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WHY DID CASINO BLACKJACK WINNINGS DROP?

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Abstract.

One issue in a court case was whether a casino's lower winnings at blackjack were consistent with historical variation. Significance tests indicated that they were not. Using explanatory variables measuring the volume of casino chips purchased, credit granted to players, player mix, and time patterns, regression analyses could not fully explain the six month dip in winnings. One proposed explanation for the dip was skimming at two particular blackjack tables. While this explanation was refuted by an analysis of extreme wins and losses by table bet maximum, a credible explanation for the winnings dip has yet to be discovered.

1. Prologue.

For over six months, a large Las Vegas casino had been experiencing sharply reduced winnings at its blackjack tables. Concerned about this turn of events, management increased surveillance of the blackjack tables, changed card shuffling procedures, moved dealers from table to table more frequently and in an unpredictable way, and ultimately dismissed the blackjack 'pit boss'. Lower blackjack winnings persisted in spite of these steps. Management believed that dealer cheating was the cause of the drop in winnings, but was not able to uncover any direct evidence of such cheating.

In frustration, the casino management decided to dismiss blackjack dealers whom it believed were most likely to be cheating. The drop in winnings was occurring for the day shift and the swing shift; winnings for the graveyard shift remained relatively constant during the period. Also, management believed that since more experienced dealers tend to be more skillful, a 'first in, first out' rule should be used in deciding whom to dismiss. There were about 40 replacement dealers that could be hired on short notice. Putting all these considerations together, the casino dismissed about 40 blackjack dealers who worked the day shift and the swing shift and who had the longest tenure with the casino. The dismissals all occurred on a single day.

The blackjack dealers who were dismissed brought suit against the casino in Federal Court alleging age and sex discrimination. All the dismissed dealers were men over forty while the replacement dealers were predominantly younger women. Attorneys defending the casino in the suit considered arguing, among other things, that there was a 'compelling business

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necessity' to take such drastic action. I was consulted by defense attorneys as part of their assessment of the viability of this argument. I describe here the analysis I carried out and my conclusions as part of this consultation.

2. A Statistical Description.

The decreased blackjack winnings for the two shifts occurred between March and August of one year (the 'suspect' period); the forty dealers were dismissed in early September. To investigate blackjack winnings, I used daily blackjack data for the two calendar years including the suspect period which occurred during the second year. My assignment was to (i) assess whether the dip in winnings in the suspect period was real since Blackjack is, after all, a game of chance, and (ii) if the dip was real, determine whether there was a plausible explanation for it using information that was available to the casino management.

The casino determines its gross revenues by taking inventory of chips, cash and credit slips at the end of each shift. The crucial quantities of interest for management for each shift are: (i) the Win per shift, defined as the net change in the casino inventory of chips, cash and credit slips from blackjack play during the shift, (ii) the Drop, defined as the dollar value of chips purchased at the blackjack tables during the shift and the Win%, defined as Win/Drop. The Win% is continually monitored by management, and historically is between 15 and 20 percent for most casinos.

Finding winning strategies for playing Blackjack has been studied for many years. For example, Thorpe (1966) and Griffen (1986) derive optimal playing strategies for blackjack and compute win probabilities under given conditions. We are interested in the total win across different hands, different players and their playing strategies, and different bet sizes. To simplify, the expected win in n hands is approximately nwb where w is the house edge and b is the average bet size. Griffen (1987) estimates the $w = -.005$ under the optimal strategy and $w = .016$ if averaged across the public's playing strategies. However, we have, at best, only indirect information about the distributions of n and b. Such indirect information includes the number of blackjack tables open during a shift, the speed of play, and the bet size minimums and maximums. Given this situation, I decided to do a statistical analysis of winnings, rather than attempt to construct a probability model.

3. Analysis and Results.

3.1 Testing Whether the Winnings Dip is Real. Figure 1 gives the daily Win and Win% and Figure 2 aggregates these quantities by month for the two calendar years. As a comparison number, the horizontal line in each plot gives the mean value for year 1 of the quantity being graphed. During the suspect period, Win and Win% are substantially lower compared to both before and after, but the pattern is not a consistent one. Table 1

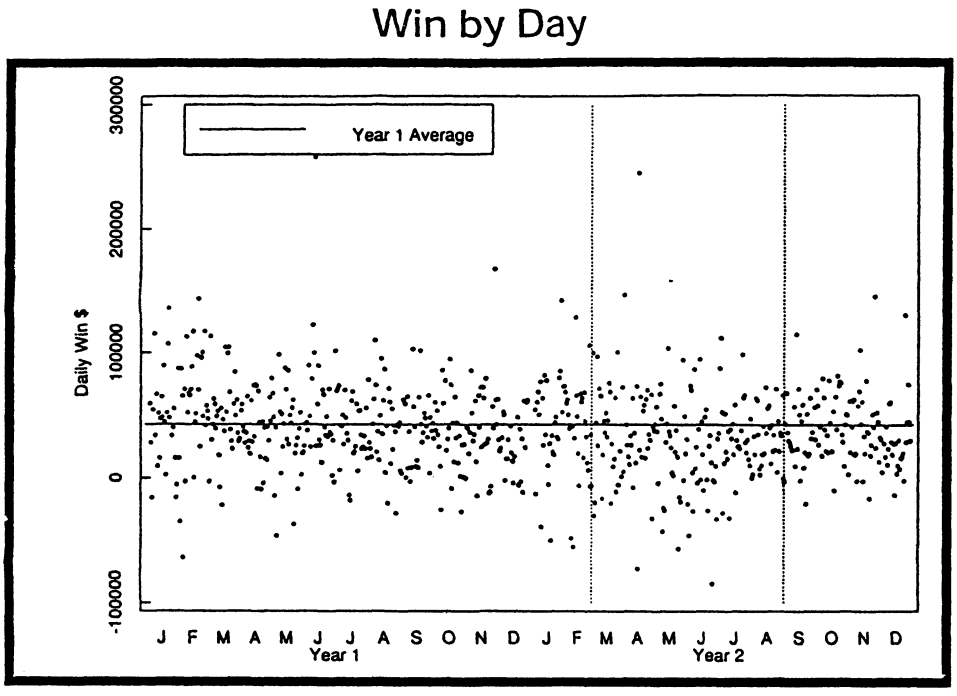
Table 1
WILCOXON RANK SUM TESTS COMPARING WIN AND WIN%

	Swing Shift		Day Shift	
WIN	Year 1	Suspect	Year 1	Suspect
Mean Rank	367.7	337.3	388.6	324.2
Significance Probability (1-tailed)	0.055		0.000	
WIN%				
Mean Rank	379.0	335.1	392.2	319.5
Significance Probability (1-tailed)	0.013		0.000	

reports the results of Wilcoxon Rank Sum Tests comparing year 1 and the suspect period for Win and Win%. (We did not compare the suspect period to the following four months since, under the dealer cheating hypothesis, a change could be interpreted as the result of eliminating the dealers who were cheating or as the result of those who were cheating or skimming not being the terminated dealers, but stopping their activities on the assumption that the winnings dip would be blamed on the terminated dealers.)

The Wilcoxon significance probabilities suggest that the winnings dip during the suspect period is real as compared to the previous year, particularly for the day shift. However, differences in Win may not indicate abnormal changes in the probability of winning individual hands, since changes in Win could also be caused by variation in player volume and in the distribution of the size of bet per hand. Significant values of the Win% have less obvious alternative explanations. This seems persuasive evidence that something was going on; that is, the dip in winnings was 'real' rather than simply the result of the random outcomes of blackjack hands.

3.2 Looking for an Explanation: Regression Analysis. To aid in assessing whether management could have come up with a plausible explanation for the blackjack winnings decline prior to taking the step of dismissing the dealers, I familiarized myself with casino operations. I will not report here on our less formal investigation of chip transfers, counting room operations, blackjack table surveillance, and other aspects of casino operations beyond saying that there were no apparent irregularities during the suspect period. I describe here our statistical investigation of factors that might explain the decline in blackjack winnings.



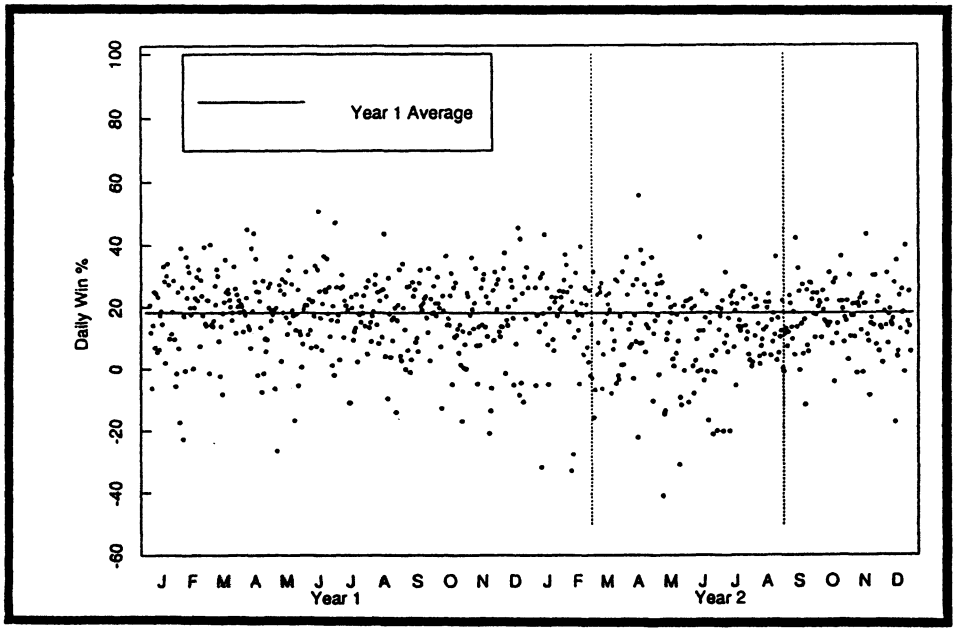
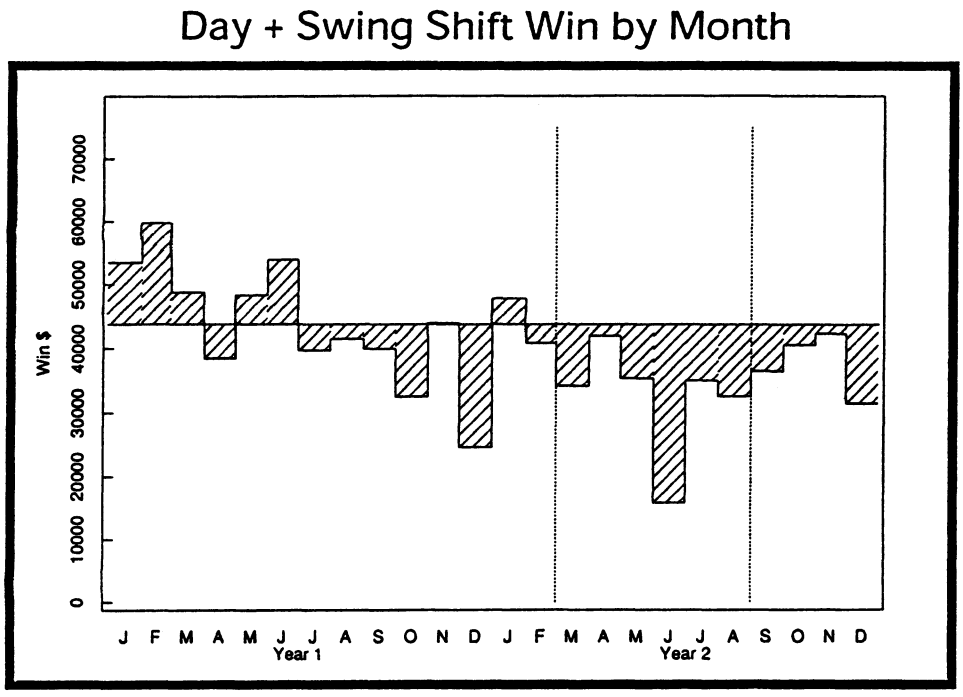


Figure 1. Daily Blackjack Winnings.



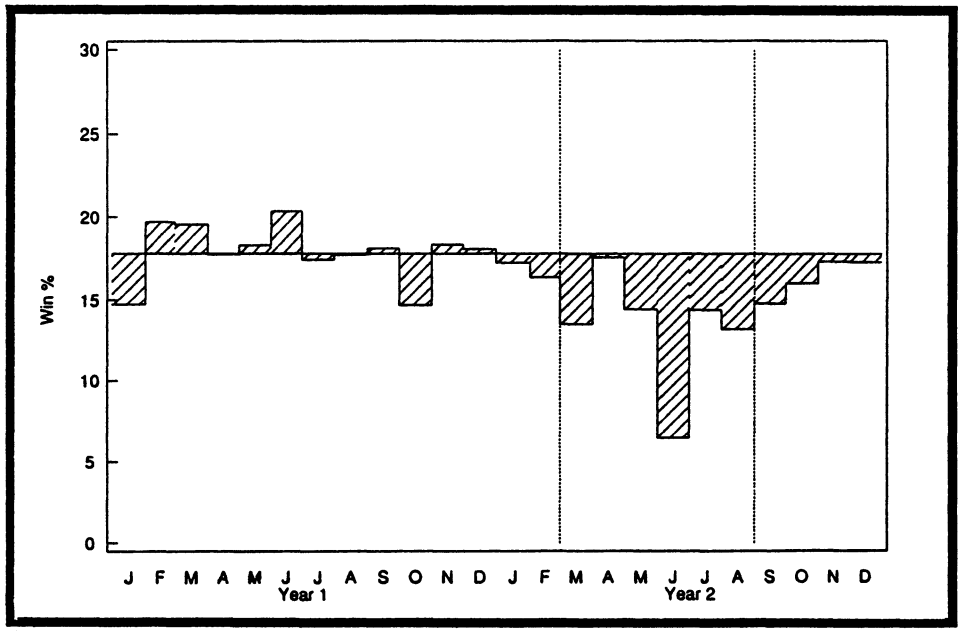


Figure 2. Monthly Blackjack Winnings.

The distributions of Win and Win% are affected by player volume, bets per player, bet size and player skill. The casino keeps data that are proxies for all these factors. Changes in Win% can also be caused by chip purchasing behavior. To see how this might happen, suppose that all players doubled the dollar volume of chips they purchased at the blackjack tables, but did nothing else differently (same size bets, same number of bets, etc.). In this situation, Win% would be halved as a consequence of the dominator, Drop, being doubled. Our analysis strategy was to carry out a regression analyses using Win and Win% as outcome variables and the proxies as explanatory variables. Our reasoning was that if the winnings dip could be predicted well by a regression using these proxies, an experienced casino management should have been able to figure out what was driving their decreased winnings and perhaps do something about it.

Table 2 gives definitions for the available explanatory variables. 'Markers' are a partial proxy for large bets, since the casino normally will only grant credit to players they believe to be 'high rollers'. The hotel guest variables indicate player mix which effects blackjack play. (Indeed, rumor has it that after the Joint Statistical Meetings in Las Vegas in 1980s, the casino winnings were so poor that they declined to host the statisticians again.) The time variables were used to pick up possible day of week and seasonal patterns as well as differences between winnings in the suspect period and adjoining periods.

Exploratory regressions were fit using various subsets of the potential explanatory variables and transformed versions of them. Diagnostic plots and measures as well as measures of fit (e.g., Mallow Cp) were used to guide model selection. Table 3 reports the regression models for Win and for Win% for the two shifts that we finally settled on.

The Win regressions used a weighted least squares fit with weights proportional to Drop. Explanatory variables for Win include Drop, marker variables, customer mix and time variables. No seasonal or day of week effects were evident. Both day and swing shift regressions had about the same proportion of variance explained (.37). Coefficient estimates for the suspect period and the before period are negative for both shifts, indicating lower than average Win as compared to the previous year. If one interprets these coefficient estimates as the unexplained shortfall, the swing shift Win was and estimated \$4800 per day lower and the day shift Win was an estimated \$3600 per day lower than the previous year. Summing over the two shifts and the six months in the suspect period yields an estimated shortfall of \$1,530, 000.

The coefficients of the Win% regressions exhibit similar patterns to those for Win for the two shifts although the hotel guest mix variables mostly drop out. In contrast to the Win regressions, one seasonal/day of week indicator, being on a weekend, was helpful for the swing shift regression. The proportion of variance explained was similar for the swing and day shifts

Table 2
VARIABLE DEFINITIONS

CASINO	
Drop	cash or credit purchases of blackjack chips during shift (00,000)
Win	net inventory change in chips, cash, credit slips (00,000)
Win%	Win/Drop
Marker count	Count of the number of markers (credit slips) used to purchase chips (0)
Marker issued	Dollar value of markers used to purchase chips (00,000)
Marker net	Marker issued minus dollar value of markers paid off by the player during play (00,000)
Ntable	number of tables open during shift
HOTEL GUEST	
Convention	number of convention hotel guests (00)
Group	number of organized gambling group hotel guests (00)
Compliment	number of complimentary hotel guests, usually active gamblers (00)
Regular	number of other hotel guests (00)
Capacity%	% of room capacity occupied in hotel (00)
TIME (all 0-1 variables)	
Day of week	Sunday, Monday through Thursday, weekend
Month	month of year
Before	January, February of year 2
Suspect	March through August of year 2
After	September through December of year 2

(.165 and .185 respectively). Coefficient estimates for the suspect period and for the before period are negative for both shifts, indicating an lower than average Win% as compared to the previous year. If one interprets these coefficient estimates as the unexplained shortfall, the swing shift Win% was an estimated 5.9 percentage points per day lower and the day shift Win was an estimated 5.3 percentage points per day lower than the previous year.

My conclusion from this analysis is that while the regression analysis pointed to some factors known to management than helped explain the shortfall, even after these were accounted for, there was still a substantial unaccounted for drop in blackjack winnings.

Table 3
REGRESSIONS OF CASINO WINNINGS

Dependent Variable	Win		Win%	
Shift	Swing	Day	Swing	Day
Drop	.268 (.00)	.162 (.00)	.083 (.02)	.008 (.79)
Marker Count	−.017 (.00)	−.022 (.00)	−.009 (.01)	−.019 (.00)
Marker Issued	−.262 (.00)	−.213 (.00)	−.235 (.00)	−.190 (.00)
Marker Net	.829 (.00)	.915 (.00)	.574 (.00)	.794 (.00)
Ntable	−.004 (.08)	−−	−.000 (.81)	−−
Convention	.006 (.00)	.003 (.05)	−−	−−
Complimentary	−.060 (.00)	−−	−.042 (.00)	−−
Capacity %	−.082 (.27)	.087 (.06)	−−	−−
Weekend	−−	−−	−.060 (.00)	−−
Suspect	−.048 (.02)	−.036 (.02)	−.059 (.00)	−.053 (.00)
Before	−.075 (.01)	.009 (.69)	−.042 (.09)	.015 (.56)
After	.022 (.29)	−.010 (.54)	.014 (.46)	−.026 (.16)
Constant	.085 (.00)	.011 (.49)	.182 (.00)	.143 (.00)
R-square	.378	.368	.165	.185
F-Probability	.000	.000	.000	.000
RMSE	.183	.150	.170	.172
No. of Observations	711	714	709	713

NOTES: (1) Win regressions fit with weighted least squares with weights equal to Drop.
(2) Significance probabilities from t-statistics are in parenthesis.

4. The Trial.

At trial, I testified that (i) the casino's blackjack winnings had dropped by a statistically significant amount during the suspect period, and (ii) that in spite of an extensive analysis I could not identify factors that fully explained the shortfall. Plaintiffs did not attempt to refute my analysis. However, several possible causes of the drop in winnings were offered up by the plaintiffs' side. They argued that management should have known about these causes and acted on them. I offered evidence in refutation of several of their explanations. The one I will discuss one here is alleged skimming at two particular tables.

Plaintiffs' expert, an accountant, asserted that cheating or skimming at two specific blackjack tables for May through August on the day shift and

for March through June on the swing shift caused a $1.2 million loss, almost all of the shortfall. He identified these tables and these months by searching through the winnings data to find tables and months that had frequent large daily losses in the suspect period and only occasional large daily losses during year 1. He then took the difference between year 1 winnings and years 2 winnings for the months in question, summed them and asserted a $1.2 million cheating 'effect'.

Table 4

100 EXTREME WINS AND LOSSES BY TABLE

SWING SHIFT	WINS		LOSSES	
	Year 1	Suspect	Year 1	Suspect
A	13	20	15	23
B	10	21	13	12
C	23	19	14	17
D	26	27	14	22
Medium Stakes	21	8	26	13
Other	7	5	18	13
Total	100	100	100	100

DAY SHIFT	WINS		LOSSES	
	Year 1	Suspect	Year 1	Suspect
A	19	15	17	19
B	14	17	15	11
C	19	21	17	27
D	21	32	28	20
Medium Stakes	20	9	18	15
Other	7	6	5	8
Total	100	100	100	100

Since I suspected that the losses at these two tables might be part of a pattern for high stakes tables generally which would produce relatively large daily wins and losses, I asked casino management classify blackjack tables by betting limits. Their three categories consisted of four 'high stakes' tables, four 'medium stakes' tables and the remaining tables. I analyzed the data using six groups of tables consisting of 4 groups of 1 high stakes table each, 1 group of the four medium stakes tables and 1 group of the remaining tables. Since my objective was to understand the distribution of extreme wins and losses, I looked at how the 100 extreme daily wins and losses for year 1 and for the suspect period were distributed across the six groups. These tabulations are given as Table 4. The distribution of the 100 extremes across the high

stakes tables (A,B,C,D) appears fairly even considering the number of days (100). This uniformity is confirmed by significance tests. Three out of the four chi-square tests comparing the distribution of the 100 extremes across the two periods separately for wins, for losses and by shift did not achieve statistical significance ($p = .10$). For the case that did achieve significance ($p = .04$), the dominating term in the chi-square calculation was from the medium stakes table group. The two tables singled out by the opposing expert are labeled 'A' and 'C' in Table 4. I believe that the 'effect' he observed resulted from the higher variance of Win produced by the larger bets at the high stakes tables.

In sum, I was able to reject several explanations for the shortfall, but was never able to come up with a causal explanation. So, unfortunately, I am unable to answer the question posed in the title of this paper.

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