A Table for Pure Cubic Fields

Ken Nakamula

Let K be a cubic number field with discriminant $D < 0$. In the previous paper [3], we have utilized an elliptic unit for the simultaneous determination of a fundamental unit e and the class number h of K. The significance of the method is that no calculation in K is necessary; exactly, one can determine e and h by some arithmetic of an imaginary quadratic field $Q(\sqrt{D})$ and by approximate evaluation of the Dedekind eta function. As a process of the algorithm is fully explained in [3] with numerical examples, we do not repeat it here.

For pure cubic fields, our method has been implemented as a FORTRAN program on HITAC M-280H, Computer Center, University of Tokyo. It is arranged to apply to any pure cubic field K with $300 \le |D| \le 4800000000$ provided that C.P.U. time is not restricted. As a preparation, a primitive ring root modulo p defined in [3] was computed for any prime number $p < 40000$ beforehand, which took 327 seconds of total C.P.U. time. Further, the constants π and $\pi\sqrt{-3}$ were computed to 40000 decimal digits precision in advance, which took 1766 seconds of C. P. U. time.

By means of the program, we have actually computed e and h for 358 pure cubic fields with $300 \le |D| \le 1153200$, which took 25758 seconds of total C.P.U. time, though 1350 seconds of C.P.U. time was required for 157 fields with $300 \le |D| \le 270000$. It should be mentioned that there are several numerical results for the class number h as in [2] and others. But, on the other hand, there exist quite a few numerical results for the fundamental unit e, not the regulator, only as in [1] and [5]. Therefore, it will be useful to present precise e in a table. By calling the program deposited in Computer Center, University of Tokyo, anyone can get extended data.

Now, let us describe the table below which has appeared in [4]. Every pure cubic field K is represented by a unique pair (a, b) of coprime square free natural numbers a, b, $a > b$, such that the defining equation of K is given by $X^3 - ab^2 = 0$. Then $D = -3f^2$, where $f = ab$ or $3ab$ respectively when $a^2 - b^2$ is or is not divisible by 9. Starting from $f = 10$, namely excluding the cases $(a, b) = (2, 1)$, $(3, 1)$, the table is given in increasing order

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of f. More than one pair (a, b) may ordinary belong to one f, therefore the heading of a field is denoted by $f(a, b)$. Together with h, the integer part B of its upper bound in [3] is also listed. We represent e by its minimal polynomial over $\mathbf{Q}$, or equivalently by its absolute traces $\mathrm{Tr}\,(e)$ and $\mathrm{Tr}\,(1/e)$, which frequently exceed the size of a line (each with 40 decimal digits divided into four 10 decimal digits blocks) and are listed in a series of lines, denoting by the symbol $|_-$ that the succeeded number may continue from the former line.

References

[1] Angell, I. O., A table of complex cubic fields, Bull. London Math. Soc., **5** (1973), 37–38.
[2] Barrucand, P., Williams, H. C. and Baniuk, L, A computational technique for determining the class number of a pure cubic field, Math. Comput., **30** (1976), 312–323.
[3] Nakamula, K., Class number calculation of a cubic field from the elliptic unit, J. Reine Angew. Math., **331** (1982), 114–123.
[4] ——, Elliptic units and the class numbers, Dissertation, Tokyo Metropolitan University, 1985.
[5] Wada, H., A table of fundamental units of purely cubic fields, Proc. Japan Acad., **46A** (1970), 1135–1140.

Department of Mathematics
Tokyo Metropolitan University
2–1–1 Fukazawa, Setagaya-ku
Tokyo 158, Japan

f(	a,	b)	h	B	Tr(e)	Tr(1/e)
10(	10,	1)	1	2	23	-7
14(	7,	2)	3	3	5	-1
15(	5,	1)	1	2	123	3
17(	17,	1)	1	3	972	54
18(	6,	1)	1	3	327	3
18(	3,	2)	1	2	165	3
19(	19,	1)	3	4	14	2
21(	7,	1)	3	3	12	6
22(	11,	2)	1	4	4007	113
26(	26,	1)	3	4	27	9
30(	5,	2)	3	4	33	3
33(	11,	1)	2	5	267	3
35(	35,	1)	3	7	278	-22
37(	37,	1)	3	7	300	30
39(	13,	1)	3	7	282	-12
42(	14,	1)	3	5	87	3
45(	15,	1)	2	8	16203	3
45(	5,	3)	1	6	4432323	3243
46(	46,	1)	1	7	49347147	-12417
53(	53,	1)	1	10	33 9046360794	-1033020
55(	55,	1)	1	12	9884 2021680603	19792083
58(	29,	2)	1	9	2 4942723393	225507
62(	62,	1)	3	11	26787	3
63(	7,	3)	3	8	2280	48
63(	21,	1)	3	9	5115	-141
65(	13,	5)	3	11	34583	263
66(	22,	1)	3	8	2379	69
69(	23,	1)	1	8	6500020803	-124197
70(	14,	5)	3	12	66153	3

f(a, b)	h	B	Tr(e)	Tr(1/e)
71(71, 1)	1	15	536506681 9658449446	4158678672
73(73, 1)	3	13	299784	462
77(11, 7)	3	14	968652	1818
78(13, 2)	3	7	627	3
82(82, 1)	1	10	764 496692903	4290653
86(43, 2)	3	11	58655	− 427
87(29, 1)	1	14	30859962 5272805067	− 967384317
89(89, 1)	2	14	1124105870	62570
90(30, 1)	3	8	2433	3
90(6, 5)	3	7	813	3
90(10, 3)	3	12	174963	3
90(15, 2)	3	9	6483	3
91(91, 1)	9	17	243	27
93(31, 1)	3	13	303627	− 1101
94(47, 2)	1	16	12 4350210671 6566153875	− 6 5549344767
99(33, 1)	1	10	4581 2022222387	11226603
99(11, 3)	1	17	71 1151496843 0361185667	− 15 2124172197
102(34, 1)	3	14	1002459	1839
102(17, 2)	3	9	7347	3
105(7, 5)	3	12	226803	843
107(107, 1)	1	17	1132 4778785520 7200198554	63 8744633822
109(109, 1)	3	19	307464555	32835
110(22, 5)	3	14	1510853	− 1537
114(19, 2)	6	13	915	3
114(38, 1)	3	11	87213	− 453
115(23, 5)	1	23	36 1910325442 6950624525 3793113963	221631 9490328403
117(13, 3)	3	12	386382	1236
117(39, 1)	6	14	1587	− 69
118(118, 1)	2	22	69869 0155699793	46647527

$f(\ a,\ b)$	h	B	$\mathrm{Tr}(e)$	$\mathrm{Tr}(1/e)$
122(61, 2)	3	20	1552657419	77265
123(41, 1)	1	18	27935 9128734569 2762343187	− 63 5974112517
126(7, 6)	6	13	1137	3
126(42, 1)	3	10	63507	3
126(21, 2)	3	12	498963	1137
126(14, 3)	9	12	75	− 15
127(127, 1)	3	26	65 2228204107	− 1233357
129(43, 1)	12	19	147	− 21
130(65, 2)	3	22	1 4346816123	231663
134(134, 1)	3	23	6 2652683111	− 270703
138(23, 2)	3	18	321468657	− 24837
141(47, 1)	2	17	168 8832878307	− 1776597
143(143, 1)	3	21	7305401975	170855
145(145, 1)	1	22	13 8033627374 6460422324 2217970363	− 175274 7877187997
153(51, 1)	3	18	323539923	− 33045
153(17, 3)	9	24	5664	− 150
154(154, 1)	3	19	2155235855	− 91627
155(31, 5)	6	25	948398	1718
158(79, 2)	6	26	2143593	− 405
161(161, 1)	3	29	11782 6640534538	− 4032288
163(163, 1)	3	25	153 8810995062	− 260952
165(11, 5)	3	15	14500203	3
166(83, 2)	1	17	111241 6979919685 3740837863	− 298 2473893363
170(170, 1)	3	24	52 6804122603	1363233
170(17, 10)	6	26	251 5372623503	− 2647747
171(19, 3)	6	17	18300	174
171(57, 1)	6	27	4382904	3252
174(58, 1)	6	14	2787	3
177(59, 1)	1	15	648 5508371392 0533152619	13 9002682179

f(a, b)	h	B	Tr(e)	Tr(1/e)
179(179, 1)	1	27	4697646427 2595871534 1256873558 7792733582	− 2763719683 3896595164
181(181, 1)	3	39	1696672432 9842756278	− 8201089222
182(14,13)	9	22	4917	3
183(61, 1)	6	16	11715	3
186(31, 2)	9	11	75	15
190(19,10)	3	19	3321469803	− 35907
190(190, 1)	3	34	7236150 1175463503	117911153
194(97, 2)	3	23	32 5313434467	1137411
195(65, 1)	9	20	2328	48
197(197, 1)	1	40	9782302011 6714372594 |- 1295149693 8016021302 4596010639 2662391504	− 5406043315 1793416416 8686175498
198(66, 1)	6	18	28515	3
198(11, 6)	3	12	1587765	3
198(22, 3)	6	14	3567	3
198(33, 2)	3	23	27 0981041763	− 673593
199(199, 1)	9	27	29927	119
201(67, 1)	6	16	12867	3
202(101, 2)	1	25	47447574 6967248662 3095330624 4129043603	984103271 6778062103
203(29, 7)	6	30	46114295	− 4057
205(41, 5)	10	31	44283	3
206(206, 1)	3	19	5808269975	− 24067
207(69, 1)	1	30	121466 |- 0511160461 2750826388 5358696186 8382357203	412 6083442768 2024178203
207(23, 3)	8	22	14907	3
210(10, 7)	9	14	213	3
210(35, 2)	9	20	2103	3
210(70, 1)	9	21	3363	3
215(215, 1)	21	28	108	18
217(217, 1)	27	36	108	− 18

f(a, b)	h	B	Tr(e)	Tr(1/e)
222(37, 2)	6	26	5634807	3
222(74, 1)	3	13	3076923	-2883
226(226, 1)	1	27	19	-5 6490011642 0417725483
			-4968979780 3020755192 8411124247 7571357023	
230(115, 2)	3	26	2587 2045662883	9818013
231(77, 1)	3	31	1125643 2105981219	-120698421
233(233, 1)	4	38	38651 9830942502	22794626
234(13, 6)	9	28	55155	-285
234(78, 1)	3	26	2945 4411508707	-6402237
234(39, 2)	3	19	5684635245	-126357
234(26, 3)	6	14	4215	3
235(235, 1)	7	38	229338383	-6577
237(79, 1)	6	23	1111362	876
238(34, 7)	3	29	90929 3328534153	36062715
238(119, 2)	3	34	35426340 7402940283	1104494457
246(41, 2)	6	22	487083	987
249(83, 1)	2	40	22 5364130322 4520242220 2960545459	343821 8857205739
251(251, 1)	1	47	921 9791805735 6818290585 3059333019	1814044 9510728002 5695711723 1069746902
253(253, 1)	1	48	-4987444955 4517291982 5120011217 5745477474	-31751343 7672595087 1904211158 7253369123
			-1967132081 5826221916 0393015840 3930786667	
255(85, 1)	3	35	197668504 1175799323	1337122083
255(17, 5)	6	22	495723	3
258(86, 1)	9	18	1695	-21
261(87, 1)	1	24	260897735 2161908820 5268028290 8703054003	-607646122 3311460485
261(29, 3)	1	25	3080015310 4077633984 9394633986 3753114243	-1 0697290046 8980300213
262(262, 1)	1	23	15343041 1177096520 3592239798 9572413603	781088510 8881794865
266(133, 2)	9	38	4947869	-3721
266(38, 7)	9	31	257319	-765

f(a, b)	h	B	Tr(e)	Tr(1/e)
269(269, 1)	7	34	64626984	810
271(271, 1)	9	35	1509554	− 358
273(13, 7)	9	29	162672	− 114
274(137, 2)	1	25	4775897815 8447925190 4789160583 7226596147	− 1 3819460378 3504400237
278(278, 1)	3	37	495648338 7910006003	− 1 0375211997
279(31, 3)	3	35	667203077 5963526820	− 4390213506
279(93, 1)	3	21	27 0419268864	− 48066
282(94, 1)	3	23	288 3907966593	− 3386253
285(19, 5)	3	37	8481142137 9849138483	9263774523
285(95, 1)	3	24	1072 9181614803	5601963
286(22, 13)	9	44	58634447	− 11899
287(287, 1)	3	48	890857 4899025624 1212808003	− 1419 7992610533
290(58, 5)	3	39	7 0186281234 9613241043	2 3749642803
291(97, 1)	3	40	31 2020627490 8675259402	− 3 8673754104
295(59, 5)	1	38	76 6712923892 7988597331	15 8955110448 6514374719 3494137603
298(298, 1)	1	31	|- 2102644450 0477239661 9910885279 5139823403 4 2844183823 |- 4165019766 1555457258 6490262358 0103779773 397 0475111714 6369505590	151249 4028908477 5150457355
299(23, 13)	3	42	2451595439	36 2421925394
301(43, 7)	9	52	695872 9527448527	96815
302(151, 2)	3	29	11 3620775789 2910811027	143034651
303(101, 1)	2	26	1178	− 2 2775249517
305(305, 1)	21	40	25 9614281307	− 22
306(102, 1)	3	21	2247267	− 371787
306(17, 6)	6	23	9 5714014179	3
306(34, 3)	3	20	8265	− 264381
306(51, 2)	12	29	19555250 5565363739	3
307(307, 1)	6	64	684212449 5261475203	289386435
309(103, 1)	3	35		− 719918085

f(a, b)	h	B	Tr(e)	Tr(1/e)
310(155, 2)	3	30	1322603 8114358253	187822803
314(314, 1)	3	45	22458 2118820437 7391688435	284 1222983763
315(15, 7)	3	28	137399 4979849803	− 72485277
315(105, 1)	6	34	2522651043	90723
315(21, 5)	9	40	18708528	5178
315(35, 3)	3	30	2616571 5863719323	− 320967357
318(106, 1)	6	32	512069043	− 25755
318(53, 2)	6	18	77277	3
319(29,11)	1	47	|− 7535771 9233813554 9137286056 9258147284	|− −− 484765231 4445965378 4083788323 9561660885
322(23,14)	3	33	119492501 3417916749	2138987399
323(19,17)	3	50	33818918 3026418868 7951140552	6184 3767392502
323(323, 1)	3	61	2337 2686868446 7058474086 1010795982	− 416735 5395831946
329(47, 7)	3	43	7259 0350777922 1861706055	− 169 9383455785
330(110, 1)	9	18	1653	3
330(11,10)	18	38	2973	3
330(55, 2)	9	27	80193	3
333(37, 3)	3	37	1 2842590810 5135493140	2 1928149570
333(111, 1)	3	40	88 1532185705 1132413736	13 0577959002
334(334, 1)	1	35	10148969 5234270128	
335(67, 5)	3	43	|− 5539278468 5742515409 7613562544 0099303467	637008994 5923548785 0750819033
339(113, 1)	4	46	5899 0836949556 3137002963	− 151 9274395077
341(341, 1)	3	47	1445106617 5713291267	7388789763
342(19, 6)	6	30	734976 6062553784 9318684982	1648 5544689254
342(114, 1)	3	18	159711267	15393
342(57, 2)	9	22	1 2399716847	184683
342(38, 3)	27	35	9237	3
345(115, 1)	3	27	147	21
			90999 0439767003	− 59477997

f(a, b)	h	B	Tr(e)	Tr(1/e)
346(173, 2)	1	55	- 7 2055235081 5315824937 1487756793 7987047713 1545079034 6927451807 1488201400 5433320483 3263664023	- 369713 3076265792 3543552196 4533434242 4532912203
354(59, 2)	6	27	41704743	5667
357(17, 7)	3	42	3027 5365718355 6530798115	- 108 0405307797
357(119, 1)	3	34	696531198 7935592203	5138840787
359(359, 1)	4	51	30 4512217399 7391737502	6 4289715336
366(122, 1)	12	33	45753	3
369(123, 1)	2	29	9635 9076671144 4639304203	- 176 8621557885
369(41, 3)	4	41	14113294 9410148923	295648707
370(37,10)	27	39	263	- 7
370(370, 1)	9	50	2192725083	89913
374(34,11)	3	30	9047206 0986604707	578918343
374(187, 2)	3	59	1470 0340122718 4644326070 6667008817	1954754 5683928463
377(377, 1)	3	69	424229567 5541231791 7819845885 4126102787	- 3619990369 6209321813
379(379, 1)	3	72	2	- 2 2031827439 7425775730
382(191, 2)	1	50	- 3161 7860847829 2062170026 1814338242 9534367463 3976857279 5251818223 9019171365 7864653107	1208431961 3983784539 1329524161 0849373957
385(77, 5)	9	62	34 4222173188 1822389607	20
386(386, 1)	3	52	2194091710 1102001928	- 1164402
387(43, 3)	3	37	8 9918568992 5465058652	- 76543 2168672079
387(129, 1)	6	36	1 4406044664	7797286686
390(13,10)	18	28	393	77934
390(130, 1)	9	23	17553	3
390(26, 5)	9	38	12320103	3
393(131, 1)	2	39	2073 8128079463 4759045466 9267379003	- 9007089 3115241997
395(395, 1)	3	57	169 3838664823 4128975869 7495175403	- 2494163 8468744317

f(a, b)	h	B	Tr(e)	Tr(1/e)
397(397, 1)	3	45	316217 3825734962 2388221259	480 0830954547
399(133, 1)	9	38	13780182	192
399(19, 7)	18	48	33834	192
402(67, 2)	6	27	58861245	8043
403(31, 13)	9	48	1340236314	−73116
406(406, 1)	3	46	854562 1338337305 3443657753	−1010 465964 5497
410(205, 2)	3	65	8763103 6757173870 3679853033 8570027873	458897038 8433577303
411(137, 1)	1	34	293415813 5421145986	
413(413, 1)	3	55	−4665997986 1052961086 4135727898 9827718403	3307881754 2135685782 8250845403
414(138, 1)	3	29	15 0237127601 9079255160 6277143151	334890 8598584687
414(23, 6)	6	31	2257574 0500292547	76289853
414(46, 3)	6	36	521386635	37263
414(69, 2)	3	20	1 4976731523	−78243
415(415, 1)	1	56	13 3466065203	−357693
			229428 8084704145	31134121
			−2117664740 7803338108 3107442146 4633667830	1644617237 8135175545 5584959859 7712103363
			−3367941118 5961880075 9884322265 6088774443	
417(139, 1)	6	51	29464 1381424012	28209342
418(209, 2)	3	37	12 0634683548 6987686803	−6 8372845197
418(38, 11)	6	60	12193530 9761656515	128378253
422(422, 1)	21	35	675	−45
423(141, 1)	8	53	27 2825578947	761403
423(47, 3)	7	27	3807003	3
426(142, 1)	6	32	1656057963	42177
426(71, 2)	3	25	21743 2350489567	−14185797
427(61, 7)	9	70	2296 3962457847	8358503
429(13, 11)	6	33	3625513755	34323
430(86, 5)	3	65	10676146 4666412288 3584961677 7500801613	274085333 4284465403
431(431, 1)	4	47	2 3624453132 0817258240	−1 7982166050

f(a, b)	h	B	Tr(e)	Tr(1/e)
433(433, 1)	6	79	325 4732895265 3744053987	− 22 4832448221
434(31, 14)	9	55	3 9323870081	− 281353
435(29, 5)	3	28	787452 8857844883	147673803
438(73, 2)	3	27	201452 8344326787	− 56131887
438(146, 1)	3	37	19 7409448866 3092671125	4177976883
442(442, 1)	3	63	2158265 8881729789 5892320354 3318631113	− 282216523 6142411065
442(26, 17)	6	40	25 2198406463	643997
446(223, 2)	6	53	201553 2999834753	− 78272997
447(149, 1)	2	35	2 5790990044 5589950677 7096520467	290620 3169871123
449(449, 1)	2	85	4059 9085483014 3213151487 3049980434	− 5158876 2755578469 6501014963 7497675486
451(451, 1)	1	50	− 9114810402 3589562969 5199489690 5025110990 10090144	− 6348
453(151, 1)	6	49	− 0181711071 3199803324 7576536963 1160560601	1169966180 2975585843 9707882706 5530695417
454(227, 2)	1	33	− 2474386883 4050344646 8887887308 5157001687	− 25636173
455(65, 7)	9	70	17665 9664333883	− 8858056 6294388375 5989577561
458(458, 1)	6	37	3176063 9399221060	4082958
462(22, 7)	9	35	− 6766991933 6916888567 7633999554 2930719909	272055
462(77, 2)	9	33	3678 4486294128	− 3693
462(14, 11)	18	28	3 9147293523	− 2307
465(155, 1)	6	63	6819123 1945947 465	3
467(467, 1)	1	75	242172926 5683290163 20 6192565288	− 3108182757
469(469, 1)	9	77	− 6025226117 0023573166 8246369754 5940460420 2270602850 0185145442 3353202514 1433160437 2639715810 9309352722 8861852221 0007725986 75165 3739187616	− 9845046643 8377225931 2463497864 1104767772 904575 2543629224 7957570960 4701702
470(94, 5)	3	34	2721379471 4126077583	1 0417768553

f(a, b)	h	B	Tr(e)	Tr(1/e)
471(157, 1)	3	43	120360 9239885195 4238085235	559 8457510299
473(43, 11)	3	52	1 0411596219 5775128087 7492291959	−88963 6386803161
474(158, 1)	3	25	26836 5460054947	3032643
477(159, 1)	3	47	24907218 3700777971 9399232299	−5188 2512848701
477(53, 3)	9	57	9 888764586	−239928
478(478, 1)	1	35	60 7813533611 8455754044	
	-		2682813395 2541223607 3586691830 5578136227	− 8 6636721518 7234670736 9259665037
482(241, 2)	6	35	1 3151506599	169227
483(23, 7)	3	37	65 4040651499 1102525915	− 14 3168348397
485(485, 1)	3	48	120090608 0290099567 5387776838	−5869 5449815152
487(487, 1)	18	63	1282059	− 765
494(26, 19)	9	69	2804 8259235987	− 2989503
494(494, 1)	9	51	9220010751	180027
495(165, 1)	9	29	392043	3
495(15, 11)	3	42	47852 0403618191 6492255883	425 9027401203
495(55, 3)	9	56	7 0745720763	− 498957
495(33, 5)	3	65	125375672 9290875785 293528663 0471210203	1843047597 0133814403
498(166, 1)	6	23	5302707	3
501(167, 1)	1	54	621050 0640303132	
	-		6161103562 8827556350 7362478867 5341597327	0998512457 5334586075 5547057337 4410542237
	-		4816722532 9679837287 8103562731 1528000987	162430924 6314002934
503(503, 1)	4	81	738961 1056481879 1192415988 6942431560	− 55 0495454037
505(505, 1)	7	91	761 7663132702 9415538403	
506(23, 22)	15	35	13665	3
510(34, 5)	18	31	1023	3
510(85, 2)	9	28	349863	3
514(514, 1)	14	50	1776387	3
515(103, 5)	3	100	1492167204 9747220806	
	-		8720945822 4991612985 8419081068 2734652203	− 6764844528 5107934240 5129206957

K. Nakamula

f(a, b)	h	B	Tr(e)	Tr(1/e)
517(47,11)	1	62	2229679903 7321028878 1453079247	- 61890 0072839113
			|- 0853661276 4147597975 6512586035 3434266478	4011279397
			|- 3689235807 9956737960 2462928257 1440863427	- 903635 9902 2565257698 2977281245
518(259, 2)	9	50	8013947801	- 174673
518(74, 7)	9	67	1444 8755130617	- 6459457
519(173, 1)	1	38	3896485 9172522644 7783545358	2308 8332900786 2214486860 0011828939
521(521, 1)	1	78	|- 5082191231 6000709524 7311395842 0230004259	
			34975 3264 9001367504	
			|- 1772423415 0299401104 7801440008 7962810317	- 3175911993 1830886132 6698636521
			|- 2642053832 7688331576 4623262555 5276694664	4639365042 7437039853 3028614269 7142489730
			|- 3853825961 3700468285 4456125327 1515175564	
522(174, 1)	12	60	706334907	46983
522(29, 6)	3	53	14 7948446679 5658457111 5751552797	- 12270 2960176197
522(58, 3)	3	26	134660 7522263235	- 60510237
522(87, 2)	3	27	577155 9515223603	146577603
523(523, 1)	9	56	11 9103468671	486047
526(263, 2)	1	63	29 5366431255 5758934503 4693590098	- 1077841 3545603858
			|- 5264831803 6847894937 1127378798 8477737051	5729464081 8670119161 6063298566 2181366397
			|- 3987050045 3839106561 7636396751 1970039603	3 8959618953
530(530, 1)	3	35	4 3217304328 0389173403	- 958 7245519894 1109340087
530(53,10)	3	77	345352	1354
531(177, 1)	1	48	|- 0933341139 0726124002 7439138257 4568476143	7354313992 6160875261 2822153132 1560245219
			458999	
531(59, 3)	1	70	|- 9459610388 1491148208 5596864397 0592498560	- 111 5078586956 2593763948
			|- 1356525509 5531283145 7224626001 8132349979	1941060970 0438816197
			71778	
			|- 3810195462 4937323164 1861563291 7582602219	
			|- 1392164380 0379069442 8591806163 0066468911	
			|- 5836839266 7293446367 0690569447 5366738803	

f(a, b)	h	B	Tr(e)	Tr(1/e)
533(41, 13)	3	98	-6819713884 6665061223 167133535 7639529396	-2122558936 9037377348 9203019368
534(178, 1)	3	32	1265404234 2313106883 5172919747 1069870686	4675968867
534(89, 2)	6	60	63303756 8942394453	743488203
541(541, 1)	9	77	211442 1789772226	-292678
546(91, 2)	18	39	7647	3
546(26, 7)	9	44	681717039	19659
546(182, 1)	27	44	867	-51
549(61, 3)	3	39	1605 0278440823 3236630587	-68 6880032397
549(183, 1)	3	82	1270051209	
553(79, 7)	9	80	-3118002906 9476713578 6023865740 2818483003	-68131 8521558780 7313529997
554(277, 2)	3	42	926486 4950581008	-190542858
555(37, 5)	30	61	95291 5367178399 5846038299	-318 1497987651
555(185, 1)	6	39	4443	3
557(557, 1)	2	66	35 9999640003 1485686226 4007229403	-355197
558(31, 6)	6	34	-0369740808 9029211716 6289433673 3775571042	512883347 7172839349 0223809874
558(186, 1)	6	33	1 5987647487 6456323379	-200877
558(93, 2)	3	65	513000334 7387003362 2087725543 7347597157	-110481
558(62, 3)	6	45	3141 5323928979	-4261232171 1336722277
559(559, 1)	9	110	68 7310503524 0563067846	-10405581
561(187, 1)	3	50	9537569289 8150397941 2234310643	-8 9899520338
561(17, 11)	3	59	180445 4287618548 5833937737 5955207299	-194175 0750355413
562(281, 2)	1	52	1067 2558831428	-77043809 7914987301
			-4418244984 0512631546 3718890083 7241873507	-410408
			-0260119863 5417107451 4941172000 6731324185	-2709851254 2437050634 4919094726 8997767337

f(a, b)	h	B	Tr(e)	Tr(1/e)
565(113, 5)	1	60	7761080 2755341304 1529335269 4829014132 0765325820 9228341390 9727275356	−5571 4284647560 3829031916 5327048297 5268534097 7534219221
566(566, 1)	6	75	7456596363 8285233614 6208082191 9189442503	19 0324962147
570(95, 2)	9	36	212 9655405163 5906988143	− 5127
570(38, 5)	18	40	1249683	3
573(191, 1)	2	50	10263	− 111 5365634724 6684904317
574(41, 14)	6	57	42028 0165527691 5831217150 4955692436 0160999107	−602445715
577(577, 1)	15	84	9953855 9976839063	106680
579(193, 1)	3	45	1 1919640674	− 5988 4837892877
581(83, 7)	3	82	9322442 7396797430 4580101947 1154927640	− 6716 0486063547 4599882913
582(194, 1)	3	42	6040395661 2713530772 2792773592 2737412383	842 3863326543
585(15, 13)	3	77	204403 0566033257 9593349787 2756578	− 176 7396843369 7508111397
585(195, 1)	6	68	2852156757 2064700530 9006490162 4901939203	− 1 5217282077
585(39, 5)	15	38	3 1023888620 2173470883 42123	3
585(65, 3)	3	43	521456 5339050354 6632995083	− 1371 3908251677
586(586, 1)	2	63	10399456 3924939727	− 635695878 9731911617 4443667183
590(295, 2)	3	43	8008457332 7841497356 4226659540 7827005383	− 988 6433334117
593(593, 1)	2	83	485825 8824820844 3070105573 106661 7672847480 8801177973 1781222341	− 2061479 0569718764 0578288784 8523939920
595(595, 1)	6	67	2561673572 3328182107 5216452551 8815973724	2 3668683503
595(35, 17)	3	99	1 4156911557 0769877903 4759507794 3492481255	607268297 6387132244 5554976903
598(299, 2)	3	89	3942343384 4275710634 7253963936 4093304023 4665 6860683346 8107167373 8001469350 3449787386 2912722003	− 6617473 3204286146 4473193247

f(a, b)	h	B	Tr(e)	Tr(1/e)
602(602, 1)	9	59	80 3066704299	1100349
603(67, 3)	6	84	202302 3685983568 1084633475	619 1286797883
603(201, 1)	3	49	8487449240 6062532855 3226431907	176888 0981878923
606(202, 1)	3	62	36508007 5027598841 5374295100 0791985003	1137243729 6267732753
609(203, 1)	9	60	154 2186887484	2465322
610(122, 5)	3	54	484 7124734121 6244575408 9596697363	3960277 3270160103
611(611, 1)	3	70	382	
613(613, 1)	3	64	− 6477633468 2125371029 0699999030 5462700483 −	29 9458159978 4533670187
615(205, 1)	3	34	566840004 3316031383 0075033847 3927846414	1777603097 6665692590
618(103, 2)	6	47	2 5039586519 4693972603	3 0955154163
			15911 6736538317	13324083