

# The Role of Roguery in the History of Probability

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*Abstract.* The English literature on gambling is examined from the early sixteenth to the mid-eighteenth centuries to try to discover the relationship between gambling and the development of the probability calculus. Throughout this entire time period, there is an overwhelming preoccupation in the literature with cheating at games of chance. The acts of cheating remain constant through time. However, the methods of cheating take various forms: for example false dice, legerdemain at cards and dice, perfect shuffles and card counting. Some probability calculations begin to creep into this literature near the end of the seventeenth century. What is demonstrated in this paper is that, contrary to the accepted folklore and even though there is some evidence that gamblers did have a concept of probability, gambling itself provided very little stimulus to the development of probability theory. In the other direction, the development of the probability calculus had a profound effect on gambling, namely in the formulation of a strategy of play. These strategies, first devised by Edmond Hoyle in the mid-eighteenth century and applied initially to the card game of whist, used very simple results in probability. As a result of this historical analysis, it is necessary to reanalyze the events surrounding the emergence of probability in the seventeenth century.

*Key words and phrases:* Card and dice games, cheating, emergence of probability, gambling literature.

## 1. INTRODUCTION

It has often been said that the birthplace of probability is to be found in gambling and games of chance; see Maistrov (1974, pp. 7-8) for several sources for this idea. The usual proof of this statement is taken from the fact that many of the early probabilists, beginning with Pascal, Fermat and Huygens, analyzed games of chance through probability. The facts used in the proof are indeed true; but there are several disturbing inconsistencies:

Although lottery problems were the stock-in-trade of the probabilists, mathematicians played a largely peripheral role in designing the lotteries. (Daston, 1988, p. 144)

Marcus Aurelius was so obsessed with throwing dice to pass the time that he was regularly accompanied by his personal croupier. Less reputable gentlemen are also well documented. Someone with only a modest knowledge of probability math-

ematics could have won himself the whole of Gaul in a week. (Hacking, 1975, p. 3)

... [T]he passion for gambling was hardly an invention of the seventeenth century, and so could not have been the catalyst that transformed qualitative probabilities into quantitative ones. (Gigerenzer et al., 1989, p. 3)

If we assume, as is commonly done, that probability owes its origin to gambling, it would be necessary to explain why gambling, which had been in existence for six thousand years, did not stimulate the development of probability theory until the seventeenth century, while in that particular century the theory originated on the basis of the same games of chance. (Maistrov, 1974, p. 8)

It should be noted that several reasons have been put forward to explain why the development of the probability calculus was held up until the seventeenth century, but none of these explanations seems satisfactory. See Maistrov (1974, pp. 14-15), Hacking (1975, pp. 2-6), Garber and Zabell (1979) and Gigerenzer et al. (1989, pp. 2-3) for details. It is much more fruitful to examine what motivated the development of the probability calculus rather than what hindered it. In

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this regard, the role of gambling as a motivating force to the development of probability is examined.

To give some further insight into the question of what role gambling and games of chance have played in the development of probability, gambling literature from the sixteenth to the eighteenth century was examined. The time span of this study covers what might be called the crucial time period; it begins well before, and ends well after, the accepted time for the emergence of probability, the mid-seventeenth century.

The method used for analysis of this literature is taken from Braudel (1980) who sees three levels of history. At one level, there is an almost unchanging history, relatively constant over long periods of time. Braudel calls this the *longue durée*. In the context of this study, the *longue durée* is associated with the phenomenon of gambling through the ages. Throughout history there is a desire by many to gain an advantage, by legitimate means, or otherwise, at games of chance. There is also a desire to protect oneself by neutralizing the advantage that others may attempt to gain. This latter desire is achieved by having a fair game in which the chances of winning are equal for all who play the game. At the next level of history, there is a gently changing history of groups and of groupings. In the current context, at this level there are the various methods of cheating, and other, possibly legitimate, methods of gaining an advantage, that have come in and out of vogue as well as the responses to these methods by the gambling community. Braudel's third level, which he calls *l'histoire événementielle*, is the stuff of traditional history. These are the major events of history. In the relationship between probability theory and gambling, the major events are the emergence of probability in the mid-seventeenth century as seen in the literature of probability and the application of probability theory to gambling strategy, an event that did not occur until the mid-eighteenth century with the publications of Edmond Hoyle. Two lines of questioning are pursued in this paper. The first is the effect of the *longue durée*, the desire to gain an advantage, on the other two levels of history: the changing, but recurring, techniques of cheating and the development of probability theory. The second, and central line, is the relationship between these last two levels.

There are several ways, legitimate or illegitimate, to gain an advantage at cards and dice. In card games, the illegitimate means include marked cards and the use of sleight of hand or legerdemain to deal, for example, from the bottom of the deck. Other tricks include the use of mirrors placed on the table in order to see the faces of cards dealt face down. Among the legitimate means to gain an advantage are various card counting techniques. With regard to dicing, the usual methods of cheating include the use of false or crooked dice

and legerdemain. Legerdemain in dicing would include techniques such as sliding the dice across the table so that they do not roll in a haphazard way. In both card and dice games it is also to a player's advantage to have a thorough knowledge of the rules of the game and to have some knowledge of probability theory. Knowledge of the rules protects oneself against being put at a disadvantage by an improper manipulation of the rules; a familiarity with probability theory can enhance the strategy of play. Each of these techniques of gaining an advantage is examined in the gambling literature up to the last half of the eighteenth century.

There are several bibliographies of gambling and various games of chance. Of interest to this study are bibliographies by Jessel (1905) and Horr (1892), which were both reprinted in one volume in 1972, another bibliography by Hargrave (1930) and a more recent bibliography by Powell (1972). In addition, some material related to cheating also shows up in conjuring books; a bibliography of English conjuring books is given by Stott (1976) and a very scholarly survey of early conjuring books is given by Hall (1973).

## 2. CARDANO

Interestingly, the most complete gambling manual until the eighteenth century was written in manuscript form during the mid-sixteenth century, at a time near the beginning of the period under study. This was Cardano's *Liber de ludo aleae* (see Ore, 1953, pp. 183–241, for a translation of this work). Begun in about 1525 and revised until at least 1565, the manuscript was not published until 1663 when it was included among Cardano's collected works (Cardan, 1930, pp. 12–13; Ore, 1953, pp. 120–122).

Cardano's book contains descriptions of false dice and marked cards (Sections 7 and 17). There is a brief mention of the use of mirrors on rings worn by the players of card games (Section 17). Rules for the card game *primero* are given (Section 16). There are also descriptions of the use of legerdemain at cards and dice (Sections 17 and 30, respectively). Cardano mentions card counting techniques and gives the situation in which he does not consider this to be cheating. He says (Section 17), "Those, however, who know merely by close attention what cards they are to expect are not usually called cheats, but are reckoned to be prudent men" (Ore, 1953, p. 210). Later (Section 23) he says, "Since here we exercise judgement in an unknown matter, it follows that the memory of those cards which we have deposited or covered or left should be of some importance, and in certain games it is of the greatest importance, . . ." (Ore, 1953, p. 220). Finally, there are lengthy discussions of probability calculations related to the division of part of the stakes in the card game *primero* (Section 16), to the throw of up to three dice

(Sections 9, pp. 11–15), and to the throw of knucklebones or *astragali* (Section 31). These calculations are analyzed thoroughly in Ore (1953, pp. 143–177). What is not present in Cardano's work is the use of probability to develop a strategy of play.

A major theme throughout Cardano's book is equality. This theme is entirely consistent with the *longue durée* that has been identified in the context of gambling. At the beginning of Section 6, Cardano states:

The most fundamental principle of all in gambling is simply equal conditions, e.g. of opponents, of bystanders, of money, of situation, of the dice box, and of the die itself. To the extent which you depart from that equality, if it is in your opponent's favor, you are a fool, and if in your own, you are unjust. (Ore, 1953, p. 189)

This concept of equality is even carried over into Cardano's probability calculations. Since it would be too lengthy to explain Cardano's treatment of his concept of equality in probability and since his concept had little or no impact on further developments of the probability calculus, it is not given here. See Ore (1953, pp. 149–154) for a very good description.

### 3. THE EARLY GAMBLING LITERATURE

In this section and the next, Braudel's second level of history, the gently changing history of groups and groupings is examined. An examination of this level of history shows a consistent fascination and attention in the gambling literature over several centuries to various techniques of cheating.

A very early example in literature of cheating techniques appears in Alexander Barclay's (see Brant, 1509) translation of Sebastian Brant's *Narrenschiff*, or *The Ship of Fools*, originally written in German in 1494 and translated into Latin the following year; see Pompen (1925, p.7). Chapter 77 of the original German and Latin versions is devoted to gambling. The general tone of the chapter is very moralistic. In his translation, Barclay added a few verses of his own to this chapter, verses which include an allusion to cheating at dice by legerdemain. The original is given first; a rendition with modern spelling and some punctuation follows immediately thereafter:

Tournynge the dyse somtyme by policy  
Them falsly settinge assaynge if they may  
Some vyle auauntage for to obtayne thereby  
or  
Turning the dice, sometime by policy  
Them falsely setting, assign if they may  
Some vile advantage for to obtain thereby

The phrase "turning the dice by policy" means to have the outcome determined; "policy" means "to organize

and regulate the internal order of" (*Oxford English Dictionary*, Simpson and Weiner, 1989). Another phrase "falsely setting the dice" means an unfair throw of the dice.

By the end of the sixteenth century there was a blossoming of this type of literary reference to cheating. These references eventually included and expanded upon all the methods of cheating at cards and dice enumerated by Cardano. In fact, a whole literature grew up in England devoted to the exposure of cheating and roguery of all types. Brown (1914) attributes the growth of this literature to the rise of London in the latter part of sixteenth century as a leading city of European commerce. With the rise of London, there was an influx of people into the city. This included both foreign merchants and members of the English upper and middle classes, with the attendant influx of disposable wealth. This in turn attracted thieves and rogues of various types to try to get a "piece of this action." Discussions of Elizabethan rogues and roguery literature may be found in Aydelotte (1913), Chandler (1907), Clark (1983), Judges (1930) and Salgãdo (1977).

What follows in the remainder of this section is a description of some of the contents of this literature on roguery. Many of the techniques of cheating enumerated here remain in vogue today. For example, Maskelyne (1894) describes in a more modern context many of the same methods of cheating that were used in the sixteenth century. Note that Maskelyne's work has remained relevant today; it was reprinted by the Gambler's Book Club of Las Vegas in 1971.

#### 3.1 Cheating at Dice

Contemporary with Cardano, there appeared in England several books or pamphlets which mention cheating at dice. A very early reference for this time period appears in a book on archery by Roger Ascham (1545). In the book, there is a comparison of the relative merits as a pastime of archery and games of chance including cards and dice. In his condemnation of dicing Ascham mentions loaded dice which he describes as "dise stopped with quicksiluer and heares [hairs]." The dice with mercury in them are weighted to favor one of the faces. Hair or short bristles attached to one or more faces or edges of a die would impede the normal roll of the die on a table. Ascham also mentions other false dice generally called "dise of a vantage," and refers to one type known as "flattes." These latter dice are not perfect cubes. Further, Ascham lists various sleight of hand techniques with true dice: "if they be true dise, what shyfte wil they make to set y<sup>e</sup> one of them with slyding, with cogging, with foysting, with coytinge as they call it" (Ascham, 1545, p. 25).

A more detailed list of false dice appears in a 1552 pamphlet entitled *A manifest detection of the moste vyle and detestable use of Diceplay, and other practices*

of the same. The authorship is attributed to Gilbert Walker. Written as a dialogue between two individuals, the pamphlet also describes some games for which various false dice are useful and mentions that these dice are taken in and out of the game by sleight of hand. For example, in a game called novem-quinque, the object of the game was to stay away from 9's and 5's. This could be achieved by introducing dice called barred cater-treys. These dice were constructed so that the numbers 3 and 4 almost never showed (i.e., the trey or three and cater or four were barred from showing) and the numbers 1, 2, 5 and 6 almost always showed. To allay any suspicion, every so often a die called a flat cater-trey would be introduced. With this die the numbers 3 and 4 usually showed so that there would be about a 50% chance of throwing a 5 or 9 with one barred and one flat cater-trey.

Walker's (1552) pamphlet provides a hint of evidence that gamblers in the sixteenth century had a rough concept of probability. After one of the speakers in the dialogue has the methods of cheating explained to him, this speaker concludes, "I feel well that if a man happen to put his money in hazard, the odds is great that he shall rise a loser" (Judges, 1930, p. 44). Another piece of evidence, which is highly debatable, comes from the interpretation of a single obscure word. Within Walker's list of false dice is one called a "demi" or "demi-bar." The *Oxford English Dictionary* (see Simpson and Weiner, 1989) says that the origin of demi-bar is obscure and defines it only as a type of false die. However, Judges (1930, p. 525) interprets the word as a "false die with only half the usual bias."

Walker's (1552) pamphlet was very influential in the roguery literature which blossomed about fifty years later. For example, the use of the barred cater-trey is mentioned in Greene's (1591) *A Notable Discouery of Coosenage* (Judges, 1930, p. 123). It is also interesting to note that Greene refers to false dice as "dice of advantage" (Judges, 1930, p. 121). Walker's pamphlet was plagiarized with minor alterations and published later under the title *Mihil Mumchance* (see Anonymous, 1597). Thomas Dekker (1608) also borrowed Walker's list of false dice and Samuel Rid (1612) copied the story of the novem-quinque game.

False dice and legerdemain at dice are mentioned in a mid-seventeenth century pamphlet published under the pseudonym of Leathermore (Leathermore, 1668, 1669). Similar to the earlier rogue literature, this pamphlet was specifically written to warn the public of the monetary dangers involved in gambling. The pamphlet also contains the following probability statement, repeated in the 1669 edition, which makes little sense:

If what has been said, will not make you detest this abominable kind of life, will the almost certain losse of your Money do it? I'll undertake to demon-

strate, that 'tis ten to one you shall be a loser at the years end with constant Play upon the square. — If then 20 persons bring 200 *l.* apiece, which makes 4000 *l.* and resolve to play; for example, three or four hours a day for a year, I'll wager the Box shall have 1500 *l.* of the Money, and that 18 of the 20 persons shall be losers.

I have seen (in lower instances) three persons sit down at twelve penny *Inn* and *Inn*, and each draw 40 shillings apiece; and in little more than two hours, the Box has had 3 *l.* of the Money, and all the three Gamesters have been losers, and laugh'd at for their indiscretion. (Leathermore, 1668, p. 8)

No further development is made so that the conclusion of the ten to one odds seems to come from nowhere. The reference to the "Box" taking the money gives the impression that there is a house which takes a percentage of the game. However, in the dice game *Inn* and *Inn*, there was no house; see Cotton (1674) for a description of this game. In the context of the above quotation, the gamester whose turn it is to throw the dice could: (a) win the pot with probability 2/27; (b) add twelve pence to the pot with probability 35/54 or (c) lose the pot to the other two gamesters with probability 5/18. What the quote does show is a general awareness of the concept of probability but little or no adeptness at the calculus of probability on the part of the writer.

Although the methods may have changed, cheating to gain an advantage at dice has always been present. Mention has already been made of Maskelyne (1894) in the late nineteenth century. In the mid-seventeenth century, the Marquis of Worcester (Dircks, 1865) claimed that he had invented a dicing box which could conceal false dice. By knocking the box against a table, the fair dice would be hidden and the false dice revealed.

### 3.2 Cheating at Cards

Walker's (1552) pamphlet is devoted mainly to the exposure of methods of cheating at dice. There is also a short description (see Judges, 1930, p. 44) of cheating at cards by marking the cards and by stacking the deck. Conversely, Greene (1591) deals mainly with cheating at cards and devotes very little space to cheating at dice. The chief method that Greene exposes for cheating at cards involves several confederates and the use of legerdemain to complete the sting. There is a long description (Judges, 1930, pp. 126–130) of how the dupe (called a cony in Elizabethan terminology) is relieved of his money in a card game. What is interesting in this description is that again there is evidence that gamblers had some vague feeling for probability. Greene ventures some odds that the dupe thinks he has to win (Judges, 1930):

The cony, upon this, knowing his card is the third or fourth card, and that he has forty to one against the barnacle, pawns his rings, if he have any, his sword, his cloak, or else what he hath about him to maintain the vie [stake]; and when he laughs in his sleeve, thinking he hath fleeced the barnacle of all, then the barnacle's card comes forth and strikes such cold humour unto his heart that he sits as a man in a trance. . . . (p. 130)

The description of the game played is too obscure to be able to make some firm probability calculations. Judges's (1930, p. 499) analysis of the game, which appears to be reasonable, puts the cony's odds of winning without the legerdemain at about 10 to 1.

Other descriptions of legerdemain at cards are given in Scot (1584) and Rid (1612).

### 3.3 Card Counting

With incomplete information, card counting along with some simple calculations using probability, can be useful in developing strategies of play. Thorpe (1966) provides a very modern example of this for blackjack.

In 1583, the philosopher and ex-friar Giordano Bruno arrived in England. For a year or two prior to his arrival, Bruno had been publishing material on the art of memory (see Yates, 1966). Soon after his arrival he published in England a massive volume on memory. Bruno's techniques were soon taken up by a Scotsman named Alexander Dicson. About ten years later, Hugh Platt (1594), who had taken memory lessons from Dicson, reported that gamblers were using Dicson's technique to memorize the placement of the cards around the table. It is interesting to note that when members of the gambling community find a technique which they think might help to provide an advantage, they are quick to seize the opportunity.

Other techniques of card counting might be considered less than honest. The Marquis of Worcester in 1663 (Dircks, 1865) described a glove with knots in the fingers so that a player could keep track of the 6's, 7's and aces he has discarded during a game of primero. Edmond Hoyle (1744), on the other hand, proposed a scheme of card placement in the hand to help remember which cards had been played in whist.

### 3.4 Rule Books

In the mid-seventeenth century rule books for various card games began to appear. The reason for their publication is best put in the preface of one of the rule books itself (Anonymous, 1651): "If you therefore but observe the Rules, and Maximes here delivered, you shall avoid all the quarrels, which usually arise amongst Gamesters for want of being thoroughly informed in the Game. . . ." Further, Cardano (Section 6, see Ore, 1953) noted that distractions during a game put a

player at a disadvantage, either by ruining his concentration or by making him susceptible to the use of legerdemain.

The earliest rule book in English that has come to light so far is for a game for two players called picquet or picket (Anonymous, 1651). The game was played with 36 cards, the standard deck with the 2's, 3's, 4's and 5's removed. The game was begun by cutting for the deal. Twelve cards were dealt to each player and the remaining twelve put in a pile. The player who was not the dealer, called the elder, then had the choice of discarding from one to eight cards and then replacing them with cards from the pile. If the elder chose to discard less than eight cards, he could look at the remainder of the eight cards not chosen and replace them on the pile. Once this was done, the dealer could discard and pick from the pile. Again, there is some evidence of a concept of probability. Early in the rule book it is noted that "it is a great disadvantage to be the Dealer." In view of this disadvantage the deal was rotated at the end of each game.

There is another rule book from this period for a card game called ombre; see Anonymous (1665). This book contains the rules of the game and some advice on drawing trumps as in modern day bridge.

## 4. EIGHTEENTH AND LATE SEVENTEENTH CENTURY MANUALS ON GAMBLING AND GAMES OF CHANCE

By the late seventeenth century, the literature on gaming went in two different but related directions. Along one avenue, authors began to draw together various pieces of advice and rules for games in order to write general gambling manuals. In a slightly different direction other authors wrote biographical sketches of various cheats and sharpers. The purpose of this latter type of literature was to warn the public about these types of gamblers in an entertaining way. The former branch of this literature will mainly be followed here.

The first of these general gambling manuals was Cotton's (1674) *The Compleat Gamester*. The rules for several card and dice games are given. Crooked dice and legerdemain at dice are also described. Some of the material in this book is possibly original, but much is plagiarized from other sources. For example, Chapter I of Cotton (1674) relies heavily on Leathermore (1668), Chapter VI on the game of picket or picquet is taken directly from Anonymous (1651) and Chapter VIII on ombre is a condensation of Anonymous (1665). There is one probability calculation, and a very simple one, in this book. It is a demonstration in Chapter XXXIV that throwing a seven with two dice has greater probability than of throwing either a six or an eight. This demonstration along with an anecdote was given to advise against betting on seeing a six versus seeing a

seven or on seeing an eight versus a seven with even odds. While there have been some hints of a modest knowledge of probability in earlier works, this is the first example, other than Cardano, of a knowledge of the probability calculus in the gambling literature. In view of Cotton's penchant for borrowing his material from elsewhere, the probability calculation may actually predate 1674. The same story, with different amounts of detail and different sets of characters, is given in Lucas (1714, pp. 203–204) and Seymour (1734, pp. 71–72).

Near the end of the seventeenth century the probability calculus was beginning to make some inroads into the gambling literature. These inroads were all in the spirit of Cardano's use of probability; games were evaluated to see whether they were fair. The first major use of probability in the English gambling literature was Arbuthnot's (1692) translation of Huygens' (1657) *De Rationciniis in Ludo Aleae*. Although not in the same genre of the early roguery pamphlets and books, and later gambling manuals such as Cotton (1674), Arbuthnot's book has some of the elements of a gambling manual. In the Preface, Arbuthnot states that

[M]y Design in publishing it, was to make it of more general Use, and perhaps persuade a raw Squire, by it, to keep his Money in his Pocket; and if, upon this account, I should incur the Clamours of the Sharpers, I do not much regard it, since they are a sort of People the World is not bound to provide for.

Later in the same preface Arbuthnot says

[T]hat only which is left to me, is to wager where there are the greatest numbers of Chances, and consequently the greatest probability to gain; and the whole Art of Gaming, where there is any thing of Hazard, will be reduc'd to this at last, viz. in dubious Cases to calculate on which side there are the most Chances; and tho' this can't be done in the midst of Game precisely to an Unite, yet a Man who knows the Principles, may make such a conjecture, as will be a sufficient direction to him; and tho' it is possible, if there are any Chances against him at all, that he may lose; yet when he chuseth the safest side, he may part with his Money with more content (if there can be any at all) in such a Case.

In this latter quotation, Arbuthnot mentions, perhaps for the first time, that probability could be used to develop a strategy of play. He also notes the associated drawback—it is difficult to carry out the necessary calculations in one's head while at the gaming table.

Arbuthnot's translation of Huygens' original work was fairly popular since it went through three or four editions [see Arbuthnot (1692, 1714, 1738); the present

author has been unable to find any reference to a third edition]. Moreover, Huygens' work was also translated by two others, Harris (1710) and Browne (1714). The former translation forms an article in an early English encyclopedia of the arts and sciences; John Harris was secretary to the Royal Society. It should be noted that in the days of hand-set type, a new edition of a book did not necessarily mean a revision and/or enlargement of an earlier edition. A printer would set the type for a book and then print a certain number of copies. Since printing plates with set type would be expensive to store, the type would be disassembled after the print run so that it could be used for another book. If the book were popular so that demand exceeded supply, the type would be reset and a new edition would be printed without any change to the contents of the book. Such is the case with Arbuthnot's first two editions; they are virtually identical.

At least one of the English translations of Huygens' book on probability was viewed as competition by one of the writers of roguery literature. Lucas (1714, pp. 284–285), in a gambling anecdote, pointed out one of the shortcomings of the book for gamblers. This is that it was much more profitable to learn the techniques of cheating than it was to learn the intricacies of performing probability calculations. Moreover, Lucas may not have believed in the correctness of Huygens' calculations. Lucas relates in his anecdote:

[He] was resolv'd to try his luck at Dice; but also not finding out the Art of how many times one may undertake to throw 6 with one Dye, or at how many times one may undertake to throw 12 with 2 dice, or with how many Dice one can undertake to throw 2 sixes at the first Case, according to the 10th, 11th, and 12th Propositions of *Hugen's Treatise, De rationciniis in ludo Alea*, and finding his Rules of calculating chances most false and erroneous, he damn'd the Authour for as great a Blockhead as he was a Fool, in loosing his Money upon such conceited Whims; therefore learning the more profitable and surest way of tricking both at Cards and Dice, in which the Adversary could make no Calculation of Chances, he became so expert in the Dexterity of flipping Cards, not coggng a Dye, that in 4 Years time he was worth 6000 Pounds: But at last playing at *Hazard* with one Sir *Edward Payne*, of whom he won 560 Guineas, the losing Gentleman finding some sinister Practices in his Play, which created a Quarrel, they fought a Duel the next Morning, in which Beau *Hewit* was kill'd in Hyde-Park, in 1702; and so there was a deserved End of the Gamester. (Lucas, 1714, pp. 284–285)

Probability calculations and statements in the gambling literature began to appear regularly in the first



quarter of the eighteenth century. Some may have been influenced by Arbuthnot, Browne and Harris; others, most certainly, were influenced by DeMoivre's (1718) publication in English of the *Doctrine of Chances*. An anonymous pamphlet of 1719 on betting on the lotteries (Anonymous, 1719) was based, in part, on results from DeMoivre. Near the beginning of this pamphlet, the author states:

I have often wish'd to see this Subject [gaming] well treated of by some able Hand; in the mean time, 'till a better undertakes it, I intend to bestow a few Thoughts, once a Week, upon it, as soon as the Town fills, to put some stop, if possible, to this growing Evil. And the shortest and most effectual Method I can think of, to put an end to this Vice, will be to set in a clear and distinct Light, this great Truth that the Mathematicians have demonstrated, *viz.* That whoever has the least Advantage in Gaming, will win all the Money he plays for (for less than one Farthing Advantage in a Guinea, is enough to win all the Money in the Bank of England, if it was infinitely greater) so consequently, he that has the least Odds or Disadvantage must lose all. (Anonymous, 1719, p. 3)

The substance of this quotation refers to the problem of the duration of play, which is solved as Problem XLIII in DeMoivre (1718); see Todhunter (1865) and Hald (1990, Chap. 20) for general discussions and solutions to the problem. What is also hinted at in this quotation is that there seems to have been two solitudes, the mathematicians or probabilists in one corner and the gamblers in a totally different one. The bulk of the pamphlet is devoted to an analysis of an insurance scheme that a lottery player could buy on tickets in a government run lottery. The unknown author uses probability and expected values to show how disadvantageous it was to participate in the insurance scheme.

Another anonymously authored publication, this one appearing in 1726, used probability calculations as part of its arguments to promote fairness in games of chance; see Anonymous (1726). Many elements of the earlier gambling literature appear in this book, but in the context of the time. As before, there are descriptions of false dice, how to palm fair dice and manipulate the dice box to the dice caster's advantage and of marked cards. One new element is a description of perfect shuffles for use in the game of bassett (pp. 89–93) and a description of how to stack a deck of cards in the game of faro (pp. 63–69). Diaconis, Graham and Kantor (1983, Section 3) mention that this is the earliest reference that they could find to perfect shuffles; they also provide several references to perfect shuffles in the gambling and conjuring literature of the nineteenth and twentieth centuries. Further in Anonymous (1726), probability is used to analyze the dice game

hazard and the card game faro. In hazard, there were two players, the caster, the person throwing the dice and the setter, his opponent. Following the rules of the game and using simple probability calculations, the unknown author of this book calculated the expected return on a £100 bet to each of the players and showed that the game favored the setter. With regard to faro, the author (p. 51) says that there are three reasons why the game was popular: it was easy to learn; it appeared fair; and it was a very quiet game. He then goes on to demonstrate how the odds of winning vary throughout the game and states (p. 57) at the end that the bank, as the game was played in England, has an advantage unlike some other countries where the bank has no advantage.

One of the great plagiarists in the gambling literature is a man writing under the name of Richard Seymour. His book, eventually entitled *The Compleat Gamester*, went through eight editions between 1720 and 1754 with many revisions and enlargements. The present author has seen only three editions of this book. The second edition (Seymour, 1720) contains mostly the rules of ombre adapted from Anonymous (1665), picquet adapted from Anonymous (1651) and chess. There is some discussion of simple methods of cheating such as looking at the bottom card on the deck when the cards are cut. The fifth edition (Seymour, 1734) is greatly expanded with much more material on cheating at various games of chance. Some of the new material is definitely taken from other authors. As mentioned previously, the advice and explanation about throwing a seven versus a six or eight in the throw of two dice is taken from Cotton (1674). There are discussions of a perfect shuffle similar to Anonymous (1726). The eighth edition of *The Compleat Gamester* (Seymour, 1754) is again an expansion of previous editions. The extra material in this edition borrows heavily from Edmond Hoyle's work.

A revolution in the literature of gambling occurred in the 1740's with the publication of several books by Edmond Hoyle. A bibliography of Hoyle's work or writing based on his work to 1850 is given in Rather and Goldwater (1983). Hoyle's first book (Hoyle, 1743a, b) was on the subject of whist, the forerunner of bridge. The book opens with some basic questions of play whose solution requires probability arguments. These include, for example, finding the probability for any particular player that out of two, or three, given cards in the deck, the player's partner holds one of them. Then throughout the entire book, the simple situations and calculations given at the beginning of the book are used to explain the best strategy of play in whist. The same formula is repeated in later books on other card games, for example, quadrille (Hoyle, 1745) and picquet (Hoyle, 1746). Hoyle's books provide a solution to the problem of finding a strategy of play that Ar-

buthnot (1692) only hinted could be done. With Hoyle, a general strategy is worked out in advance of the play of the game.

Hoyle's books were so popular that they went through several editions very quickly – fifteen editions by 1770 (Hoyle, 1770). In addition, the books were widely plagiarized, including that of Seymour (1754), very soon after their initial publication. Hoyle (1743b) and Hoyle (1745, 1746, 1747), which were all bound together as one book in the copy seen by the present author, contain various notices, with threats of lawsuits, concerning the fraudulent copying of the book. One of the editions on whist was translated into French (Anonymous, 1765).

Edmond Hoyle also wrote a book which, as noted in its preface, described in plain and simple language how to carry out elementary probability calculations for use in games of chance. The book went through at least three editions (Hoyle, 1754, 1761, 1764). Ever wary of plagiarism, Hoyle placed a notice in the 1754 edition warning that the book was only a true copy if it had been signed overleaf by the printer. With the exception of Chapter 8, the book contains very simple explanations of probability calculations related to dicing games and card games such as picquet and whist. From the discussion in Chapter 8 of the book, Hoyle appears to have learned probability by reading DeMoivre's work, probably DeMoivre (1738). In the eighth chapter, Hoyle gives various numerical examples to the solution of the following lottery problem: "In a Lottery to find out the Number of Tickets which is requisite to entitle You to a Prize, upon an equality of chance." To find the answer in any particular situation, Hoyle uses the solution of Problem V of DeMoivre (1718, 1738). Hoyle provided no reasoning behind the solution probably because it involved the use of logarithms.

The use of probability and other mathematical methods to develop a strategy of play in games of chance has continued in the gambling literature to today. Several books on strategy of play today in various games have been published in the recent past; see Gardner (1980, Chap. 3) for a partial bibliography. One of the recent gambling crazes is the lotto or number lottery. Currently, *Books in Print* (see Bowker Company, 1991) lists over 70 books under the subject heading "lotteries." Of the books listed at least 25 appear to be devoted to mathematically based strategies of play.

## 5. DISCUSSION

Several observations and conclusions can be drawn from this examination of the gambling literature. The first to note is that the earlier description of the *longue durée* in gambling is confirmed. The literature of gambling from at least the sixteenth century is devoted

primarily to variations on the general themes of equality and inequality in play. Some gamblers were very quick to seize on any new scheme or method which might give them an advantage in play. The most obvious, the most profitable and with some practice, the easiest way to gain an advantage is through cheating in one form or another. The practice of cheating remained constant over the entire span of the literature reviewed; only the games played and the techniques used to cheat in these games changed over time. The story of the changing nature of the techniques to gain an advantage is Braudel's second level of history.

From the mathematical perspective, the main event of Braudel's third level of history, *l'histoire événementielle*, is the publication of Huygens' (1657) *De Ratiociniis in Ludo Aleae*. Although there is some evidence that gamblers had a concept of probability, the calculus of probability had very little impact on gambling practice until well into the eighteenth century. As evidenced by the publication of Arbuthnot's (1692, 1714, 1738) *Of the Laws of Chance*, in part a translation of Huygens (1657), there was some interest in probability theory from the gambling community. In the first quarter of the eighteenth century, there was a burst of activity in probability theory from the mathematics community. Much of this work dealt with games of chance or used cards and dice to describe the problem. Hald (1990, pp. 191–192) calls this time the "Great Leap Forward." It was at the end of this period that simple probability calculations began to creep into the English gambling literature. Although the probability literature itself progressed and became much more complicated, the calculations in the gambling literature remained at a very simple level. What led to the flowering of the use of probability calculus in gambling, simple as the application was, was the discovery by Hoyle in the mid-eighteenth century that the probability calculus could be used to develop a strategy of play during the course of a game. In the context of this analysis, this was the second main event of Braudel's *l'histoire événementielle*.

What then are the links between gambling and the development of the probability calculus? Without much further analysis of the early literature of probability only a conjecture can be given. The basis of the present conjecture is the recognition that advances in mathematics can come about in two ways. The first route to a mathematical discovery is followed when an individual is faced with a practical problem. Its solution requires the development or discovery of new mathematical tools or results. On this route the problem leads to the mathematical discovery. On the second route the discovery comes first and the applications are found afterward. Here a mathematical discovery is made only because someone finds a particular problem interesting or challenging. The motivation here is more



of the nature of philosophical satisfaction than practical consideration. On either route, the mathematical discovery is considered a major one if it opens up whole new areas of enquiry or solves several longstanding questions.

The major discovery of 1654 by Pascal and Fermat and in Huygens' work three years later was the solution to the problem of points. The problem is described succinctly by Hald (1990):

Two players A and B, agree to play a series of fair games until one of them has won a specified number of games,  $s$ , say. For some accidental reason, the play is stopped when A had won  $s_1$  and B  $s_2$  games,  $s_1$  and  $s_2$  being smaller than  $s$ . How should the stakes be divided? (p. 35)

The Pascal–Fermat and Huygens solutions seem to fall along the second route rather than the first. One piece of evidence to support this claim is that Huygens' work was originally published in a book of mathematical exercises (van Schooten, 1657). More supporting evidence comes from viewing the problem of points in a gambling context. The problem of points, as stated, does not appear to be a usual, or even practical, gambling problem since among almost all serious gamblers play would continue until the pot was won. However, there may have been a gambling origin to this problem. For example, Cardano (see Ore, 1953, pp. 174 and 208) describes a situation in the card game *primero* which is related to the problem of points. At a certain point in the game, the players have the option of dividing part of the pot. It is of interest to note that there were accepted conventions under which the division was made which were loosely related to the probability of winning. Cardano analyzed these rules and found that they actually favored the player with the smaller probability of winning. Whatever the origin of the problem of points, it appears that the problem was one of mathematical rather than practical interest.

What gambling and games of chance did provide to the mathematicians was not the motivation for work, but models under which various fairly complicated and challenging combinatorial problems could be set. Other areas, music for example (Edwards, 1987, p. 47), provided alternate, but perhaps less challenging, models for combinatorial problems.

The major breakthrough in the solution to the problem of points was the discovery of the concept of expected value. Holgate (1984) has argued that Huygens' discovery of expected value was influenced by Huygens' own work in physics on the movement of bodies after collision. In view of Holgate, one might argue flippantly that the origins of probability theory are to be found in physics. On looking beyond the flippancy, it may be argued that Holgate does point us in the right direction. It would be fruitful to look

at the emergence of probability in the context of the nature of pure mathematical discovery and the subsequent applications which are made of the discovery.

What has been done in this paper is only a fraction of what could be done. To examine the question of the development of the probability calculus as a pure mathematical discovery with subsequent applications, the early literature of probability must be reexamined. When the analysis of a game of chance is encountered in this early literature, questions such as the following must be asked: Is the calculation relevant to the play of the game or to the gamblers involved? If so, in what way? With regard to the literature on gambling, only works in English, probably only a fraction of the total literature available, have been examined. Further, several questions remain outstanding in the English gambling literature. For example: Are the probability calculations in the gambling literature correct? And are they relevant?

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