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A MODEL FOR RESIDENTIAL FATAL FIRE IN HONG KONG

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A nonlinear model which relates area, population density and number of fire stations with the number of residential fatal fires in Hong Kong is introduced. The model can be interpreted as a relationship between the number of fatal fires and response time. Based on the data collected from 17 out of 19 districts in 1984-1988, the four parameters in the model are estimated by the method of least squares with 78 percent variance explained. Statistical inference of the estimates is provided by bootstrap technique. Based on the model, some guidelines for allocating fire stations can be recommended.

1. Introduction. A fire once started should be extinguished as soon as possible or else it will burn out all inflammable material in its reach and cause tremendous damage. In most cities, fire stations (or firehouses) equipped with firemen and apparatuses (together called a fire company) are responsible for extinguishing fires. Response time is the amount of time from the reporting of a fire to the arrival of the first fire company at the fire scene. Response time has three components: dispatching time, turnout time and travel time, among which the travel time can be reduced by scattering the locations of fire stations throughout a region. There are two major categories of fire damages: property and life losses, both of which, by common sense, should be closely related to travel time. To analyze fire protection, it is essential to understand the relationship between those fire damages and travel time. Nevertheless, most existing models, such as those used in Kolesar (1979 a,b), Rider (1979) and Walker (1979), use travel time as their criterion for analysis, but without constructing its relationship with the property and life losses. In this paper, we build a model to relate the number of fire stations with the number of residential fatal fires for Hong Kong and illustrate the use of the model to suggest how to choose among 8 out of 19 districts to allocate fire stations for the minimization of residential fatal fires.

2. The Situation in Hong Kong. Each of the 19 districts in Hong

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Kong will be abbreviated by one to four uppercase letters throughout this paper. We will use the average data, collected from 1984 to 1988 to reflect the situation in a district. A study by Kolesar (1979a) shows that a district's average travel time to fires is proportional to the square root of the district's area divided by the fire station count. Let this square root be r. Graph 1.1. exhibits r for 18 districts; one is excluded because it consists of many disconnected islands and consequently its average travel time cannot be proportional to r. As we can see from that graph, the variation of r is substantially large — the existing allocation policy of fire stations in Hong Kong is not to equalize travel times. In 1984–1988, there were totally 133 fire fatalities and 102 fatal fire incidents in Hong Kong. From Graph 1.3, where fire fatalities and fatal fire incidents per 100,000 population per year are broken down for the 18 districts, we can see that the variation of fatal fire incidents is smaller than that of travel time (see Graph 1.1); thus, the existing fire policies are more likely to equalize fatal fires than travel time. Also from Graph 1.3, the ratio between the number of fire fatalities and the number of fatal fires is not substantial, and the fires with more than one fatality appear to be very rare for most of the 18 districts. So we will restrict our study to fatal fire incidents only.

When fatal fires are mentioned, population density shall be investigated. Graph 1.2 shows the population density substantially varies among districts.

3. The Model. Among the 102 fatal fires observed from 1984–1988, 83 of them occurred in residential buildings, 14 in commercial/industrial buildings and 5 in others. For the reason of homogeneity, we only model the 83 which occurred in the residential buildings and call them residential fatal fires.

DEFINITIONS.

- F number of residential fatal fires.
- P residential population (in 100,000).
- f F/P.
- A area (in km^2).
- p P/A.
- S number of fire stations.
- r square root of A/S.
- $f(r,p)$ the residential fatal fires per 100,000 population for given r and p.

The model is

$$f(r,p) = a \cdot p^b + c \cdot r^d \cdot p^{-d} + \varepsilon \qquad ((1))$$

for some $a, b, c > 0$ and $0 < d < 1$. Note that $f(0,p) = a \cdot p^b$. By $F = f \cdot A \cdot p$, (1) implies

$$\partial F/\partial p = a \cdot A \cdot (1+b) \cdot p^b + c \cdot A \cdot (1-d) \cdot (r/p)^d > 0$$

which says F increases with p, a reasonable statement.

4. Estimation and Statistical Inference. Using the average data from 1984 to 1988, we estimate a, b, c and d to minimize $\sum \varepsilon^2$ without imposing any constraint on their values. Since YMT (Yau Ma Tei) will increase the minimal $\sum \varepsilon^2$ by almost 100 percent, it is excluded from analysis. The results of estimation are $\hat{a} = 0.4874, \hat{b} = 0.5576, \hat{c} = 0.0016, \hat{d} = 0.8019$ which gives $1 - \sum \varepsilon^2 / \sum (f - \overline{f})^2 \approx 0.78$. The data fits (1) pretty well and $\hat{a}, \hat{b}, \hat{c}, \hat{d} > 0, \hat{d} < 1$ holds as required by (1). Let us call $\hat{f}(\hat{a}, \hat{b}, \hat{c}, \hat{d};\ p_i, r_i)$ the estimated residential fatal fires for the ith district and display them against observed ones by population density and r, respectively on Graphs 2.1 and 2.2. In these two graphs, the variable 'population density' is arranged in order but not in scale, because if so, the large variation of population density would make the graphs difficult to read.

To obtain some statistical inferences on the above estimates, we use the bootstrapping technique in Efron (1982 p. 35–36) for $B = 200$, $n = 17$, to estimate the distributions of $\hat{a}, \hat{b}, \hat{c}$ and $\hat{d}$. The resulting frequency distributions of the 200 estimated a's, b's, c's and d's are shown on Graph 2.3 and 2.4 except the 6 large $\hat{c}$'s which are 5.79, 1.24, 0.64, 0.04, 0.03 and 0.02. From those distributions, all the estimates for prob ($\hat{a} > 0$), prob ($\hat{b} > 0$), prob ($\hat{c} > 0$), prob ($\hat{d} > 0$) are equal to 1 and for prob ($\hat{d} < 1$), 0.765; thus, the inequalities in (1) are well supported by the data except for $d < 1$, for which the estimated prob ($\hat{d} < 1$) is rather low, 0.765. This shortcoming may be improved if a larger sample can be obtained. Another way to make use of those estimated distributions is to list out each distribution's 0.05-quartile and 0.95-quartile. These are [0.3881, 0.5906], [0.3162, 0.8151], [0.0004, 0.0105], [0.5668, 1.2771] for $\hat{a}, \hat{b}, \hat{c}$, and $\hat{d}$, respectively. As one can see the difference between the 0.95-quartile and the 0.05-quartile for $\hat{c}$ is 0.0101 which is about five times larger than our estimate for c, 0.0016. The value of c is thus largely uncertain. Fortunately, $\hat{c} > 0$ is always true. In the next section, we will illustrate applications in which only the positive sign of c matters.

5. Illustration: Allocation of Fire Stations. The model (1) indicates that the effects of travel time can be measured in life loss. Since travel time is a component of response time, the model has a broad range of applications in fire services such as dispatching time, turnout time, reporting time and the allocation of fire stations. However, the shortcoming is that the residential fatal fire count is only part of all the fire losses concerned. The model's application has to be restricted to those districts where residential fatal fires are of major concern. In this section, we illustrate applications in allocating fire stations to 8 districts which have a population density less than 10,000 per square km.

Let $F_i^k(n_i)$ denote the expected residential fatal fires of the ith district when the total fire stations in the eight districts is k and in the ith district is n_i

respectively. Now, if there is one more fire station available then the question is which district this fire station should be allocated to so as to minimize the total expected residential fatal fires. This question can easily be answered by using (1). Since $F_i^k(n_i) = f((A_i/n_i)^{1/2}, p_i) \cdot p_i \cdot A_i$ is a strictly decreasing function of n_i, the district chosen for the additional fire station must be the one which maximizes $\{F_i^k(n_i) - F_i^{k+1}(n_i+1) = c \cdot A_i^{1+d/2} \cdot p_i^{1-d} \cdot [1/n_i^{d/2} - 1/(n_i+1)^{d/2}] : i = 1, \cdots, 8\}$. Since c is independent of i, the choice of district will not be affected by the value of c as long as $\hat{c} > 0$. As of now, there are 22 fire stations allocated to those districts. If we use the above method and start with one station in each district i.e. $k = 8$ and recursively allocate fire stations to each district by incrementing k up to $k = 22$, then an optimal allocation can be obtained as follows:

	YL	TP	N	SK	TM	ST	TW	S
no. of S	4	4	3	3	3	2	2	1

The existing allocation is

	YL	TP	N	SK	TM	ST	TW	S
no. of S	3	2	4	2	3	2	2	4

If the existing allocation cannot be changed and there are five more fire stations available then by applying the above method, the five fire stations should be allocated to districts by the following order: TP, SK, YL, TP, ST.

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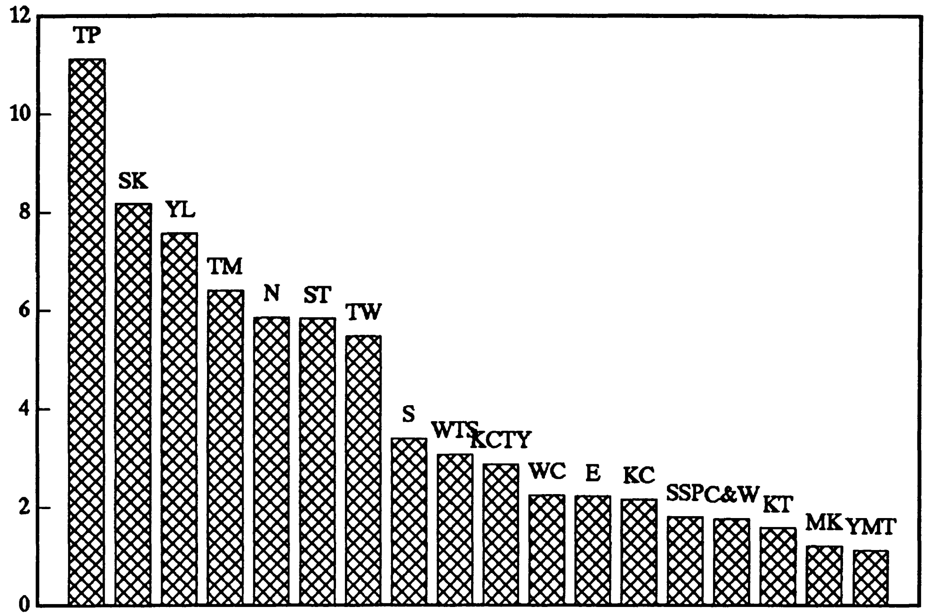
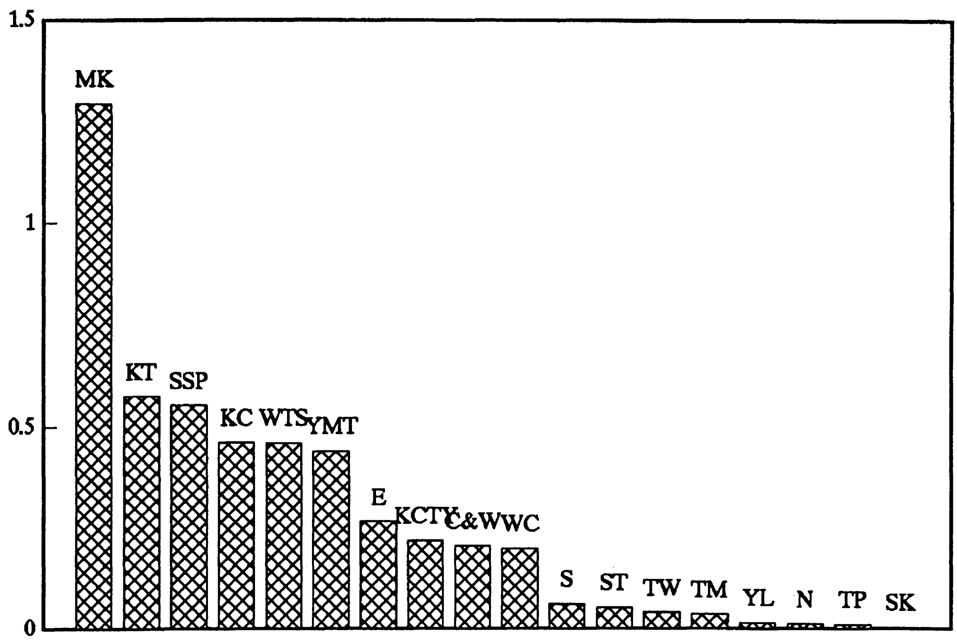
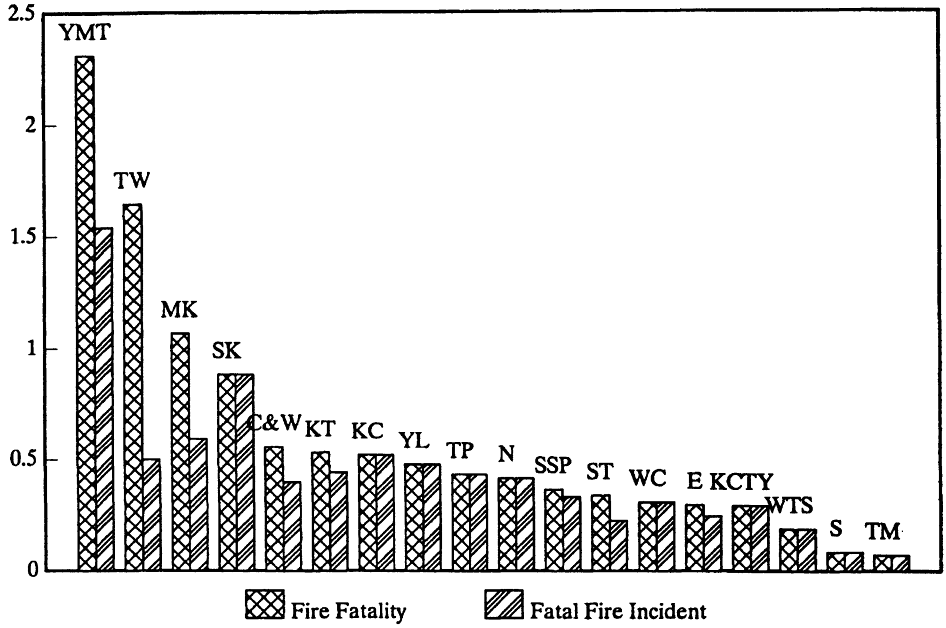


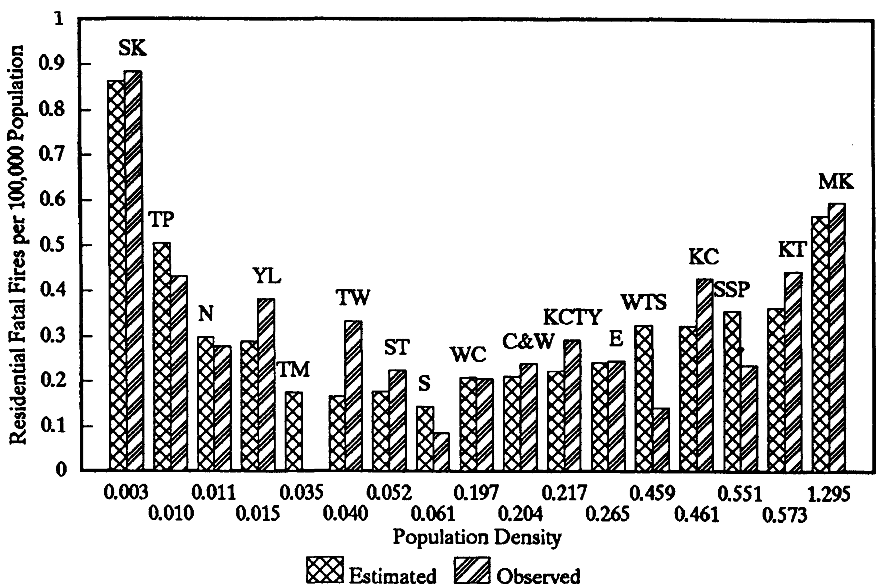
Graph 1.1. Square Root of Area Divided by Fire Station Count



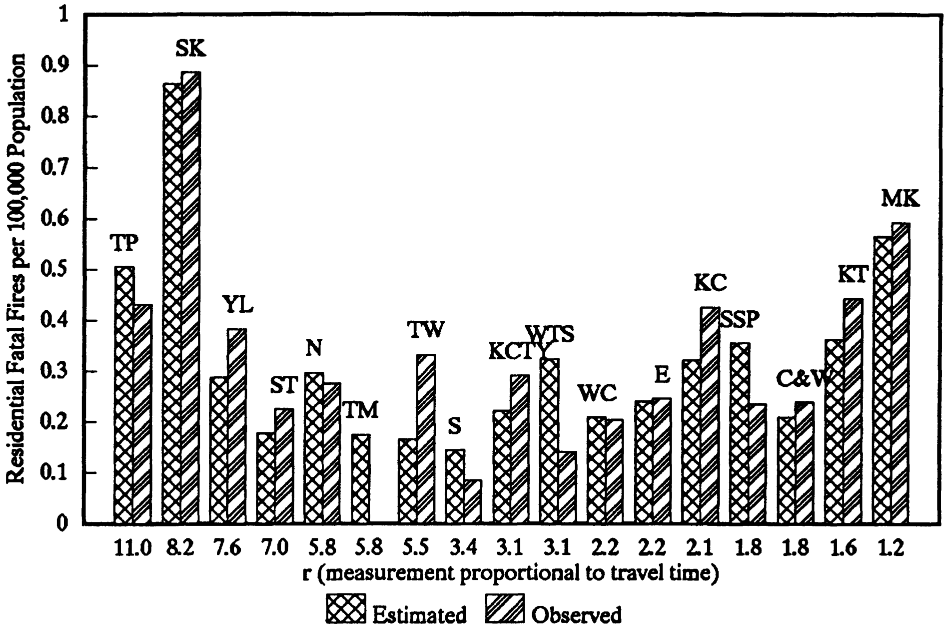
Graph 1.2. 100,000 Population per Sq.Kilometer



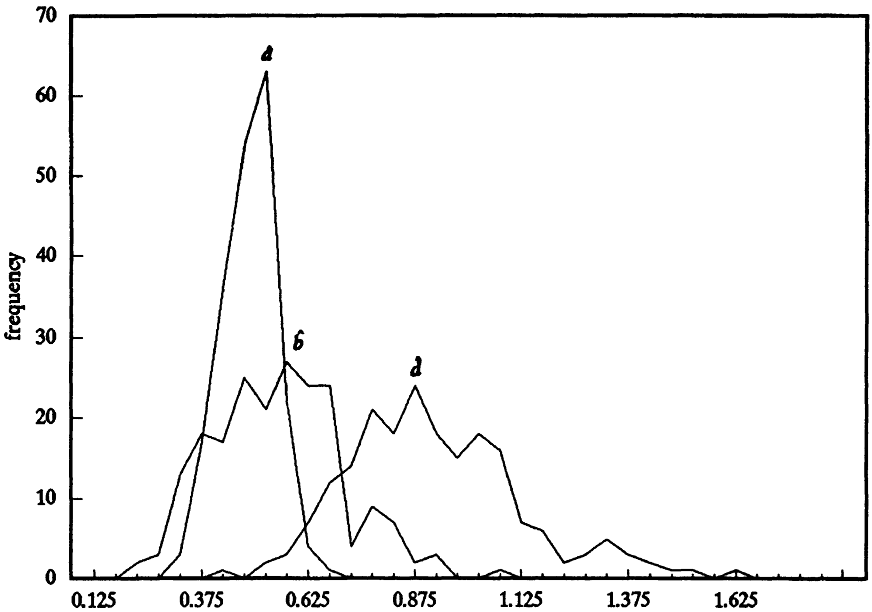
Graph 1.3. Fire Fatalities & Fatal Fire per 100,000 Population



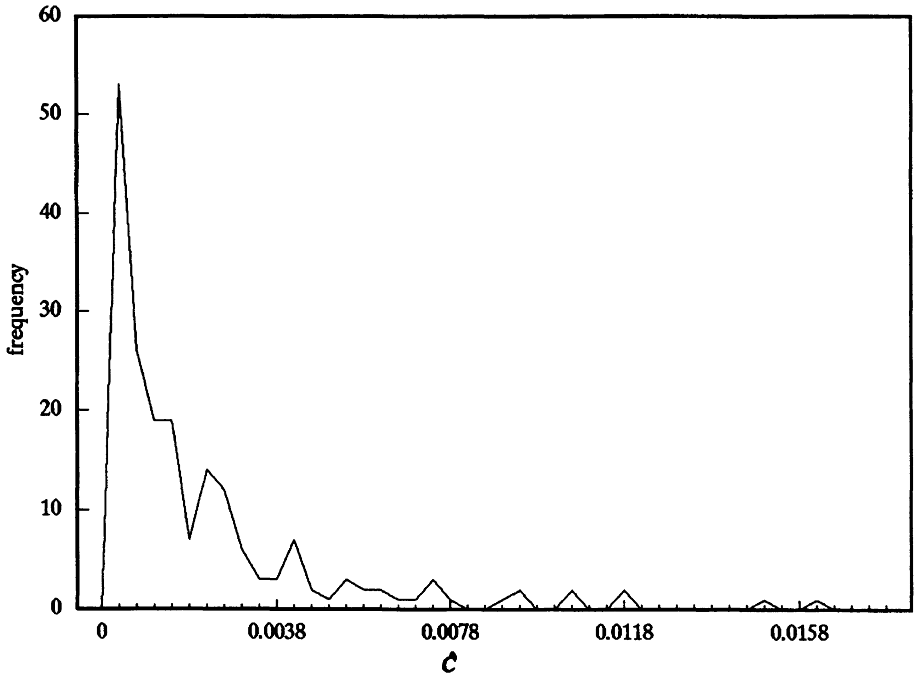
Graph 2.1. Residential Fatal Fires per 100,000 Population by population density — Estimated vs. Observed



Graph 2.2. Residential Fatal Fires per 100,000 Population by r — Estimated vs. Observed



Graph 2.3



Graph 2.4

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