyo) with Dairoku Kikuchih, Rikitaro Fujisawa
1898-1900	23-25	Studied in Berlin University with Frobenius, H.A.Schwarz
1900-01	25-26	Studied in Göttingen University with Klein, Hilbert
1901	26	Returned to Todai (associate professor)
1903	28	"Über die im Bereiche der rationalen complexen Zahlen Abel'schen Zahlkörper" Dr.Sc. (Todai)
1904-36	29-61	Professor at Todai
1914-18	39-43	World War I
1920	45	"Über eine Theorie des relativ Abel'schen Zahlkörpers" ICM in Strassburg "Sur quelques théorèmes généraux de la théorie des nombres algébriques"
1922	47	"Über das Reciprocitätsgesetz in einem beliebigen algebraischen Zahlkörper"
1925	50	Member of Japan Academy
1932	57	ICM in Zürich
1939-45	64-70	World War II (Japan involved in 1941-45)
1940	65	Awarded a Cultural Medal "Kaiko to Tembo" (Reminiscences and Perspectives)
1955	80	International Symposium on Algebraic Number Theory, Tokyo-Nikko
1960	85	Deceased on February 28 in the Hospital of Todai

article "On the life and works of Teiji Takagi." I show you two portraits of Takagi, taken from both editions of his Collected Papers. This one from the first edition dates from the 1920's. When I met him for the first time, he looked like this. The other one dates from the time after the World War II. Both are accompanied with his signatures in Chinese

The photos have been removed due to copyright issues.

and Latin characters, which are the same, taken from the first edition. You see that Takagi had a very beautiful handwriting.

Now let me give some comments on the table I have just shown you. The year of birth of Takagi, 1875 A.D., is called in Japan "Meiji 8 nen" or the 8th year of Meiji, i.e. the 8th year since the coronation of Emperor Meiji, 1868, around which time the modernization was started in Japan. The modern Japan was thus still in a very young age. Gifu prefecture is in a mountainous region in the central Japan. He was born in an agricultural area of this prefecture, finished the primary school at his native village, the secondary school at Gifu, chief town of the prefecture, the High School, i.e. the senior secondary school at Kyoto and then he came to Tokyo to study in the unique university in Japan at that time. It was called just the "Imperial University", but renamed the "Tokyo Imperial University" when another Imperial University was created in Kyoto in 1897, renamed again "the Tokyo University" dropping the word "Imperial" after World war II. In Japanese it is called "Tokyo Daigaku" or "Todai" in abbreviation.

Professors Dairoku Kikuchi and Rikitaro Fujisawa, representing the first and the second generations of mathematicians in modern Japan, taught then at Todai. Kikuchi had been educated in England from his very early age, and returning to Japan just after graduation from Cambridge, was named immediately professor at Todai. Later he was named president of Todai and other universities, minister of education and ennobled baron. Fujisawa studied in British and German universities and obtained his Ph.D. at German university of Strassburg. He transferred to Japan the atmosphere of research in German universities, but in later years, he became a senator and more interested in administrative affairs. We had to wait until the third generation to find a Japanese mathematician who created such a work as the class field theory, which attracted the attention of the world's mathematical community.

Fujisawa had studied in Berlin and received influence of Kronecker; thus he knew the importance of algebra. Takagi joined his seminar and studied about abelian equations by the French text-book of Serret. The first 2 volumes of German text-book of H. Weber arrived at Tokyo toward that time, which Takagi read with great interest. He learned also in Todai the theory of elliptic functions in the style of Jacobi. After graduation, he continued his study for one year in the graduate school, then he was sent to Germany by the government. He went first to Berlin,

but Kronecker, Weierstrass and Kummer were already dead and Schwarz was old. He found interest in the lively lecture of young Frobenius. Then he went to Göttingen, where Klein and Hilbert were at the top of their activities. In attending the lecture of Klein and the weekly meeting of mathematicians from all parts of Europe, Takagi realized that the mathematics he had learned in Tokyo was about half a century behind the mathematics which was studied by contemporary researchers. He had then certainly studied for himself the Zahlbericht of Hilbert. Apparently, he did not attend the ICM at Paris in 1900, but I am sure that he was interested in the 23 mathematical problems presented there by Hilbert, particularly in those inconnection with Kronecker's Jugendraum (dream in his younger day.) He chose as the subject of his research, a special case of this problem when the ground field is the Gaussian field $\mathbf{Q}(\sqrt{-1})$. When he told this to Hilbert, he was encouraged by Hilbert's response "That's a fine subject!" as he related us later. He succeeded to bring this research to a happy end, and presented it to Todai as his doctoral thesis: "On the abelian field on the field of rational complex numbers" which was accepted of course. In its preface, we see a word of acknowledgement by Takagi to Hilbert.

Takagi was nominated to an associate professorship of Todai while he was in Göttingen, then to a professorship two years later in 1904, in which position he remained 32 years until 1936, the year in which he attained the age limit of 60 years fixed at Todai. The World War I took place during these years which interrupted the scientific correspondence between Europe and Japan. It is during this period that Takagi worked on his research in the class field theory. In 1920, two years after the end of War, he could complete his results in his important paper: "On the theory of relatively abelian number fields" whose contents are just Takagi's class field theory. The first ICM after the War was organized at Strassburg in the same year. Takagi attended it and gave a report in French "On some general theorems in the theory of algebraic numbers" which appeared in the Proceedings of this Congress as an article of only 4 pages, containing, however, in addition to all the essential results of his theory, a presentation of an important problem to extend this theory to the general galois, not necessarily abelian, case; which is still unsolved today. Unfortunately, the reconciliation between France and Germany was not yet perfect at that time, and the German mathematicians were not invited to this Congress. As the algebraic number theory was not

much studied then outside Germany, the report of Takagi at Strassburg Congress did not have an immediate response. I suspect, however, that the problem posed by Takagi at that occasion attracted the attention of Artin, who had an idea to introduce a new kind of L-series for galois extensions generalizing Hecke's L-series for abelian extensions. Two years later in 1922, Takagi published his paper "On the law of reciprocity for arbitrary algebraic number fields" based on the results of his theory. Suggested by the form of the reciprocity law given in this paper, Artin has found the form of the general reciprocity law which retains his name, from which the classical reciprocity laws for power residues would follow as simple corollaries and the new kind of L-series for galois extensions would be expressible by Hecke's L-series. Thus Artin published in 1924 his idea of new L-series together with the formulation of his general reciprocity law, proposed then as a conjecture. As well-known this law makes the isomorphism theorem of Takagi's class field theory more precise by means of the "Frobenius automorphism", concerning which Frobenius had given a conjecture in 1896. This conjecture was proved by Tschebotareff in 1926 using the so-called "crossing method with cyclotomic extensions" as was remarked by Schreier in the same year and which enabled Artin to prove his general reciprocity law by the same method in 1927. As soon as his proof appeared in Hamburger Abhandlungen, Takagi made a review article in the newly published journal (in Japanese) of Physico-Mathematical Society of Japan in which he expressed his admiration to Artin's paper as one of the most prominent results in recent times in the algebraic number theory. This was, so to speak, the corner stone of Takagi-Artin's class field theory.

On the other hand, the German mathematical Society asked Hasse to give a report on the class field theory in its annual meeting in 1925, which was published in 1926 in a written form, followed by a more detailed description with proofs published in the next year. This "Klassenkörperbericht" of Hasse contributed to the greater diffusion of this theory in world's mathematical community. (After the discovery by Artin of the general reciprocity law, the second part of this report appeared in 1930.)

May I speak now a little of myself ? I entered Todai as a student in 1926, just the year in which the first part of Hasse's report arrived in Japan. I knew of course nothing of the class field theory when I entered Todai, but I was immediately fascinated by Takagi's lecture and

his personality, and after following his lessons for two years, I began to understand what was said in Hasse's report. In the third and last student year, each student had to join a seminar of a professor, in which he had to begin his research. I requested then Professor Takagi to join his seminar and was delighted in learning that I was admitted by him. Thus I began my research work in 1929, the year of my graduation. I remained in the graduate school for one and half more years after which I went over to Hamburg to study with Artin, where I had a good luck of making acquaintance with Chevalley.

I stayed in Europe in 1931-34, during which period an ICM was organized in Zürich in 1932. Takagi was invited there as one of the vice-presidents (together with mathematicians like Hilbert and Hadamard. The president was, by the way, R. Fueter, veteran in the field of complex multiplications.) I participated in this Congress together with Chevalley who made acquaintance with Takagi at this occasion and handed over a copy of his thesis to Takagi after a suggestion of Artin. His thesis, in which he tried to arithmetize the proof of the class field theory (without, however, attaining then a complete success which he achieved later in 1940) appeared thus in 1933 in the Journal of the Faculty of Science of Todai, the same journal in which Takagi's main work had appeared in 1920.

In 1935, I was named associate professor at Todai and charged with direction of exercise of Takagi's lecture for the first year students on infinitesimal calculus. I was lucky, to make acquaintance with excellent students in this class: Kunihiko Kodaira, Yukiyosi Kawada, Kiyosi Ito ... who became later well-known mathematicians. They followed Takagi's lecture only for one year, but I am sure that they received his deep influence. His lecture was, by the way, published as a well-known book (in Japanese) "Kaiseki Gairon" (Course of Analysis) and served greatly for raising upward the general level of Japanese mathematics.

In Germany, the nazis took the power in 1933, and the exodus of European intellectuals to America began toward that time. Still earlier, the Japanese army had started its invasive maneuvers in China. The world in the 1930 was thus rather in a tumultuous situation. The World War II began in Europe in 1939, in which Japan engaged formally in December 1941. In 1940, Japan was still before the formal engagement in the War, and celebrated the so-called 2600th year of its foundation after traditional history. In that year, the government of Japan decerned

Takagi with a Cultural Medal, as its highest recognition for cultural achievement, as it noticed the respect and honor paid to him by the international scientific community.

I am sure that Takagi was well aware of the importance of what he had done, but we have never heard him speaking boastfully of his proper work. However, when we requested him just after he received the Cultural Medal, to talk on the memories of his research and of his views on the future of mathematical studies in Japan, he was willing to accept our proposal and gave a very interesting and important lecture: "Reminiscences and Perspectives (in Japanese: Kaiko to Tembo)" at the Department of Mathematics at Todai in December 1940. Fortunately, the content of this lecture was noted and published in 1941 and we can read it now in an appendix to his another masterpiece "Topics from the history of mathematics in the 19th century (in Japanese: Kinsei Sugaku Shidan)" in the Iwanami Collection of Classical Books together wirh comments by Mitsuo Sugiura. I have given an English translation of some passages of this lecture in my article "On the life and works of Teiji Takagi" in the second edition of his Collected Papers, but otherwise there has been only text in Japanese of this lecture until we could find quite recently a French text in an article entitled "Takagi Teiji et la découverte de la théorie du corps de classes" by Dr. Pierre Kaplan, Professor at the Faculty of Science of the University of Nancy, versed also in japanese language and staying in Japan, more precisely in Ebisu, Tokyo, since two years as French director of the Maison franco-japonaise, in the review "Ebisu" no.16 published by this Maison. To those among you who are more familiar with French than with Japanese language, I would like to warmly recommend to look at that article. I would like also to read some passages of that lecture by Takagi in my English translation later in this talk.

In 1945 toward the end of World War II, the house close to Todai, where Takagi's family had lived since 1902, was burnt down by American bombardment. Takagi was forced to retire to his native village in Gifu prefecture, but he came back to Tokyo after 2 years, and lived together with the family of his eldest son. In 1952, he had to endure the sad event of the death of his wife, who had given him a prosperous family. I believe his second portrait dates from that period. In 1955, ten years after the end of the War, we could organize an International Symposium on Algebraic Number Theory, Tokyo-Nikko, where we could invite ten

mathematicians from outside Japan, including Artin, Chevalley, and Weil. Takagi attended this Symposium as honorary chairman. He did not give any official talk, but we were happy to see him enjoy private conversations with these mathematicians.

Five years later in 1960, we had to see him off from this earth at the age of 85 years. A few days after he had been hospitalized at the hospital of Todai stricken by a cerebral apoplexy, he went calmly away. His funeral service was very beautifully served without religion.

Now I would like to read some passages of his lecture "Reminiscences and perspectives" in my English translation as far as the time permits.

First from his reminiscences of his studies in Götingen around 1900: "At the time when I studied in Germany, Göttingen was perhaps the only place in the world where research on algebraic number theory was going on. Thus, when I told Hilbert that I wanted to study this theory, Hilbert did not seem to believe me immediately... He invited me one day to follow him on his way home. During our talk, I told him that I was studying the special case of "Kroneckers Jugendtraum" where the ground field is the Gaussian field, i.e. I was dealing with complex multiplication of the lemniscate function. He said to me: 'Oh! that's fine,' and stopped at the corner of the street crossing with Wilhelm-Weber Strasse where he drew on the earth two figures, one of a square and another of a circle, figures related to lemniscate function, which we find in the work of Schwarz, saying 'you certainly know this as you have studied with Schwarz.' I remember that place even now."

Next, from the reminiscences of the time of World War I: "I am of a nature which needs a stimulus in order to work. There are now quite a number of Japanese mathematicians, but in these days, we had few colleagues. Neither had I heavy duties. You might imagine that I did research on class field theory in those carefree days but it was not quite so.

The World War I started in 1914. This gave me a stimulus, so to say a negative stimulus. No scientific message reached us from Europe for four years. Some said that this would mean the end of Japanese science. Some newspaper articles showed 'sympathy' with Japanese professors from losing their 'jobs.' This made me aware of the obvious truth that every researcher should make research for himself, independently of others. Possibly I would have done no research for myself, but for World War I."

Lastly, on the class field theory: "Concerning the class field theory, I should confess that I had been misled by Hilbert. Hilbert considered only unramified class fields. From the standpoint of the theory of algebraic functions which are defined by Riemann surfaces, it is natural to limit considerations to unramified cases. I do not know precisely whether Hilbert himself stuck to this constraint, but anyway, what he had written induced me to think so. However after the cessation of scientific exchange between Japan and Europe owing to World War I, I was freed from that idea and suspected that every abelian extension might be a class field, if the latter is not limited to the unramified case. I thought at first this could not be true. Were this to be false, the idea should contain some error. I tried my best to find this error. At that period, I almost suffered from a nervous breakdown. I dreamt often that I had resolved the question. I woke up and tried to recover the reasoning, but in vain. I made my utmost effort to find a counterexample to the conjecture which seemed all too perfect. Finally, I made up my theory confirming this conjecture, but I could never get rid of the doubt ..."

The talk of Professor Takagi continues still further, but I am afraid my time is running out. I have now to stop here my talk. Thank you very much for your kind attention!

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