EDMUND HUSSERL: A PHILOSOPHER FOR ALL SEASONS?

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Edmund Husserl, a mathematician by training, student and later assistant of Karl Weierstrass, abandoned mathematics for philosophy upon writing his *Habilitation's* thesis in the last discipline, specifically, in what is now called 'philosophy of mathematics'. His first major work in philosophy, his *Philosophie der Arithmetik* of 1891 is an expansion of his *Habilitation's* thesis of 1887 *Über den Begriff der Zahl.* In 1900–1901 he published his philosophical masterpiece, the *Logische Untersuchungen*, a work still not completely studied by scholars, in which he dealt with problems in the philosophy of logic, philosophy of mathematics, philosophy of language, ontology, epistemology and — in the relatively unknown Sixth Logical Investigation — specifically, with the epistemology of mathematics. In later works, i.e., essentially after his turn to transcendental phenomenology, he not only continued to deal with problems in the above mentioned areas, but also dealt with problems related to the ontology of nature and cultural objects, with the problem of the so-called 'life-world' and the philosophy of history, with ethical problems and with what he considered the foundational philosophical discipline, namely, phenomenology. Very few philosophers have dealt with so many philosophical issues and in such depth as Husserl, and very few philosophers have drawn the attention of such a diversity of scholars as Husserl has.

However, there does not exist in English either a work that does justice to all these varied aspects of Husserl's philosophical enterprise, or a book that treats in depth those aspects of this enterprise of special interest to the readers of this journal, namely, Husserl's views on logic, mathematics and the epistemology of mathematics. The Smith & Smith book in *The Cambridge Companion to Philosophers* series tries to do justice to this 'philosopher for all seasons', but it falls short, very short, especially in the areas of logic, mathematics and the epistemology of mathematics.

The anthology, which seems quite uneven in quality, begins with a long introductory essay by the editors and a sort of second introductory essay by J. N. Mohanty. Both essays give a general overview of Husserl's philosophical enterprise, and they do it, in general, very well. E. g., Smith and Smith point out correctly (p. 5) that, contrary to a common opinion among philosophers — they don't name them, but we will: E. W. Beth in *The Foundations of Mathematics*, Dagfinn Føllesdal in *Husserl und Frege*, Michael Dummett in *Frege: Philosophy of Language*, Hans Sluga in *Gottlob Frege*, and Christian Thiel in the Introduction to the *Centenarausgabe* of Frege's *Die Grundlagen der Arithmetik*, among others —, Frege played an insignificant role in Husserl's abandonment of the psychologism of *Philosophie der Arithmetik* to embrace Platonism in the first volume of *Logische Untersuchungen*. On this issue, they also mention (p. 18) that as Mohanty showed in his *Husserl and Frege* of 1982, Husserl drew the distinction between sense and reference already in 1891 independently of Frege, thus, a year before the publication of Frege's 'Über Sinn und Bedeutung'. They, however, do not mention — although they should — that the reviewer also showed in a paper published in *Kant-Studien* in 1982, not only that Husserl made the distinction between sense and reference independently of Frege — it was already present, although not in its full strength, in a paper of 1890 published posthumously —, but that Frege was well aware of it. (As Claire Ortiz Hill reminded me recently in a personal communication, so aware was Frege of the coincidence between his and Husserl's distinction that in a letter of August 1919 to Paul F. Linke he borrows Husserl's examples of the *Logische Untersuchungen* to illustrate his distinction.) On the other hand, Frege must also have obtained the distinction around 1890, since it is already present in 'Funktion und Begriff', published in January of 1891, i. e., two months before Husserl's relevant work, namely, his review of the first volume of Ernst Schröder's *Vorlesungen über die Algebra der Logik*. Smith and Smith also point out (p. 10), although very timidly, some sort of connection between Husserl and Carnap. This is an issue that has not been sufficiently discussed in the literature, but when it is finally discussed thoroughly, it will become clear that Husserl's influence on Carnap was much greater than what the latter ever dared to acknowledge.

Other points Smith and Smith make in the introductory essay seem less convincing, e. g., their assertion (p. 32) that Husserl's model of knowledge formation is essentially the same as that of Quine. (And they probably think that they are doing Husserl a favor!) On this issue, they mention that Husserl recognizes various kinds of intuition and mention some of them, but fail to mention categorial intuition. A remark on p. 36 concerning higher level acts of intuition explains that omission. It seems that Smith and Smith assimilate categorial intuition to eidetic intuition, or intuition of 'essences', a conflation that could be the root of much confusion in the efforts of many Husserlian scholars to understand Husserl's conception of mathematical knowledge.

Mohanty's paper 'The Development of Husserl's Thought', although in essence still an introductory essay because of its general scope, is clearly more concerned with details than the Smith and Smith introductory essay. Mohanty's main concern in the paper is the development and the continuity of Husserl's thought from *Philosophie der Arithmetik* to *Formale und transzendentale Logik* of 1929, a work that together with the *Logische Untersuchungen* and *Ideen I* form the trilogy of Husserl's most important works. Mohanty underscores correctly (pp. 46–47) that Husserl's publications (including his *Erfahrung und Urteil* published a year after his death) are based on work done many years — sometimes more than a decade-earlier. He also stresses the continuity of Husserl's thought, which is somewhat distorted by the accounts of the development of Husserl's philosophy based on his alleged conversions. With respect to the areas of interest for us here, namely, logic and mathematics, it should be pointed out, in contrast to Mohanty, that the abandonment of a Brentanian form of psychologism between 1891 and 1894 is too significant to be ignored, even though Husserl's goal of giving an epistemological clarification of the foundations of logic and mathematics remains present from the beginning to the end of his philosophical enterprise.

There are a few other points on which we cannot agree or, at least, are inclined to disagree with Mohanty. On p. 48 Mohanty says that Husserl seems to have been working on the planned second volume of *Philosophie der Arithmetik* as late as 1894, and adds that one of the reasons for the abandonment of that project was that he realized that a universal arithmetic could not be based on the notion of cardinal number. But he seems to have known this since 1890, as attested in a letter to Carl Stumpf included as an Appendix to the first part of his posthumously published *Studien zur*

Arithmetik und Geometrie. On the other hand, although Mohanty correctly mentions (p. 53) that it was the influence of Bolzano, Lotze and Leibniz, and not Frege's, which were decisive for Husserl's rejection of psychologism, he says that Husserl developed his idea of a pure logic by the end of the 1890s. This is an important inaccuracy. The first volume of the *Logische Untersuchungen* was completed in 1896, and Chapter 11, i.e., the last one — in which Husserl offers his views on logic and mathematics — seems to have been the first to be written. It must have been finished by 1895. The final point that we want to make against Mohanty concerns his assertion on p. 69 that *Formale und transzendentale Logik* develops a philosophy of logic in a totally new direction. Moreover, he says that 'the idea of a formal ontology as the correlate of formal logic (and formal mathematics which for Husserl really belongs to formal logic) is developed by bringing in the notion of a pure deductive theory and the idea of a definite manifold . . . .' These passages are at best confusing. Firstly, apart from some change in terminology, a clearer distinction between logic as a theory of deduction and what Husserl called 'logic of truth', and the emphasis on completeness (both semantic and deductive, which he did not distinguish), the conception of logic and mathematics in *Formale und transzendentale Logik* is the same as that presented in Chapter 11 of the first volume of the *Logische Untersuchungen*, the transcendental turn notwithstanding. Moreover, contrary to what Mohanty seems to say, for Husserl mathematics is formal ontology, and the term 'formal ontology' is already used in §10 of *Ideen I*.

Other papers of the Smith and Smith volume which refer in one way or another to Husserl's conception of logic and mathematics are those of Hintikka, Simons, Tieszen and Fine. We will comment briefly on them and then make an exposition of what is totally missing in the Smith and Smith volume, namely, Husserl's views on logic and mathematics. (This is no contradiction!) Jaakko Hintikka's paper 'The Phenomenological Dimension' is an interesting and provocative one. He begins with a critique of Føllesdal's well known interpretation of Husserl's notion of 'noema' as a generalization -according to Føllesdal — of Frege's official notion of sense ('official' since we have shown elsewhere that he had two different notions of sense). On this issue it should be said that in any case Husserl generalized his own notion of sense and that such a generalization is already present in the Fifth Logical Investigation, where he speaks of the 'matter' of acts (e. g., perceptual acts) as a sort of generalization of the sense of meaning acts. Contrary to the editors, Hintikka seems to recognize (p. 87) the importance of categorial intuition in Husserl's philosophy. Although he does not discuss the notion in detail, it surfaces once more on p. 96 in his comparison of Husserl with Russell. This comparison is somewhat sketchy, but deserves some attention. He argues (p. 95) that there is a striking similarity between Husserl's and Russell's aims. Moreover, on p. 96 Hintikka says that in Russell's relatively recently published *Theory of Knowledge*, written in 1913, there is an excellent counterpart to Husserl's categorial intuition, when he asserts that we should include logical forms as a further class of objects of acquaintance. Hintikka adds (p. 96) that 'our immediate knowledge of them is the "categorial acquaintance" or "logical intuition" (to mix their terminology) which is a counterpart to Husserl's categorial intuition in Russell'. Hintikka asserts correctly (p. 100) that many of Husserl's ideas — including that of categorial intuition (see p. 101) — are more clearly understood in their application to logic and mathematics than in their application to other areas of knowledge. Apart from missing any reference by Hintikka to Claire Ortiz Hill's comparison of Husserl's and Russell's views in her *Word and Object in Husserl, Frege and Russell* (Athens, Ohio 1991), we do not have any serious misgivings with respect to Hintikka's paper, which is one of the most interesting in the Smith and Smith volume.

Kit Fine's paper 'Part-whole' is one of the few bright stars (from the standpoint of Husserl's relevance to logic and mathematics) in the Smith and Smith volume. As Fine maintains (p. 463), the Third Logical Investigation 'is perhaps the most significant treatise on the concept of part to be found in the philosophical literature'. Fine not only offers a clear exposition of Husserl's ideas on parts and wholes, filling the lacunae present in Husserl's schematic treatment and clarifying the relations between different concepts, but also develops the theory further. We think that this paper could very well serve as a basis for future developments in this area to which Husserl attached such a great importance but that has been neglected by the logico-mathematical literature. For Husserl the notions of part and whole were as fundamental mathematical notions — and, hence, as capable of a mathematical treatment — as the notions of set and cardinal number. We have only two small misgivings with respect to Fine's paper. First, we miss any mention of possible connections with other authors, e. g., the very plausible influence of Husserl on Leśniewski's mereology, or the fact that with Frege, and contra Schröder, Husserl clearly distinguished the part-whole relation from the membership relation and contra Frege considered the part-whole relation as capable of a fruitful mathematical treatment. Secondly, we miss any special discussion of what Husserl called 'extensive parts and wholes', which from the standpoint of formal structure appear as the most simple and fundamental of the parts and wholes considered by Husserl. It seems that other parts and wholes can be obtained from extensive parts and wholes by means of more structure. Fine asserts (p. 475) that in his discussion of parts and wholes Husserl seems to foreshadow the structure of a relative closure algebra and, thus, that of a relative topological space. Related commments are also made on p. 477. The relation between Husserl's ideas and general topology are worth pursuing (and there is work in progress by Barry Smith and others). Although we have not written anything about it, we have thought since the early 1970s that the most fundamental topological notions, e. g., those of a topological space, a basis for a topological space, neighborhood and cover could be defined in terms of the notions of extensive parts and wholes without any use of the notion of set. In the case that such a program could be carried out and the basic theorems derived without using the notion of set, we would obtain more conceptual clarity by knowing exactly when the notion of set is indispensable. (Perhaps such a program could even open the doors to a new foundation for mathematical analysis different both from the standard and the Robinsonian.)

In his paper 'Meaning and Language', Peter Simons tries to present Husserl's conception of language both in the *Logische Untersuchungen* and after his 'turn' to transcendental phenomenology. Although many of his observations are insightful and correct, his exposition is plagued by a fundamental misunderstanding of one of Husserl's most important semantical distinctions. Let us mention first some interesting points made by Simons in his paper. Firstly, Simons correctly mentions (p. 106) that Husserl's treatment of language is not for its own sake but to support his conception of logic. On p. 113 Simons asserts correctly that 'Husserl was the first modern philosopher to formulate the principle of congruous replacement as definitory for such [namely: meaning] categories'. Moreover, on the same page he correctly underscores that contrary to Frege, Husserl did not proclaim a correlation between dependent meanings and dependent objects, respectively, independent meanings and independent objects, since he was conscious of the existence of counterexamples to such a correlation. This, of course, as Simons observes (p. 120), helped Husserl avoid the difficulties confronted by Frege in 'Über Begriff und Gegenstand'. On p. 133 Simons argues that 'Husserl's view of the relationship between logic and mathematics is much more akin to that of model theory than to that of his logicist contemporaries like Frege and Russell'. His comparison on the same page of formal apophantics with proof theory is more or less adequate, whereas his comparison of formal ontology with model theory is, as he himself suspects, overhasty. In general, with some limitations, Simons' observations on Husserl's views on logic and mathematics are the less inadequate that the reader can find on this issue in the Smith and Smith volume.

Notwithstanding such positive aspects of Simons' paper, there is a misunderstanding of a basic distinction in Husserl's semantics of sense and reference — to use the more familiar Fregean terminology — that seems inconceivable in a scholar who is writing precisely on Husserl's semantics in a book like the Smith and Smith volume. Engaging in a bit of historiography, in the First Logical Investigation Husserl contends (as he is going to contend from there on) that the referent of a statement is a state of affairs. However, the example that he offers to illustrate his assertion that two different but related statements can refer to the same state of affairs is not only inadequate but dangerously confusing, namely, that the statements 'a is larger than b' and 'b is smaller than a' refer to the same state of affairs. This example not only does not mix well with the example he had given to illustrate the fact that two proper names can have different sense but the same referent, namely, 'the looser of Waterloo' and 'the victor of Jena' — which is completely similar to Frege's 'the morning star' and 'the evening star', and, as we mentioned above, was also used by Frege (with an unessential modification) in a letter of 1919 —, but is used by Husserl in other contexts to illustrate precisely the case in which two statements that refer to different states of affairs have in common the same situation of affairs. It seems that when Husserl wrote the First Logical Investigation (probably around 1896-1897), he had still not distinguished between states of affairs and their referential basis, namely, situations of affairs. This distinction is made for the first time in the Sixth Logical Investigation (§ 48) and is usually illustrated by Husserl by means of examples like the above mentioned. In his *Vorlesungen über Bedeutungslehre* (pp. 29-30) Husserl precisely points out that in the First Logical Investigation he had confused the notions of state of affairs and situation of affairs, which he had clearly separated later in the Sixth Logical Investigation. Simons falls on p. 112 into the same confusion as Husserl in the First Logical Investigation. But what is worse, on p. 124 he refers to a passage on p. 98 of *Vorlesungen über Bedeutungslehre* in which Husserl uses the same example — this time correctly to illustrate the case in which two statements refer to different states of affairs but these two states of affairs have the same situation of affairs as referential basis. Simons clearly confuses this Husserlian distinction with the Frege-Husserl distinction according to which two expressions can have different senses but the same referent, and probably thinks that Husserl has simply changed his terminology. Simons misses the point completely on this distinctive Husserlian distinction of the utmost importance for Husserl's semantics and beyond. We cannot dwell on this issue here, but simply briefly illustrate via examples the difference between the two distinctions. '9 – 1 > 5 + 1' and '7 + 1 > 4 + 2' are two statements with different sense but the same Husserlian state of affairs as referent, since their difference consists in the fact that the first one contains a name of the number 8, where the second contains another name of the number 8, and a name of the number 6, where the second contains another name of the number 6. Each of the two inequalities can be obtained from the other by a transformation of statements that replaces expressions with expressions having different sense but the same referent. On the other hand, '7 + 1 > 4 + 2' and '4 + 2 < 7 + 1' do not differ in the same way. They cannot be obtained from each other by such sorts of transformations. They refer to different states of affairs (in this case relations), namely, to the state of affairs that the number 8 is greater than the number 6 and, respectively, to the state of affairs that the number 6 is smaller than the number 8. Of course, those relations are inverse relations of each other, but nonetheless different relations. They have in common a sort of (abstract) proto-relation, and that is precisely what Husserl calls a 'situation of affairs'. Husserl considered this distinction of the utmost importance and fruitfulness as is shown by some remarks made in *Vorlesungen über Bedeutungslehre* (pp. 101–102) concerning its possible application to physical contexts, and the fact that it is present even in his very late *Erfahrung und Urteil* (p. *285f.*, p. *296f.*) on which he was working with the help of his assistant Landgrebe, at the time of his death. Elsewhere we have tried to exploit the importance of that distinction for the semantics of mathematical statements.

Richard Tieszen's 'Mathematics' is by far the worst paper of all the 'contributions' to the Smith and Smith volume. If Simons' confusion between state of affairs and situation of affairs was astonishing coming from a presumed specialist in Husserl's philosophy of language, it is insignificant in comparison with Tieszen's ignorance of the most basic aspects of Husserl's views on mathematics and mathematical knowledge. These facts seem more disturbing, since Tieszen appears to be the 'official' exponent of Husserl's thought on mathematical issues, as is also evidenced by his book *Mathematical Intuition*, his paper in L. Haaparanta's anthology, *Mind, Meaning and Mathematics* and his paper in M. D. Resnik's *Mathematical Objects and Mathematical Knowledge*. Before we discuss Tieszen's presentation of what he considers Husserl's views on mathematics and mathematical knowledge to be, let us begin with a positive note, mentioning Tieszen's exposition of some of the difficulties that face some better known philosophies of mathematics.

At the very beginning of his paper (p. 439) Tieszen mentions some of the well known difficulties faced by nominalists. They have to explain why it is that mathematics is 'so much different from the way it appears to be', and why it is that the language of mathematics is so misleading. A more traditional argument against nominalism mentioned on the same page by Tieszen is that 'many mathematical propositions are about infinite sets of objects' and 'they do not seem to be reducible to a language that refers only to spatio-temporal particulars (presumably finite in number)'. Against formalism, Tieszen offers (pp. 440–441) the traditional argument that if mathematics is just a "meaningless" syntax, then it is not clear 'why mathematicians are so interested in some systems of sign-configurations but not others'. Against pragmatism of the Quinean sort, Tieszen argues correctly (p. 441) that if only the practice of the applications of mathematics (e. g., to physics) can validate mathematics, then it is not clear why many areas of pure mathematics are obvious. Moreover, he points out (pp. 441–442) also correctly, that the fruitfulness (in applications) does not play any decisive role in many cases when mathematicians introduce definitions, rules or axioms. Against conventionalism, Tieszen argues (p. 442) that 'it also fails to recognize any form of evidence unique to mathematics'. Finally, against Penelope Maddy's so-called 'realism', according to which we have, e. g., sense perceptions of sets of physical objects, Tieszen argues also correctly (pp. 450–451), that she either confuses what is to be understood by sense perception or what is to count as a mathematical object. Many other arguments not mentioned by Tieszen could be offered against such philosophies of mathematics, but that does not concern us here. Let us see now what Tieszen says about Husserl.

The first point that should arouse the attention of a Husserl scholar are the references made by Tieszen to Husserl's texts. Husserl presented his mature views on logic and mathematics — and, thus, I am excluding his *Philosophie der Arithmetik*, which presents his views at most up to 1890 and is so dear to scholars who want to show how superior Frege was as a philosopher to Husserl — in Chapter XI of the first volume of *Logische Untersuchungen*, very briefly (his views on mathematics only) in §10 of *Ideen I*, also very briefly in §4 of *Erste Philosophie* and in much more detail in §§12–36 (i.e., the first three chapters) of *Formale und transzendentale Logik*. Moreover, a detailed exposition appears in the first part of his posthumously published *Einleitung in die Logik und Erkenntnistheorie*. His views on mathematical knowledge appear in a systematic way (so far as we know) only in §§40–52 and 59–66 of the Sixth Logical Investigation, whose second part, which includes all those sections, is titled 'Sensibility and Understanding' in a clear reference to Kant's former attempt at solving the same problem. It is simply astonishing that although Tieszen cites frequently the most varied parts of *Logische Untersuchungen*, *Ideen I* and

Formale und transzendentale Logik (and also other texts of very little relevance for the issue that presumably concerns him, e.g., *Philosophie als strenge Wissenschaft*), he never refers explicitly to Chapter XI of the first volume of *Logische Untersuchungen* or, specifically to §§67–70 of that chapter, or to §§ 12-36 of *Formale und transzendentale Logik*, or specifically to § 10 of *Ideen I* (although there are two general references to §§1–26 (in note 12) and to §§1–36 (in note 19)), or does he even mention the *Einleitung in die Logik und Erkenntnistheorie* or *Erste Philosophie*. Moreover, although Tieszen sometimes refers to the Sixth Logical Investigation, and even to some of the sections mentioned above (e.g., in notes 16, 19 and 24), he does not seem to have understood what Husserl is doing, namely, explaining how categorial objectualities and, in particular, mathematical objectualities, are constituted in categorial intuitions, and how it is that although categorial intuitions build ultimately on sense intuitions, mathematical objectualities are not founded on experience. But since after writing a book and several papers on the issue Tieszen still has not learnt what Husserl's views on logic and mathematics are, nor does he even know where in Husserl's writings he could learn about them, it should come as no surprise that he does not understand Husserl's views on mathematical knowledge. He tries to assimilate Husserl to a sort of constructivism in mathematics and argues repeatedly that for Husserl mathematical objects are in some (unexplained) way some sort of invariants in the phenomena of our experiences (see pp. 447-449, 451 and 455). (By the way, he also forces his constructivism into Gödel's views. Well, Gödelians should get accustomed to such unwanted company!) Although Tieszen does not explain it, this constant talk about invariants in our experience — if intelligible at all — seems to be interpretable as if Husserl were holding that we obtain mathematical objectualities by a process of eidetic variation. However, categorial intuition and categorial abstraction, i.e., the acts that intervene in the constitution of mathematical objectualities, are totally different from eidetic variation (and do not involve any process of obtaining invariants). By the way, curiously but not surprisingly, Tieszen never mentions the technical terms 'categorial intuition' and 'categorial abstraction' used by Husserl in the relevant sections of the Sixth Logical Investigation. Tieszen is in a similar situation to someone who writes a book presumably on Newtonian mechanics, cites Newton chaotically, but never mentions the second and third law of classical mechanics. In what remains of this study we will do what was not done in the Smith and Smith volume, namely, we will offer a brief exposition of Husserl's views on logic and mathematics, and on mathematical knowledge. In particular, it will be clear from our exposition that Husserl was a (sophisticated) mathematical Platonist from the *Logische Untersuchungen* onwards, and his so-called 'transcendental turn' did not push him to any sort of constructivism as superficial readers like Tieszen would like us to believe. It will also finally become clear what categorial intuition is.

It makes no essential difference for our exposition if we follow Chapter XI of the first volume of *Logische Untersuchungen*, which dates approximately from 1895, or the more detailed expositions of the first part of *Einleitung in die Logik und Erkenntnistheorie*, which is based on lectures of the winter term of 1906–1907 and is, thus, contemporary to the transcendental turn of his 1907 lectures posthumously published under the title *Die Idee der Phänomenologie*, or the first three chapters of *Formale und transzendentale Logik*, which is based on later work and published in 1929. The only two differences between the exposition in *Formale und transzendentale Logik* and the other two works are, first, a clear separation between logic as a theory of deduction and a logic of truth, and, secondly, the emphasis on some sort of completeness of his system. (More on this below.) None of these differences is fundamental nor has anything to do with the acceptance of transcendental phenomenology. We will more or less follow Husserl's more concise exposition, but will refer also to the conceptual refinement mentioned above and use freely the terminology of the later work (and of *Ideen I*).

Logic, as conceived by Husserl, consists of different strata. The most fundamental stratum is that of the pure logical grammar. In this stratum we find the fundamental meaning categories, e. g., name and proposition, and determine, first, the laws that govern the formation of elementary propositions from pre-propositional elements, and, secondly, the laws that govern the formation of complex propositions from simple propositions, laws that can be iterated indefinitely. Both sorts of laws concern not the individual expressions but the categories of expressions, and are laws directed to avoid nonsense, i.e., sequences of expressions whose totality is incapable of expressing a unitary meaning. This most basic stratum of logic, which essentially studies, as Husserl says in *Formale und transzendentale Logik*, the morphology of propositions, is nothing else than what Carnap in *Die logische Syntax der Sprache* many years later called 'syntax', and the laws mentioned by Husserl are simply what Carnap called 'formation rules'.

The second stratum of logic, which presupposes the first one and builds on it, is the stratum of the laws of deduction that allow us to infer propositions from other propositions in a purely formal way according to laws, and that protects against formal countersense (i.e., contradiction). This stratum, called by Husserl 'apophantics' or 'pure apophantics' both in *Ideen I* (§ 10) and in *Formale und transzendentale Logik*, is supposed to include all the laws of (formal) deduction. As Husserl stresses in *Einleitung in die Logik und Erkenntnistheorie* (pp. 435–436), traditional syllogistics and what we now call 'propositional logic' represent only a small, although fundamental, part of this second stratum. (However, Husserl does not explicitly mention the more advanced parts of this stratum.) Once more, it should be clear that what Husserl has in mind is nothing else than what Carnap much later called 'transformation rules'. In *Formale und transzendentale Logik*, where Husserl points out more clearly the syntactical nature (somewhat obscured by his choice of terminology) of the first two strata, Husserl distinguishes from the apophantics what he calls a 'logic of truth'. The logic of truth would add to the apophantics the notion of truth and related notions. Thus, we have here the basic distinction between syntax and semantics so dear to Carnap many years later. However, Husserl does not develop further his views on the logic of truth.

In the same way in which logic is based ultimately on the notion of meaning and the meaning categories, mathematics is based on the notion of object in its utmost generality, what Husserl called the '*Etwas überhaupt*', the something no matter what (or anything whatsoever), and the formal-ontological categories that are sorts of variations or determinations of the something no matter what. These formal-ontological categories are the fundamental blocks of mathematics, and include, e.g., the notions of set, cardinal number, ordinal number, part and whole and many others. The examples that Husserl offered are not so important except to underscore that Husserl rejected all sorts of reductionism, like logicism or set-theoreticism (and he did that as early as 1890 in a letter to Carl Stumpf as we mentioned above). The formal-ontological categories originate the most fundamental mathematical structures, fundamental since each and every mathematical structure that is not fundamental is either a variation, specialization or combination of ultimately fundamental ones. Husserl's views on mathematics are clearly very similar to those of the Bourbaki school: the structures based immediately on the formal-ontological categories play the role of 'mother structures' in Bourbaki's terminology, whereas all other mathematical structures are variations, combinations or specializations of the fundamental ones. Finally, mathematics is a sort of ontological correlate of logic, and the two unite to form a sort of mathesis universalis which includes all logico-mathematical theories in the sense that each and every logico-mathematical theory obtains its legitimacy from this *mathesis universalis*, since it has to be based on one or more of the logico-mathematical fundamental theories.

There is, however, a still higher logico-mathematical stratum, namely, what Husserl calls 'the theory of all possible forms of theories' or correlatively 'the pure doctrine of multiplicities'. This stratum investigates a priori the forms of all possible theories (or possible forms of multiplicities), their connections and the possible transformations of some theories (or multiplicities) into forms of theories (respectively, multiplicities). It should be clear, as Husserl underscores in § 69 of the first volume of *Logische Untersuchungen*, that the theorems in this uppermost stratum are of a different nature than those of the stratum immediately below. They are really metamathematical or metalogical theorems of the utmost generality. In *Formale und transzendentale Logik* Husserl explicitly advocated both a sort of semantic and deductive completeness — which he, as many others before Gödel and Tarski, did not seem to have distinguished — for this uppermost stratum of the logico-mathematical 'building'. Hence, if we were to make Husserl's views completely precise, Gödel's theorems would block the fulfillment of Husserl's most radical demands. Nonetheless, one cannot ignore that the development of logic and mathematics in this century is much more akin to Husserl's views than to those of Frege, Russell and their reductionist contemporaries. Universal algebra, general topology and category theory could be seen as partial realizations of Husserl's views on mathematics, whereas some early investigations of Tarski and others on the methodology of the deductive sciences as well as recent investigations in abstract model theory are clearly 'in the spirit' of Husserl's uppermost logico-mathematical stratum, even though the full strength of this stratum should remain forever as a Kantian regulative idea.

Let us consider now very simple empirical statements like 'The cat is under the table', 'Joe or John is at the door' or 'Peter is taller than Mary'. We say that such sentences are empirical since there are possible sense perceptions that could offer evidence that the states of affairs referred to by such sentences are or are not the case. E. g., we could 'see' that the cat is under the table or that Joe is at the door. We could also 'see' that Peter is not taller than Mary. However, even in the case of such simple empirical statements as those mentioned above there are formal components that are not correlated to any sensible component of our experience. We do not sensibly perceive any correlate of the 'under', the 'or' or the 'taller than', or the 'on' or 'at the side of', or the 'and' or the 'not'. Such formal components of statements do not have any correlate in sense perception. Nonetheless, we say that we perceive (or see) that the cat is under the table and not on the table, and we say that there are perceptions, like that of seeing Joe alone at the door, that fulfill the statement 'Joe or John is at the door' but not the statement 'Joe and John are at the door', even though we cannot sensibly perceive the correlates of the particles 'or' or 'and'. Our experience is not that of sense data correlated to objects and properties of sensible objects, but a structured experience in which categorial (or formal) components are present. They are not sensibly perceived, but built on the sensibly given or, to use Husserl's terminology, they are 'constituted' on the basis of the sensibly given. (A word of caution : Constitution does not mean construction or creation. When Husserl says that we constitute an objectuality in an act of consciousness he is simply asserting that it becomes the object to which the act is directed.) Thus, on the basis of our sense perceptions of the cat and the table we constitute the state of affairs that the cat is under the table, and on the basis of our sense perceptions of Peter and Mary we constitute the state of affairs (or relation, in this case) that Peter is taller than Mary (or that Peter is shorter than Mary). Similarly, on the basis of the objects Peter, Joe, John Mary, the cat and the table, which are all sensibly perceived, is constituted the set of objects whose members are Peter, Joe, John, Mary, the cat and the table. States of affairs, relations and sets of sensible objects are not sensible objects. They are categorial objectualities built on, i.e., constituted on the basis of, sensible objects in a categorial perception. Categorial perception is a new sort of act based on sense perception, which does not affect or distort what is sensibly given, but simply constitutes new objectualities, categorial objectualities, on the basis of the sensibly given. But for the purpose of the constitution of categorial objectualities we do not even need to have sensible objects be given in sense perception. We could very well have imagined them, i.e., they could have been given in sensible imagination. Then the corresponding categorial act in which the categorial objectualities are constituted would not be a categorial perception but a categorial imagination. 'Intuition' is for Husserl a generic term that essentially includes perception and imagination. Thus, the term 'sensible intuition' is a generic term for sensible perception and sensible imagination, and the term 'categorial intuition' is a generic term — which we shall prefer from now on — for categorial perception and categorial imagination.

Once categorial objectualities immediately based on sensible objects are constituted, i.e., categorial objectualities of the first level, we can iterate the process of constitution and constitute categorial objectualities of the second level, e.g., sets of sets of sensible objects, relations between sets of sensible objects, sets of relations between sensible objects, etc. The categorial intuitions in which such objectualities are constituted are categorial intuitions of the second level. We can iterate indefinitely the process of constitution of categorial objectualities of ever higher level, and in a categorial intuition of the n^{th} level constitute categorial objectualities of the

n^{th} level. This hierarchy of constitution of categorial objectualities of any finite level has clear similarities with simple type theory and, especially, with the iterative conception of sets, and it can be shown that neither the Russell set nor the Cantor set of all sets can be constituted in this hierarchy of categorial objectualities.

To obtain objectualities free from any trace of sensible foundation a new sort of act is indispensable. This is what Husserl called 'categorial abstraction' and which is simply a process of formalization of the categorial objectualities of any level (including the first one) to obtain mathematical objectualities. If we have a set of n concrete objects or a relation between two concrete objects, we can replace the objects by indeterminates and consider simply a set of n objects or a relation between two indeterminate objects. Although Husserl does not mention it explicitly in that context, it is clear that we can also replace the relation by an indeterminate relation with the same formal properties. By means of this process any trace of sensibility disappears and we obtain pure categorial objectualities, i.e., mathematical objectualities. Thus, we can say that for Husserl mathematical intuition is categorial intuition plus categorial abstraction. This is a clearly un-Kantian response to the Kantian problem about the possibility of mathematical knowledge as independent from experience, although all our knowledge seems to have a sort of genetic origin in our senses.

We hope that the above exposition and the reference to the relevant texts will dispel any doubt about Husserl's views on logic, mathematics and mathematical knowledge. Thus, it should be clear that he was not a constructivist of any sort and, in particular, neither a Kantian nor a Brouwerian, and although he was not a logicist or a Fregean Platonist, he was a Platonist but of a different, more refined sort than Frege. Tieszen's chaotic way of referring to all but the relevant Husserlian texts serves only the purpose of creating a strawman with very little relation to Husserl's views on the subjects that concern us here.

By the way, there is still a point that needs clarification. We have argued — and it can be verified by reading the relevant texts — that Husserl's turn to transcendental phenomenology did not produce any essential change in his views on logic and mathematics, and, especially, did not turn him away from Platonism and to constructivism. The transcendental turn simply meant that Husserl conceived transcendental phenomenology as the foundational discipline for the rest of the sciences (including logic and mathematics) and, thus, when doing phenomenology — which, by the way, was meant to be a descriptive science — all existential assumptions both of the sciences and of everyday life should be 'bracketed'. That does not mean that we are to construct or invent mathematics from the standpoint of transcendental phenomenology, but just to clarify its nature. The same happens with any other discipline. And it lies precisely in the nature of mathematics that it is ontologically commited, it is in its essence formal ontology (whereas logic is in its essence formal apophantics). The standpoint of transcendental phenomenology simply confirms Husserl's analysis in *Logische Untersuchungen* of both logic and mathematics.

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