

Pao-Lu Hsu (Xu, Bao-lu): The Grandparent of Probability and Statistics in China

Dayue Chen and Ingram Olkin

Abstract. The years 1910–1911 are auspicious years in Chinese mathematics with the births of Pao-Lu Hsu, Luo-Keng Hua and Shiing-Shen Chern. These three began the development of modern mathematics in China: Hsu in probability and statistics, Hua in number theory, and Chern in differential geometry. We here review some facts about the life of P.-L. Hsu which have been uncovered recently, and then discuss some of his contributions. We have drawn heavily on three papers in the 1979 *Annals of Statistics* (volume 7, pages 467–483) by T. W. Anderson, K. L. Chung and E. L. Lehmann, as well as an article by Jiang Ze-Han and Duan Xue-Fu in Hsu’s collected papers.

Key words and phrases: Multivariate analysis, Wishart distribution, Student’s t , Hotelling’s T^2 , determinantal equation, eigenvalues, design of experiments, mathematics in China.

1. HSU’S LIFE

Pao-Lu Hsu was born in Beijing on September 1, 1910, “into a Mandarin family from the famed lake city of Hangchow” in the Zhejiang Province in Eastern China. His family was well educated: not only his father, but also his grandfather, his great-grandfather and the father of his great-grandfather, as well as their brothers and brothers-in-law. This reflected the tradition in old China of excelling in local exams, provincial exams and finally in national exams. The tradition was abandoned after China lost the war with Japan in 1895 and turned to Western methods.

Hsu was the youngest of seven children, with two brothers and four sisters. His father died when he was 14. During his childhood, Hsu moved from Beijing to Tianjin to Hangzhou and back to Beijing. His early education was with private tutors, a luxury that few people could afford at that time. He began attending school at the age of 15, and enrolled at Yenching University when he was 18. He first studied chemistry,

then decided to study mathematics and transferred to Tsinghua University in 1929. Both Yenching and Tsinghua universities had connections with universities in the United States. For example, the Yenching–Harvard Institute, founded in 1928, was designed to foster education in the humanities and social sciences in Asia.

Hsu received a bachelor of science degree from Tsinghua University, and then went to Peking University, where he was an assistant in the Department of Mathematics. He passed the examination in 1936, after which he went to the University of London to continue his studies. He served as a lecturer, obtaining a Ph.D. in 1938 and a Sc.D. in 1940. Thus, he remained in England for the four years 1936–1940. However, it is known that he spent some time in Paris (perhaps during the academic year 1939–1940) to study with Jacques Hadamard. Hsu then returned to China, which was again at war with Japan. (The Second Sino–Japanese War lasted from 1937 to 1945.) After his return, Hsu was appointed as professor at Peking University, which was relocated to Kunming during World War II.

Constance Reid writes in her biography of Jerzy Neyman: “The most outstanding of Neyman’s students at that time (eds. 1937–1938) was a Chinese, . . . P. L. Hsu. (Neyman expresses to me his admiration for Hsu with a Polish phrase which he translates—with a little bow and a gracious wave of the hand—as *Please sit*

Dayue Chen is Professor of Mathematics, School of Mathematical Sciences, Peking University, Beijing, China 100871 (e-mail: dayue@pku.edu.cn). Ingram Olkin is Professor Emeritus of Statistics and Education, Department of Statistics, Stanford University, Stanford, California 94305, USA (e-mail: olkin@stanford.edu).

ABSTRACT OF THESIS

BY P. L. Hsu

for Ph - 1938

I. CONTRIBUTION TO THE THEORY OF THE t-TEST
AS APPLIED TO THE PROBLEM OF TWO SAMPLES.

If two normal populations have a common variance, the "Student's" t-test for a significant departure between the two means has become classical. But can it also be used to test the hypotheses (i) that two normal populations are identical and (ii) that two normal populations have the same mean without assuming that they have the same variance? The writer answers this question by obtaining the exact distribution of "Student's" t and thence its power function in terms of the population means and variances. The answer is in the negative except when the two sample sizes are equal, in which case the t-test can be safely used for the purpose (ii) of testing for a significant departure between the two means even if the variances are unequal.

FIG. 1. Abstract of Pao-Lu Hsu's 1938 thesis, part I: "Contribution to the theory of the t-test as applied to the problem of two samples."

down!)" (Reid, 1982, page 153). The Neyman biography further notes that, "Hsu was, in Neyman's opinion, absolutely on a level with Wald—they were the two outstanding statisticians in the generation coming up!" Hsu was invited to lecture at Berkeley for six months with an appointment for the following year. Harold Hotelling was at Columbia at this time and suggested that Columbia and Berkeley join together to bring Hsu to each university for a semester. (Hsu was also sought after by Chicago and Yale.) Hsu accepted the joint offer and indicated that he preferred to first visit the West Coast.

Erich Lehmann in his autobiography lists Hsu as one of his three Ph.D. godfathers. At that time (1945) Lehmann was a doctoral student at Berkeley, and Neyman asked Hsu to give him a thesis topic:

Within a few days, Hsu presented me with a new possible topic: applying methods of Neyman, Scheffé and himself to some situations for which they had not been tried before. Hsu then got me started on this line of work. In a letter of January 24 to Neyman, about which I learned only much later, he wrote: "I have passed the problem of testing for independence between successive observations to Erich for his doctoral thesis. Will do all I had done independently, and then add a new part which I have not done. I hope this scheme will meet with your approval, so that Erich can look forward to the degree with certainty."

This was an act of greatest generosity. Hsu made me a present of work he had

II. ON THE BEST UNBIASED QUADRATIC ESTIMATE
OF THE VARIANCE.

Let σ^2 be the variance of a population or the common variance of several populations, and let x_1, x_2, \dots, x_n be the observational data. It is common practice to use a homogeneous quadratic function of the x_i as an unbiased estimate of σ^2 . Such an estimate is itself subject to sampling error. This paper solves the problem of estimating σ^2 by means of the quadratic form in the x_i whose variance in repeated sampling is the smallest possible. In particular a necessary and sufficient condition is obtained in order that the answer should be the classical residual sum of squares resulted from the least square method. Matrices are used throughout to simplify the otherwise heavy algebra.

FIG. 2. Abstract of Pao-Lu Hsu's 1938 thesis, part II: "On the best unbiased quadratic estimate of the variance."

planned to do himself and on which he had already obtained some results. I had hoped to see him on his return to Berkeley after the term at Columbia. However, this was not to be; in fact I never saw him again. (Lehmann, 2008, page 39)

When Hotelling moved to the University of North Carolina in 1946 to found a department of mathematical statistics, he offered Hsu an associate professorship. Hsu accepted the offer and spent the period 1946–1947 at Chapel Hill, but the pull to return to China was too strong, and he returned in the summer of 1947. He was committed to China, and wanted to participate in "the emerging new society in his homeland." On a trip across country (1946–1947) Neyman visited Hotelling in Chapel Hill and saw Hsu again, whom he hoped to entice to Berkeley. "He found the Chinese scholar miserably unhappy, disappointed in love, and desiring only to return to his native land." (Reid, 1982, page 214)

After his return to China, Hsu's research was unknown in the West. "Apart from his published papers and a few remarks given us by an old friend, we are unable to obtain further information about Hsu's life and work in the twenty-some years he lived in Peking" (Anderson, Chung and Lehmann, 1979). His colleagues at Peking University did not see him easily

either. As reported by Boju Jiang, who joined the department as a faculty member in 1957 (but did not meet Hsu until 1968), "Mr. Hsu was essentially a legendary hero, somewhat mysterious to us."

Hsu was the first teacher to offer courses in probability and statistics in China, from the early 1940s in Kunming. Kai-Lai Chung was a teaching assistant at that time, and became interested in probability by taking courses and discussing research with Hsu. For this reason Chung always regarded himself as a student of Hsu. Other students included Shou-Jen Wang, L.C. Hsu and Chin-long Chiang. Under Hsu's supervision, Zhong-Zhe Zhao completed graduate study in 1951 and was the first graduate student to major in probability in China. After three lectures given in the fall of 1955, Hsu could no longer teach in a classroom due to his poor health.

In 1956, probability and statistics (together with computational mathematics and differential equations) were identified as key subjects of mathematics to be developed with high priority in China. Only a few Chinese researchers knew probability and statistics at that time, and in order to produce qualified teachers at an accelerated pace, a special program was created at Peking University, with 34 juniors from PKU, 10 juniors from Nankai University in Tianjin and 10 juniors from Sun Yat-sen University in Guangzhou (Canton). In addition, some 10 teachers came from all over the country to audit the courses. Instructors were brought from the Chinese Academy of Science and Sun Yat-sen University. Hsu was a great teacher, and served as leader of the program. The curriculum he created there later became the national standard. Textbooks were compiled quickly, some based on the notes from his lectures and some translated from Russian. After a two-year training period, students were dispatched to other universities to teach probability and statistics. In some sense, all Chinese probabilists and statisticians are students or grand-students of Hsu. On teaching, he once said: "One can feel proud to be the advisor of a Nobel laureate. It means nothing just to be a student of a Nobel laureate."

Hsu continued his teaching by running seminars at his home. He began the practice informally in the early 1950s, and continued on a regular basis for eight consecutive years until 1964. This was very much in the Chinese tradition of private education in which students were required to learn by themselves and to present their findings each week for evaluation by the advisor. Quite often the seminar became a small class

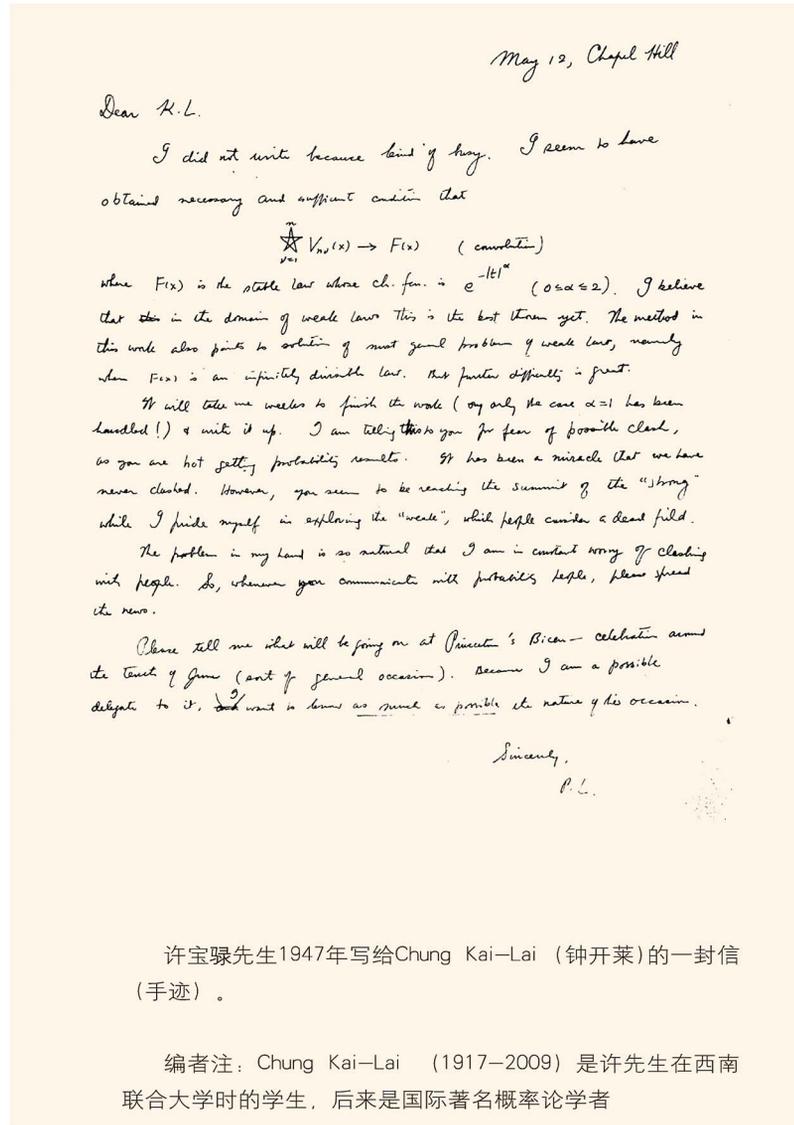


FIG. 3. Pao-Lu Hsu's handwritten letter to Kai-Lai Chung in 1947. K.-L. Chung (1917-2009) was Professor Hsu's student during the Second World War and later became a well-known probabilist.

taught by Hsu himself. Only a few people were fortunate to serve as his apprentices. During the peak period, he ran three seminars a week in the living room of his one-bedroom apartment on campus, which was about 140 square feet in size. Seminar participants were selected by Hsu himself; in contrast, his graduate students were assigned to him. Among the few photographs available today are two taken with students of his seminars in 1959 (Figures 6 and 7). The topics of the seminars covered a wide range: mathematical statistics, limit theory, Markov processes, stationary processes, experimental designs, sampling techniques, order statistics, and topology. Research conducted by members of the seminars represented the first coordi-

nated efforts in probability and statistics in China. Hsu created pen names such as Ban-cheng, Ban-guo and Ban-ji for the students to write and submit joint papers. (The Chinese word "Ban" means class.) From 1958 to 1962 he also supervised six graduate students, including Yongquan Yin.

The reader is reminded that China and the Soviet Union were in a "honeymoon" period in the 1950s, and China learned much from the Soviet Union. Following the Soviet pattern, a subdivision within the mathematics department was set up at Peking University in 1956, named *Teaching and Research Unit of Probability Theory and Mathematical Statistics*. This was the very first of its kind in China, and it evolved into an indepen-

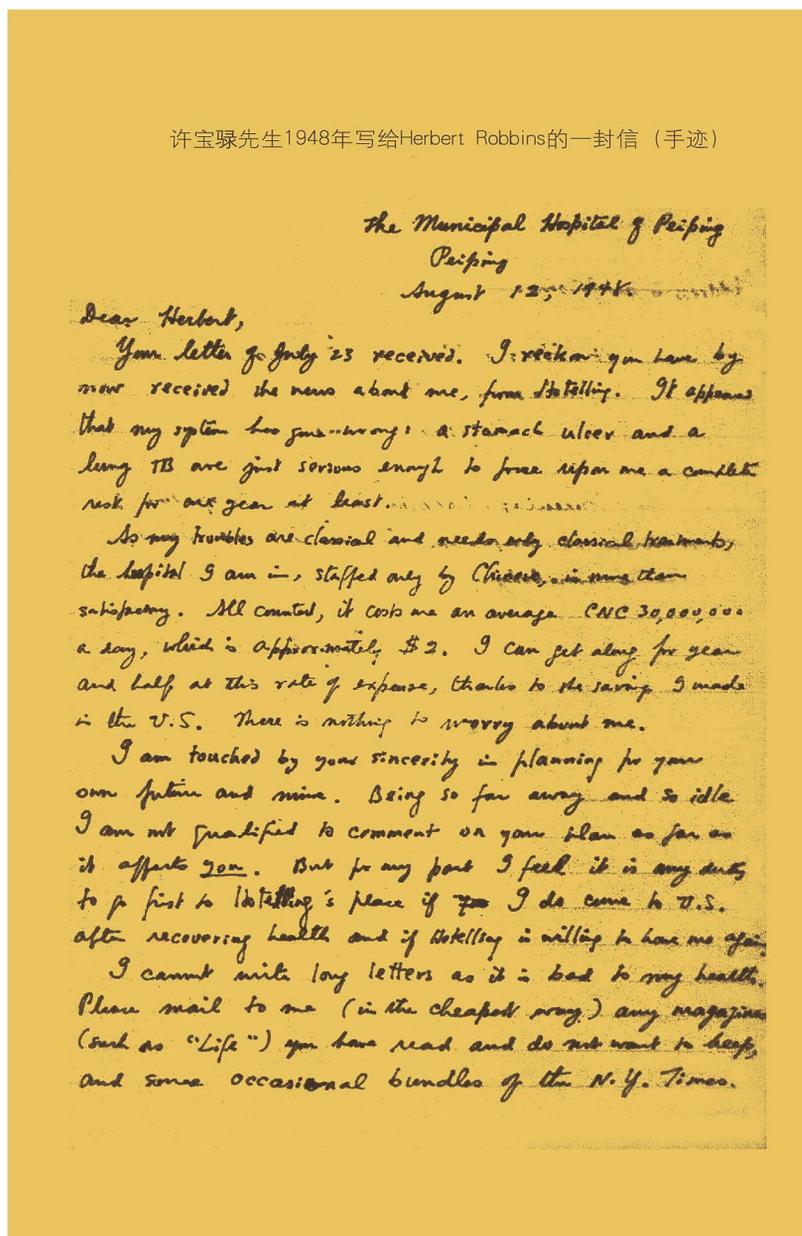
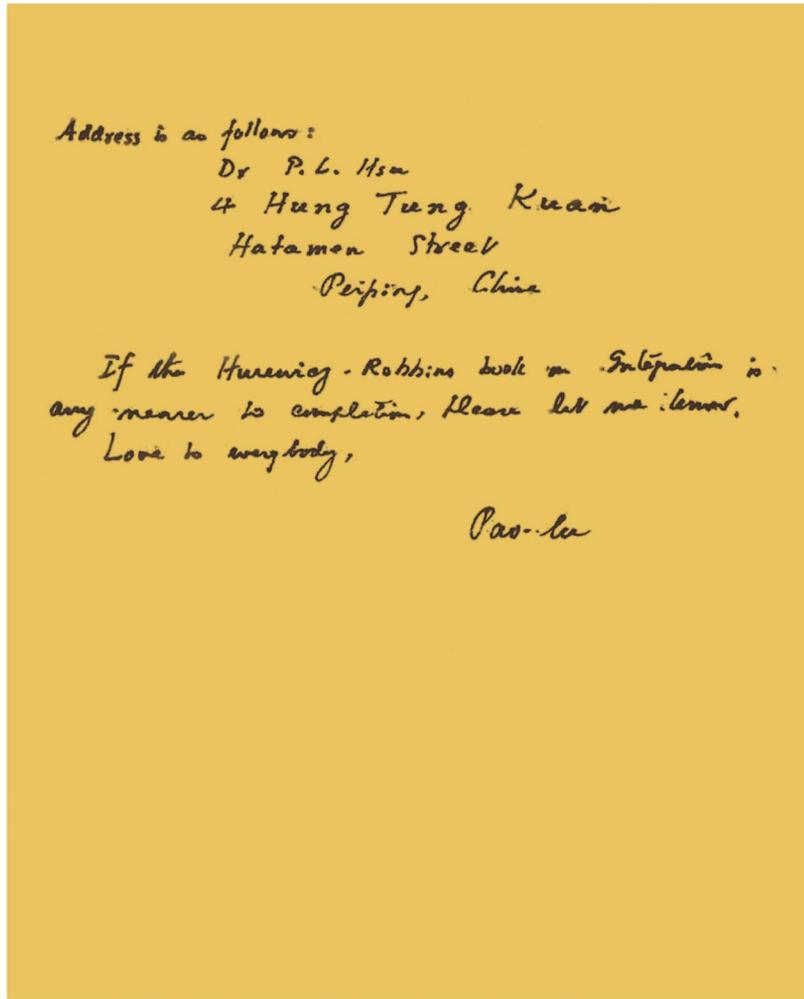


FIG. 4. Pao-Lu Hsu's handwritten letter to Herbert Robbins in 1948 (page 1 of 2).

dent department in 1985. As the founding director of the unit, Hsu watched over the career development of young colleagues, because most members only had undergraduate training. He also organized scientific exchanges with foreign colleagues, for example, Marek Fisz and Kazimierz Urbanik from Poland in 1957 and Eugene Dynkin from the USSR in 1958. Careful preparation was made before each visit in order to better understand the forthcoming lectures. Several months before these visits, Hsu would assign related papers for young faculty members and students to study.

However, although Hsu himself was immune from politics, Hsu's efforts were discounted. Seminars were ended unexpectedly because students were required to devote themselves fully to a political movement. Graduate students were selected based on political criteria and were assigned to professors, without much consideration of the academic interest and ability of the student. Some were not well prepared for graduate study. Hsu must have been annoyed by the requirement to submit a research plan each year, simply because everything had to be part of a planned economy. His so-



编者注：H. Robbins (1915–2002) 是许先生在美国任教时的同事，后来是美国国家科学院院士、著名统计学家。他一直保存着这封信，在他去世后由Stanford大学的T. L. Lai (黎子良) 教授把信转交给北京大学

FIG. 5. Hsu's 1948 letter to Robbins (page 2 of 2). Herbert Robbins (1915–2002) was Professor Hsu's colleague at Columbia University. He was an eminent statistician and a member of the US National Academy of Sciences. Robbins had kept this letter for a long time, and after his death the letter was sent to Peking University by Professor T. L. Lai of Stanford University.

lution was to propose his new papers as the research plan for the next year.

Shortly before the Communist victory in 1949, Hsu and most other professors declined the offer of Chiang Kai-shek to airlift them to southern China. He even sent a telegram to a foreign friend saying "... am happy after liberation." However, when Hsu returned from the US in 1947, China was in the middle of their civil war. His return from the UK to China in 1940 had been even worse. At that time, China was involved in

WWII and living conditions in Kunming were miserable. On May 4th of 1919, students of Peking University demonstrated in Tian-an-men Square, crying out for the adoption of principles of democracy and science. (This was the first of a number of Tian-an-men Square protests, the most recent occurring in 1989.) The May 4th Movement was a turning point in the modern history of China. It is reasonable to assume that Hsu, like many Chinese intellectuals of his generation, wished to build a strong country by introducing sci-

ence to China. This may also explain in part why Hsu submitted papers to Chinese journals.

In the 1950s Hsu wanted to create a Chinese journal in probability and statistics, as a launch pad for young researchers to publish their papers. He was prepared to subsidize the journal even with his own money. Hsu had a tremendous linguistic talent, with a command of English, German, French and Russian. He learned Russian by himself, and helped correct several textbooks by Alexander Khintchine and Vyacheslav Stepanov, translated from their original editions. Indeed, the last accomplishment of his life was to proofread a set of manuscripts scheduled for completion in one month. He looked at the task and said he could do it in ten days; he finished the job in a little over nine days.

Hsu's health had been fragile since he was young. He was 5 feet, 9 inches tall, but only weighed 88 pounds at his maximum. Because of his light weight, he was disqualified for a government fellowship to study abroad in 1933. According to medical records, he was hospitalized in 1948 and in the early 1950s, and recuperated in hospital from illnesses in 1933 and 1957. In a letter to Herbert Robbins in 1948, Hsu wrote, "It appears that my system has gone wrong, a stomach ulcer and a lung TB are just serious enough to force upon me a complete rest for one year at least." In his final decade or so he was essentially confined to bed, where he continued to read and write. He never married and lived alone.

For his important contributions, Pao-Lu Hsu was one of five mathematicians elected as Academicians in 1948. He was elected as an Academician again in 1955, along with eight other mathematicians. He was a Fellow of the Institute of Mathematical Statistics. After his death, memorial meetings were held every ten years at Peking University. In 2010, a memorial collection of papers was published, a bronze statue was dedicated, an international conference on probability and statistics was held in July, and an official commemoration was held on his centennial birthday. The Pao-Lu Hsu Lecture Series was launched at Peking University in 2009 with a roster of distinguished speakers. Tsinghua University recently also inaugurated the Pao-Lu Hsu Distinguished Lecture in Statistics and Probability, with Brad Efron as the first speaker. The P.-L. Hsu Conference on Statistical Machine Learning is now in its third year. The International Chinese Statistical Society (ICSA) has announced its intention to set up the P.-L. Hsu Award. A memorial webpage will be built as part of PKU's Department of Probability and Statistics web site, <http://www.stat.pku.edu.cn>.

2. HSU'S RESEARCH

Some insight into Hsu's views about research can be gleaned from comments made to his students, such as the following:

- "The merit of a paper is not just to get published, but is realized when it is cited repeatedly by others."
- "A good author should show the simplicity."
- "I do not want to become famous because my paper appears in a well-established journal. I wish a journal to be well established because my paper appears in that journal."

Pao-Lu Hsu authored 41 papers and three books, on a wide range of topics: limit theorems, random matrices, Markov chains, experimental designs, characteristic functions. Almost all his papers were singly authored. He published with only three coauthors: Kai Lai Chung, Tsai-han Kiang and Herbert Robbins. Hsu used the pen name Ban-cheng to publish papers with his students. As demonstrated in Table 1, his peak performance was from 1938 to 1947. In a ten-year period he wrote 22 papers, all in English. For the next 15 years (1949–1964), he wrote 13 papers of which 11 articles were published in Chinese journals. Some of his manuscripts were published posthumously, with the assistance of his students.

3. HSU'S RESEARCH IN MULTIVARIATE ANALYSIS

In addition to probability and statistics, Hsu lectured on topology, matrix theory and analysis. He did not

TABLE 1
Hsu's publications

Period	Residence	Journals		
		Chinese	Int'l	Topics
1935	Beijing	1	1	topology
1938–1940	London	1	7	statistics
1941–1945	Kunming	1	9	random matrices, limiting distributions
1946–1947	USA	0	4	matrices, complete convergence
1949–1964	Beijing	11	2	characteristic functions, experimental design, matrix transformations, Markov processes, limiting distributions
1968	Beijing	0	1	limiting distributions
Posthumously		3	0	Markov chain, random matrices, coding

have access to outside literature, and provided new proofs of some known results. He also obtained new results which still remain hidden.

3.1 The Wishart Distribution

The Wishart distribution was a focal point in Hsu's early work. He generated new derivations and obtained the distribution of the eigenvalues of the sample covariance matrix and the canonical correlations.

The joint distribution of the elements of the covariance matrix obtained from a sample of p -variate standard normal variates was obtained by Fisher in 1915 for the special case $p = 2$, and by Wishart in 1928 for general p . (Although stated for standard normal variables, by a simple transformation the variates can have a covariance matrix Σ .) What is tantalizing about this distribution is that the starting point is a set of pn ($p \leq n$) random variables in the $p \times n$ sample matrix X , and the ending point is a set of $p(p + 1)/2$ random variables in the matrix $S = XX'$. There are a number of routes and methods that might be used to make this transition. Wishart's original derivation was a geometric argument and later in 1933, together with Bartlett, he gave a derivation using characteristic functions. In 1937, Mahalanobis, Bose and Roy gave a geometric derivation of rectangular coordinates (described below), which is a stepping stone to deriving the Wishart distribution.

In 1939, Hsu gave a new derivation of the Wishart density using a clever inductive argument. The case of $p = 1$ reduces to the chi-square distribution. The essence of the induction is to go from $p - 1$ to p variables. Here Hsu uses a multivariate transformation, an area that he later developed.

In 1940 Hsu gave an algebraic derivation of rectangular coordinates which led to a general result. Suppose the joint density $p(X)$ of the qm elements of the random $q \times m$ matrix X is of the functional form $p(X) = f(XX') = f(S)$, where $S = XX'$. Let $S = TT'$, where T is lower triangular. The elements of T are called *rectangular coordinates*. Hsu gives a detailed proof that such a factorization exists; this is an example in which Hsu gives a new proof of a known theorem, in this case by Toeplitz in 1907. Hsu next obtains the Jacobian of the transformation, which leads to the joint density of the elements of T :

$$c(q, m) \prod_1^p t_{ii}^{m-i} f(TT'),$$

$$0 < t_{ii}, -\infty < t_{ij} < \infty, i \neq j,$$

where $c(q, m)$ is a normalizing constant explicitly determined.

3.2 Roots of a Determinantal Equation

In two papers of 1933 and 1936, Hotelling opened the door to consideration of the distribution of the roots of a determinantal equation $|A - \theta I| = 0$ or $|A - \theta(A + B)| = 0$, where p -dimensional random matrices A and B are independently distributed, each having a standard Wishart distribution.

In the first case suppose that the random positive definite matrix A has a density $f(A)$ that is orthogonally invariant, that is, $f(A) = g(\theta_1, \dots, \theta_p)$, where the θ 's are the eigenvalues of A . Make the transformation $A = \Delta D_\theta \Delta'$, where Δ is orthogonal and $D_\theta = \text{diag}(\theta_1, \dots, \theta_p)$ to yield the joint density of Δ and θ . Integration over the orthogonal group yields the joint distribution of the eigenvalues. At that time (1953), this integration was not very well known. Here again, Hsu found a clever way to carry out this integration. He showed that every orthogonal matrix Γ has a representation in terms of a skew-symmetric matrix Y , namely, $\Gamma = 2(I + Y)^{-1} - I$. The beauty of this transformation is that whereas the $p(p - 1)/2$ variables in Γ are not explicit, they are explicit in Y . Hsu then shows how to carry out the integration to yield a general result.

THEOREM 1. *If S is a random p -dimensional positive definite matrix with an orthogonally invariant density $f(S) = g(\theta_1, \dots, \theta_p)$, where $\theta_1 \geq \dots \geq \theta_p > 0$ are the eigenvalues of S , then the joint distribution of the eigenvalues is*

$$c(p) \prod_{i < j} (\theta_i - \theta_j) g(\theta_1, \dots, \theta_p),$$

where $c(p) = \pi^{p(p+1)/4} / \prod_1^p \Gamma(i/2)$.

To obtain the distribution of the roots of $|A - \theta(A + B)| = 0$, Hsu transforms (A, B) to (W, φ) by $A = WD_\varphi W'$, $B = WW'$, where $D_\varphi = \text{diag}(\varphi_1, \dots, \varphi_p)$. Note that $\varphi_i = \theta_i / (1 - \theta_i)$. The key problem is to evaluate the Jacobian of the transformation. Here Hsu gave an explicit derivation for $p = 3$ but stated the general result, later given in the exposition by Deemer and Olkin (1951). Hsu obtained the result for $p < n_1, n_2$ and $n_1 < p < n_2$, where n_1 and n_2 are the sample sizes that lead to A and B .

In the description above, both A and B have a central Wishart distribution. Hsu (1941) tackles the case that B has a central Wishart distribution but A has a noncentral distribution. This distribution is complicated and Hsu obtains asymptotic results. See Anderson (1979) for a more detailed description.



许宝騄先生和他指导的“统计组”（北大数学力学系1954级）师生合影。前排是许宝騄教授（中）、张尧庭老师（左二）、卢崇飞老师（左四）

FIG. 6. Pao-Lu Hsu and his group of statistics students in the Department of Mathematics and Mechanics at Peking University in 1954. In the front row are Professor Hsu (center) and his assistants Yao-Ting Zhang (second from left) and Chong-Fei Lu (second from right).

3.3 Student's t and Hotelling's T^2 Distributions

Hsu's first statistical paper was in 1938, and in it he obtained the distribution of the square of the Student's t -statistic in which the denominator is a linear combination, $as_1^2 + bs_2^2$, of the variances in the two underlying samples. This is now called the Behrens–Fisher problem.

Hotelling obtained the null distribution of the multivariate version of the Student's t -statistic in 1931, and in 1938 Hsu obtained its noncentral distribution. This

may be the earliest noncentral distribution in multivariate analysis, and was very much in the spirit of the Neyman–Pearson view. (Recall that Neyman invited Hsu to the first Berkeley Symposium in 1945. His paper was titled “The limiting distribution of functions of sample means and application to testing hypotheses.”)

Hsu also wrote about power functions of several multivariate tests, and provided a canonical form for the multivariate analysis of variance. Here the mean of the random $p \times m$ matrix Y is $EY = \Theta$, the mean of the



许宝騄先生和他指导的“排队论组”（北大数学力学系1954级）师生合影。前排是许宝騄教授（左）和胡迪鹤老师（1959年7月）

FIG. 7. Professor Pao-Lu Hsu (left), Professor Deehe Hu (right) and the Queueing Theory Discussion Group (1954 Entering Class, Department of Mathematics and Mechanics, Peking University), circa July, 1959.

random $p \times n$ matrix Z is $EZ = 0$; the rows of Y and Z are multivariate normal with a common covariance matrix Σ . The hypothesis in question is $H : \Theta = 0$.

3.4 Design of Experiments

Hsu entered into another realm, that of combinatorial analysis. Here he defines a balanced incomplete block (BIB) design in terms of a (0, 1) matrix T of dimension $v \times b$ with the properties

$$e'T = ke', \quad Te = re \quad \text{and} \quad TT' = rI + (\lambda + r)J,$$

where e is the vector of ones, and J is the matrix with all elements equal to 1. Such a BIB design is denoted by the five constants (v, b, r, k, λ) . The following is an example of his results. Suppose that a $\text{BIB}(v, *, r, k, \lambda)$ exists, Hsu shows how to obtain a $\text{BIB}(v+1, *, r, k, \lambda)$.

He then defines a *code* as a new $n \times w$ matrix M with elements $(1, -1)$. The transmitter sends only rows of M (without repetition). Let $MM' = Q = (q_{ij})$. Hsu obtains a number of results concerning the matrix Q ,

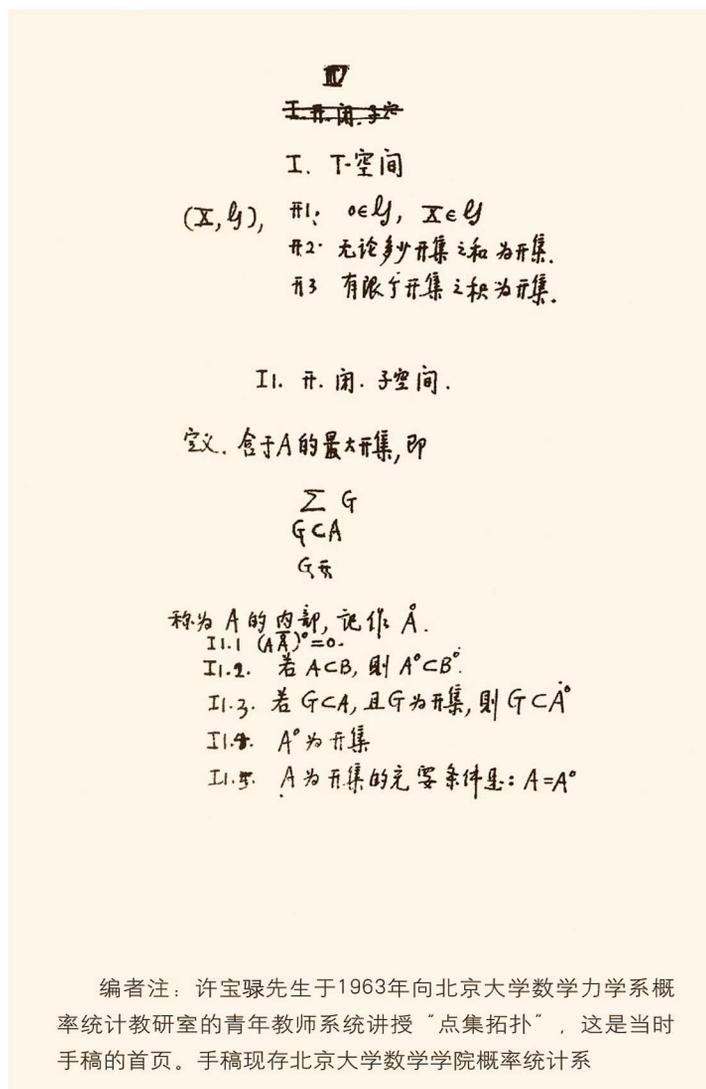


FIG. 8. First page of handwritten notes, kept at the Department of Probability and Statistics of the School of Mathematical Sciences, of Professor Pao-Lu Hsu's lectures on point-set topology in 1963 at the Department of Mathematics and Mechanics, Peking University.

such as the inequality

$$\begin{aligned} \max_{i \neq j} q_{ij} &\geq \frac{1}{n(n-1)} \sum_{i \neq j} q_{ij} \\ &\geq \begin{cases} -w/(n-1), & \text{if } n \text{ is even,} \\ -w/n, & \text{if } n \text{ is odd.} \end{cases} \end{aligned}$$

Hsu defines a *simple code* to be a matrix M for which equality is achieved, and then obtains necessary and sufficient conditions for M to be a simple code. An *orthogonal code* is one for which $MM' = wI$, where I is the identity matrix. Hsu provides an example of a 12×20 orthogonal code. It is to be noted that a $4w \times 4w$ orthogonal code is a Hadamard matrix, a subject of interest for a long time. Hsu is aware of this in-

terest (recall that he studied with Hadamard) so raises the following question. Suppose, by his construction, that we can generate an $n \times 4t$ orthogonal code: is it possible to extend this to a $4t \times 4t$ orthogonal code? He provides two counterexamples, one of dimension 4×12 and the other 12×20 . Hsu's interest in codes may stem from his study with Jacques Hademard, who connected the construction of error-correcting codes with matrices whose elements are $+1$ or -1 and whose rows are orthogonal.

4. EPILOGUE

Hsu's last paper, "BIB matrices, simple codes and orthogonal codes," was published in 1970. Zhang Yao-

ting provided this introduction for its appearance in Hsu's collected papers (page 566):

This paper was Professor Pao-Lu Hsu's last article, completed in October 1970. He died in December of the same year. The Cultural Revolution lasting already four years at the time had not yet ended. He had suffered tremendously in this calamity, and physically he was paralyzed. This paper was completed when he was bedridden. The only journal he could have access to was the *Annals of Mathematical Statistics*. It was said that when he gave this paper to Mr. H. F. Tuan, being no longer able to speak clearly, he was using his hands to express himself.

The material discussed in this article is closely related to the contents covered in his early 1966 seminars on combinatorial analysis. During the later years of his life, he was devoted to using matrices to describe and prove results in combinatorial analysis. This article is a typical representative of this idea.

ACKNOWLEDGMENT

We thank Professor Tom Fearn of University College, London, for his persistence in uncovering Hsu's history as a doctoral candidate.

REFERENCES

- ANDERSON, T. W. (1979). Hsu's work in multivariate analysis. *Ann. Statist.* **7** 474–478. [MR0527482](#)
- ANDERSON, T. W., CHUNG, K. L. and LEHMANN, E. L. (1979). Pao Lu Hsu: 1909–1970. *Ann. Statist.* **7** 467–470. [MR0527480](#)
- CHUNG, K. L. (1979). Hsu's work in probability. *Ann. Statist.* **7** 479–483. [MR0527483](#)
- DEEMER, W. L. and OLKIN, I. (1951). The Jacobians of certain matrix transformations useful in multivariate analysis. *Biometrika* **38** 345–367. [MR0047300](#)
- HSU, P. L. (1941). On the limiting distribution of roots of a determinantal equation. *J. London Math. Soc.* **16** 183–194. [MR0005576](#)
- HSU, P. L. (1983). *Collected Papers*. Springer, New York. Edited by Kai Lai Chung, with the cooperation of Ching-Shui Cheng and Tse-Pei Chiang.
- LEHMANN, E. L. (1979). Hsu's work on inference. *Ann. Statist.* **7** 471–473. [MR0527481](#)
- LEHMANN, E. L. (2008). *Reminiscences of a Statistician. The Company I Kept*. Springer, New York. [MR2367933](#)
- REID, C. (1982). *Neyman—From Life*. Springer, New York. [MR0680939](#)