N. N. LUZIN AS SEEN THROUGH HIS CORRESPONDENCE WITH P. A. FLORENSKY*

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Introduction

This paper presents new material about the early development of the career of the mathematician N. N. Luzin (1883–1950). This material comes from recently published correspondence from the period 1904–1922 [Demidov, Parshin, Polovinkin & Florensky, *1985*]. It shows Luzin's reactions as he encountered various mathematicians and mathematical subjects in the early part of his career. It sheds particular light on the period of indecision that he experienced before committing himself to a career in mathematics. In addition, it locates Luzin in a specific intellectual context of the period immediately preceding the Revolution of 1917.

This context is the Russian religious philosophical renaissance of the early twentieth century [Zernov *1963*]. Key to understanding this renaissance, and Luzin's involvement in it, is the identity of Luzin's partner in correspondence, the philosopher, theologian and scientist, Pavel Aleksandrovich Florensky (1882–1937). Florensky is one of the most important figures in this renaissance [Pospielovsky *1988*, 77]. His interests included both science and theology [Pospielovsky *1988*, 240–241, n. 64]. Indeed, one of his major contributions is an attempt to present a unified world view that includes both science and theology. This attempt resulted in Florensky's most famous treatise *The Pillar and Foundation of Truth*

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[Florensky *1970*], [Slesinskii *1984*], which played a central role in the relationship between Florensky and Luzin [Ford *forthcoming*].

Luzin is important not only for his mathematical results but also as a founder, together with his teacher Dmitrii Fedorovich Egorov (1869– 1931), of the Moscow school of the theory of functions [Demidov *1993*, 35], [Phillips *1978*, 277]. Out of this developed many of the successor schools which made the Soviet Union an important center for mathematical research in this century. Although Luzin and Egorov were neither theologians nor clergymen, they were mathematicians who shared the outlook of the religious philosophical renaissance [Ford *1991*]. Luzin was particularly influenced in this direction by Florensky. Florensky became a close friend of Luzin and influenced the direction of mathematics at Moscow University [Demidov *1986*].

Toward the end of the nineteenth century, materialist world views spread rapidly in Russia, eventually culminating in the Bolshevik revolution of 1917. The Russian religious philosophical renaissance, which begun around 1900, was a countervailing movement. Led by a few of the most significant Russian intellectuals, it was a movement away from the prevailing revolutionary ideology, toward a philosophical view much more sympathetic to religion [Zernov *1963*].

Prior to the Revolution, this renaissance was an experience limited primarily to a stratum of the Russian intellectual elite. After the violent attack on the Church following the Revolution, the turn from materialism to religion broadened, and by the late 1920s, had become a mass movement [Pospielovsky *1986*, 100–102], [Pospielovsky *1987*, 38–39]. It was lost sight of when the Church was practically annihilated by the "cultural revolution" of the 1930s, but it continued throughout the Soviet period and continues to this day.

The Revolution in 1917 had a decisive effect on the careers of Florensky, Luzin and Egorov. The Moscow Theological Academy, where Florensky served on the faculty, was closed in 1919. Florensky came to Moscow in 1921. There he worked in a series of new scientific organizations that were set up after the Revolution. His insistence on defending a religious perspective, for example by attending the meetings of various scientific commissions dressed in his priestly cassock, eventually lead to his arrest in 1933 [Pospielovsky *1988*, 77]. After four years in labor camps, he was shot in 1937 [Shentalinskii *1990*, 27], [Demidov & Ford *1966*, 147].

Egorov, who was the most prominent mathematician in Moscow in the 1920s, also publicly defended the Church. He was arrested in 1930 and died in exile in Kazan the following year [Ford *1991*, 30]. Luzin, who was the most important mathematician in Moscow in the 1930s, did not openly

defend his religious perspective. This did not spare him from being criticized for his relationship to Egorov and Florensky [Demidov *1993*, 47]. Suddenly, in July 1936, he was attacked on the front pages of *Pravda* [Levin *1990*], [Yushkevich *1989*]. Just as suddenly, the attack against him subsided [Demidov & Ford, 140]. Though deeply shaken, he was able to continue his career without serious consequences.

Discovery

Sergei Demidov is a Russian historian of mathematics. He and his colleagues are part of the living tradition of the Russian religious philosophical renaissance. They had been studying the works of the Russian religious philosophers since the early 1960s and were well acquainted with the work of Florensky. Only in the early 1980s, however, when Demidov was approached by the grandsons of Florensky and shown material from the Florensky family archive, did he learned of Florensky's connection with Luzin and Moscow mathematics.

Since then, Demidov and his colleagues have published various items from the Florensky archive and articles about the role of Florensky in the early development of the Moscow school of mathematics [Demidov *1986*], [Florensky *1986*], [Medvedev *1986*], [Polovinkin *1996*]. The correspondence may represent the most significant component of this material. It has been augmented with excellent footnotes, including significant quotes from other, related correspondence, and has been given an informative introduction [Demidov, Parshin & Polovinkin *1989*].

The strong influence of Florensky on Luzin was apparently initiated by Florensky's leadership in founding the Student Mathematical circle. As the correspondence reveals, Luzin credits Florensky with awakening his self-confidence as a mathematician. The correspondence also reveals the religious nature of the relationship. It shows the powerful influence Florensky had on turning Luzin toward an interest in the "general questions" of philosophy and science. It gives us an excellent picture of two important Russian intellectuals during the most formative period of their adult lives. This is especially true of Luzin, who is the author of 30 of the 39 letters published. Two more of the 39 are letters from Luzin's wife, as are others quoted in footnotes.

Although strongly influenced by Florensky, Luzin gradually develops as a mathematician quite independently of Florensky. As he gains confidence in his own abilities, he works out views that, on a number of issues, differ

substantially from those of Florensky. Thus, for example, they come to differ greatly on their views of infinity. We see stages of development in Luzin's career as a mathematician, including his early, unsuccessful attempt to resolve the continuum hypothesis, and his subsequent turn to trigonometric series.

The correspondence reveals that Luzin experienced a dramatic crisis when his materialist world view collapsed in 1905. This crisis culminated in the summer of 1908 and was resolved when Luzin embraced the religious philosophy of Florensky. This transformation of Luzin is paradigmatic of the experience of many intellectuals in Russia during this century.

The beginnings

Florensky entered Moscow University in 1900 and Luzin in 1901, both as students in mathematics and physics. The editors describe the bond of friendship which was created between them as lasting their entire lives [Demidov, Parshin & Polovinkin *1989*, 118].

In 1902 Florensky founded the student circle of the Moscow Mathematical Society, at which he gave lectures on set theory and discontinuous functions [Polovinkin *1986*]. In this he received strong support from N. Ye. Zhukovsky (1847–1921), member of the faculty of Moscow University and then head of the Moscow Mathematical Society. Zhukovsky, one of the most important mathematicians in Russia, would become famous for his research in aerodynamics. He developed a great attachment to Florensky.

Florensky's lectures helped to create an atmosphere favorable to the reception of the new theory of functions of a real variable, which was then being introduced in Moscow by B. K. Mlodzeevsky (1858–1923), a mathematics professor at Moscow University [Medvedev *1986*]. The importance of this student circle in the history of the Moscow school during this period has been noted, for example, by Esther Phillips [Phillips *1978*, 282].

Though an exceptional mathematics student, Florensky left the University upon graduation in 1904, despite the offer of a fellowship for post-graduate study. He had decided on a career as a theologian even before entering Moscow University. He entered the Moscow Theological Academy located in Sergiev Posad (renamed Zagorsk during the Soviet period), about 30 miles northeast of Moscow. Upon graduation in 1908, he was appointed directly to its faculty in the History of Philosophy. In his last year as a theology student, he wrote the treatise *On Spiritual Truth*. This was

expanded into what became his first doctoral dissertation in 1914, when it was published under the title *The Pillar and Foundation of Truth* [Florensky *1914*]. It has become one of the most famous works of modern Russian theology [Slesinski *1984*], [Zernov *1963*, 101].

Luzin graduated from Moscow University in 1905 and spent the next year abroad. He went abroad again during 1910–1914. He defended his superb thesis *Integration and Trigonometric Series* in 1916, and was, quite exceptionally, exempted from the usual second thesis. He was appointed as a regular member of the University faculty the following year.

We will cover the correspondence in chronological order, referring to each letter by date (corresponding to the old calendar). This allows us to follow the course of development of the relationship between them, including mathematical as well as theological and philosophical themes. We will mention several of the informative footnotes that the editors have added. Esther Phillips' account [Phillips *1978*] follows Luzin's activities during this period.

Mathematics and religion

The first letter was written on 31 May 1904 [Demidov, Parshin, Polovinkin & Florensky *1989*, 125], about the time that Florensky graduated from Moscow University. It was left by Luzin at the apartment of Florensky in Moscow. Luzin has just learned that Florensky has been offered a position as a post-graduate student at the University and rushes to give him the news.

> Florensky!
>
> Speaking on behalf of Nikolai Yegorovich Zhukovsky, I have good news for you. You have been admitted to post-graduate study at the University. Nikolai Yegorovich asks that you visit Professor Lakhtin as soon as possible.
>
> May God give you all the best.
>
> Your comrade Nikolai Luzin
>
> P.S. Arrived at your place at 4:57′3″ on 31 May 1904.

L. K. Lakhtin (1853–1927) was the faculty member under whom Florensky wrote his candidate's work entitled *The Idea of Discontinuity as an Element in a World View*. The introduction to this work has now been published [Florensky *1986*].

Florensky declined the offer and left for the Theological Academy in Sergiev Posad. This letter shows the enthusiasm of Luzin at the prospect of Florensky remaining at the University, noting even the exact second of his arrival at Florensky's apartment. The religious greeting is typical of their correspondence.

The following two letters [Demidov, Parshin, Polovinki, & Florensky *1989*, 127–128] are simply announcements of meetings of the student circle for September and October, which Luzin sent to Florensky. The next, from 19 November 1904, contained not only the program for November, but also a letter from Luzin. Luzin comments about a talk given by A. A. Volkov at the November meeting of the "big" Moscow Mathematical Society, reporting on a Congress of mathematicians held in Göttingen [Demidov, Parshin, Polovinkin & Florensky *1989*, 129, n. 4].

Luzin relates Volkov's discussion of an exchange involving Georg Cantor, the founder of set theory. Cantor had apparently announced that he was able to transform a segment without endpoints into a segment with endpoints, but had not provided a proof. Other mathematicians claimed that this would lead to paradoxes. Luzin gives this description of his response [Demidov, Parshin, Polovinkin & Florensky *1989*, 130], quoting Volkov.

> Cantor, on hearing this, became very nervous and declared that "this moment was for him the most critical in his life; that he would have to think further about it."

Florensky was the first to publish an account in Russian of Georg Cantor's set theory. His essay appeared in the fall of 1904 in a short lived journal, *The New Way*, that was published by the religious philosophical society, under the title "On Symbols of the Infinite (An essay on the ideas of G. Cantor)." The editors note that it contains the following statement [Demidov, Parshin, Polovinkin & Florensky *1989*, 132, n. 10].

> He [Cantor] gave up the chance for fame and glory, which, undoubtedly, he could have fully expected, refusing to spend his time

> writing the sort of things by which reputations are made, and turned away from fashionable problems (complex variables and the like)

After reading this Luzin reacts strongly in the letter of 19 November 1904 [Demidov, Parshin, Polovinkin & Florensky *1989*, 131].

> But you bought me to the verge of tears, Florensky. Why, why did you punch complex variables in the teeth? In public, no less. I was very glad that you pointed out all the nervous clamor that is going on in scholarly papers. That in fact people are forgetting the way of science and gravitating toward paths in which fame is sold like drinks to go. But I am deeply wounded by what you said about complex variables. You know very well the importance of the differential equation. You also know that all the methods known as of now for integrating it — all that exists is a bag of tricks — are only special cases of a unified general theory of integration — a theory exclusively confined to the soil of the complex variable; that we integrate an equation in order to know the values of the integral at at any point on the real line or the complex plane; that it is precisely this knowledge which an analytic theory of the differential equation will yield in the near future; that if this theory were well developed, we would know the critical points of the integrals and their values at regular points — we would know all that is necessary, all the properties, and "integration" as is it is now understood would become superfluous . . . . In view of this, I consider it impossible to call complex variables a fashionable problem. It is a problem of life, a problem as important as the problem of the transfinite. At least to those who value real people . . . .

Already at the beginning of the correspondence we see that Luzin does not hesitate to challenge Florensky on mathematical grounds. The final sentence, with its appeal on behalf of "real people," is presumably intended to challenge Florensky on his own ground, namely, concern for the well being of people. Note the expression here of Luzin's interest in classical mathematics, supporting the description of Luzin in [Phillips *1978*, 288, 295] as one who did not encourage the disparagement of classical mathematics, as some Petersburg mathematicians suspected their Moscow colleagues of doing.

This letter also offers insight to Luzin's mathematical thought. I am indebted to an anonymous referee for the translation of this complicated text,

but also for the explanation of its mathematical significance given in the following paragraph, which is quoted verbatim from his report.

This passage suggests that one cannot understand Luzin without a careful study of what he meant by *analytic*. Lebesgue had used this term to mean anything that was explicitly constructible — a natural extension of the term from Euler's time, where it applied to functions that are representable by power series. *Analysis* to Luzin (and Lebesgue also) was properly concerned with things that were analytic in this sense. In this passage Luzin seems to be expecting some constructible algorithm that would accept a differential equation as input and generate its solution as output. Then, truly, integration would hardly be needed, since solving differential equations — either directly through quadrature or indirectly through Fourier and Laplace transforms — is its main application. However, there is a difficulty here. Luzin refers to "differential equation" in the singular. Since Russian has no articles, I cannot decide whether he means a generic differential equation or a particular one that he considered central, such as the Laplace equation. Based on the context I strongly favor the former hypothesis.

In his essay on Cantor, the editors note, Florensky had made the following remark [Demidov, Parshin, Polovinkin & Florensky *1989*, 132, n. 11].

> I would like to say that the idea of potential infinity is a national idea of the aryans, especially Hindus, of actual infinity, of the semites, mainly the Hebrews.

Luzin expressed great interest in this and in the question of racial differences in mathematics. I am not aware that he ever pursued this interest. Florensky, however, was very interested in the *Kabbala*, writings of Jewish mystics. In a letter to Luzin in 1915 [Demidov, Parshin, Polovinkin & Florensky *1989*, 181], he said that he had spent the year studying it. "For example, your 'logic' [symbolic logic], of course, is nothing but a derivative of the Kabbala. But, my God, how much more lively, inspirational and powerful in the Kabbala than in the dull, drab works of Peano, Russell and others."

At least one letter of Luzin to Florensky in the spring of 1905 is lost. The next exchange of letters that has been preserved were written in the spring of 1906. The intervening period included the tumultuous period of the 1905 revolution, which had an effect on both correspondents. Though far

from being a revolutionary, Florensky was imprisoned briefly at the end of March 1906, for a sermon that he had delivered in protest against the execution of one of the leaders of the revolution [Demidov, Parshin, Polovinkin & Florensky *1989*, 134, n. 2]. Shortly after being released, he writes to Luzin on 3 April 1906 [Demidov, Parshin, Polovinkin & Florensky *1989*, 132–134]. By that time, Luzin is in Paris during his first commission to study abroad, and is in the midst of a deep spiritual and emotional crisis which had begun the previous year.

Florensky does not mention his arrest. He alludes to "the troubled conditions in our country" and "personal difficulties" which made it impossible for him to write, "though I wanted to many times." It is now possible to write because he has been given Luzin address in Paris by a mutual friend, Dobritsky. He writes that he is unable to do any science and indeed in the troubled times he has given up research to work on "practical matters" and "general causes." Although troubled by the political turmoil, even worse, for Florensky, is the condition of the Church. He reaffirms his decision to give up science and "to abandon everything, even life, . . . for the sake of religious interests."He asks Luzin about his activities in Paris.

> So, tell me, how are you, what are you studying and, above all, where do your interests lie. Dobritsky writes that you are studying philosophy and the like. Is it true? I would be glad if it is. And is mathematics progressing? In Paris, probably, you are listening to the likes of Poincaré and others, and so you will return as a supermodern mathematician. . . .
>
> Christ be with you! Best wishes as you ponder your thoughts and may they lead you to The Source of all truth, to complete Truth.

Spiritual crisis

Luzin's response comes three weeks later, on 1 May 1906 [Demidov, Parshin, Polovinkin & Florensky *1989*, 135–138]. In a long and emotional letter, Luzin pours out to Florensky all the agony and turmoil that have overtaken him during the past year. Luzin in the throes of an emotional and spiritual crisis that will continue for at least three years and reach its culmination in the summer of 1908. This letter is quoted at length in [Ford, *forthcoming*], where this crisis is discussed in detail.

The materialistic world view that Luzin had formerly held has collapsed, and with it much of his faith in science. No longer able to be satisfied by

science alone, Luzin had seriously considered transferring to the medical school, but decided against it. He now intends, upon return to Moscow, to attend lectures in the Department of History and Philology. He is convinced that he can no longer live on just mathematics and natural science. He does, however, retain some interest in mathematics.

> At the moment my scholarly interests are in *principles, symbolic logic, and set theory*. But I cannot live by science alone.

Luzin concludes his letter with a description of his mathematical studies in Paris. This is interesting for his impressions of mathematics in Paris at that time, but shows no steady direction in Luzin's mathematical thought as yet.

> What to write you about Paris? . . . In science, its an old trend: developing the method of successive approximations, complex variables, Fourier series; there is particular zeal for integrating the systems of equations needed for theoretical mechanics and physics. Poincaré is occupied with his celestial mechanics and (very probably, I am afraid to say) suspicious of set theory. Borel is studying it, but from a peculiar point of view: he incarnates everything in geometric groups. He believes that the *continuum* cannot be *well ordered*. I am pleased by the trend that started here in the Department of History and Philology of studying mathematics and its logic. Its talented teacher is Louis Couturat. Having joined with the British school of Russell, he is a strong defender of the theory of *sets* and regards it as the foundation of mathematics. This trend interests me very much, this trend toward symbolic logic, although . . . God only knows what it assumes! Well, I thank you with all my heart for your letter! All the best to you! May the Lord keep you!

Here we see the influence of Florensky in Luzin's interest in the foundations of mathematics and set theory. More significant is his influence in turning Luzin toward the "general questions of life." His most fundamental influence, however, will come in 1908 when Luzin's crisis reaches its peak.

Luzin's next letter is from 24 January 1907 [Demidov, Parshin, Polovinkin & Florensky *1989*, 139–140]. (The month is printed as February, apparently a misprint. The first footnote states, correctly, that it arrived in Sergiev Posad on 25.1.07.) Luzin states that he is sending books

by Couturat and Lucas. The editors [Demidov, Parshin, Polovinkin & Florensky *1989*, 140, n. 1] suggest that the Couturat book referred to might be *The Principles of Mathematics*, to which Florensky makes reference on page 688 of his *Pillar and Foundation of Truth*. As a postscript, Luzin offers heartfelt thanks to Florensky for sending along a letter from Egorov.

Florensky responds on 9 February 1907 [Demidov, Parshin, Polovinkin & Florensky *1989*, 140–141]. He asks Luzin to convey his "cordial greetings" to Egorov and Zhukovsky, especially to the latter, who had special affection for Florensky. Florensky wants to convey greetings to Lakhtin and Mlodzeevsky as well, "but I don't dare, because I feel that they have taken a great dislike for me." This may suggest that these two professors had become suspicious of the influence of Florensky on Luzin. Luzin later refers directly to such suspicions on the part of Mlodzeevsky in his letter of 19 November 1908, as will be noted below.

In the next letter, of 23 May 1907 [Demidov, Parshin, Polovinkin & Florensky *1989*, 141], Luzin asks Florensky for a "big favor:" to purchase for him a copy of the book *A Revelation in Tempest and Storm: A History of the Origin of the Apocalypse*. Written by N. A. Morozov (1854–1946), it was published in 1907, shortly after his release from a long imprisonment. Luzin asks this favor because it has already sold out in Moscow. The editors note that a criticism of this book was published by V. F. Ern, a friend of Florensky [Demidov, Parshin, Polovinkin & Florensky *1989*, 142, n. 1].

Morozov had been imprisoned from 1881 to 1905 for his revolutionary activities. An amateur scientist, Morozov had written a letter about the fourth dimension to his fellow prisoners in 1891, which was published after his release. He connected the fourth dimension with spiritualism and, on the basis of his disdain for spiritualists, decided against the real existence of a fourth dimension of space [Henderson *1983*, 246].

Luzin writes again to Florensky on 14 March 1908 Florensky [Demidov, Parshin, Polovinkin & Florensky *1989*, 142–143]. He is studying some mathematics, looking over journals in history and philology, and looking over "news of the theory of electrons." He makes clear that Florensky's influence has led him to this search for a perspective much broader than science.

> I can not become a specialist: I have been poisoned by you. I thank you for that.

He writes again a month later, on 11 April 1908 Florensky [Demidov, Parshin, Polovinkin & Florensky *1989*, 144–145]. He has been drawn in to the study of number theory "a mysterious area that envelopes me deeper and deeper." He reports that he has gotten married. His wife will develop a great appreciation of Florensky and his importance to Luzin. During the summer, Luzin writes to his wife about a visit he made to Florensky [Demidov, Parshin & Polovinkin *1989*, 149–151, n. 2]. Among the questions he discussed with Florensky was why God is a trinity, why precisely three? Luzin also gives a rather critical assessment of Florensky's work in mathematics. Although he derived "beautiful analogies" from mathematics, wrote Luzin, Florensky produced no concrete mathematical results [Ford, *forthcoming*].

The crisis that Luzin has been experiencing reaches a climax in the summer of 1908. It is resolved when Luzin has a decisive encounter with (the first version of) Florensky's treatise *The Pillar and Foundation of Truth*. Luzin writes about its overpowering impact on him in a letter to his wife on 29 June 1908 [Demidov, Parshin, Polovinkin & Florensky *1989*, 145–146, n. 3]. This letter is quoted at some length in [Ford, *forthcoming*], where the crisis and its resolution are discussed in more detail. In a letter written shortly after, on 15 July 1908 [Demidov, Parshin, Polovinkin & Florensky *1989*, 148–149], Luzin expresses his profound gratitude directly to Florensky, thanking him for his personal support when Luzin came to him on the verge of suicide.

Finding a mathematical vocation

After this, the crisis seems to recede, and some mathematical direction begins to appear in Luzin's letters. It seems that the impact of reading Florensky's thesis has led to some resolution of Luzin's spiritual crisis, and enabled him to move on in mathematics.

In a letter to Florensky of 19 November 1908, Luzin discusses conflicts within the Mathematics faculty concerning foundations of mathematics and the nature of proof, conflicts that have developed since the introduction of Cantor's set theory [Demidov, Parshin, Polovinkin & Florensky *1989*, 152–153].

> What can I tell you about our life? Among the teachers one notices confusion over the poisonous ideas of Cantor. One full professor

> complains: "At the present time it is unusually difficult to work as a researcher and as a teacher. If I am the former, then how am I to deliver lectures? To lecture with all the reservations, with all the strictures is impossible, it would be incomprehensible; but without them, I could not bring myself to give deliberately childish proofs. So we have to establish a series of provable postulates, comforting our listeners, so that when they mature they can be given proofs, based on set theory. But if I work as a researcher, it is even worse. Before taking a step, I must justify it, and as a consequence you never get beyond the first step . . . ." That is a picture for you of the contemporary mathematical interregnum. "The deterioration of minds," as B. K. Mlodzeevsky puts it.

Mlodzeevsky was the faculty member who had first introduced the new theory of functions of a real variable at Moscow University in 1900. He apparently disliked the direction of Florensky's influence on Luzin. Florensky was aware of this and had already mentioned it in the letter to Luzin on 9 February 1907, as noted above.

> By the way, I plead guilty to not having passed your greetings on to him —I was afraid. B. K., having learned that I see you from God knows where, frowns on my "generalizing" ideas, general arguments which I am not willing to do without in my work. ("There now, your friend was also interested in such ideas . . . .")

He also relays greetings from fellow students I. I. Zhegalkin and P. V. Preobrazhensky, and from Professor N. E. Zhukovsky.

This letter, nevertheless, shows Luzin's increasing interest in mathematics. He states that he has been reading Kummer's ideal theory and is "delighted." He then proceeds with a long presentation of its basic ideas, concluding with [Demidov, Parshin, Polovinkin & Florensky *1989*, 156], "I am unbelievably amused by it and could not resist sharing it with you."

In the spring of 1909 Luzin is still very much interested in "general questions." Florensky had sent him a copy of his inaugural lecture upon assuming his faculty position at the Moscow Theological Academy. Responding to this he writes on 7 April 1909 [Demidov, Parshin, Polovinkin & Florensky *1989*, 156–157] "I read it, I like it very much, though I am frightened by its deepness and the daring thoughts in some of its parts."

Luzin remarks that he is interested in Herman Grassmann's *Studies on Linear Extension* and that he wants to construct a projective geometry for transcendental curves. The editors note that such a theory was later constructed in the 1930s by L. Alfors and H. and I. Weyl under the name of the theory of holomorphic (meromorphic) curves [Demidov, Parshin, Polovinkin & Florensky *1989*, 157, n. 3].

In the next letter, of 12 April 1909 [Demidov, Parshin, Polovinkin & Florensky *1989*, 157–158], Luzin asks Florensky's advice about reading Plotinus, a neoplatonist philosopher who greatly interested Florensky.

> My goal: to become acquainted with the world view of the mystic, the strong man, who is no stranger to the deep logical work required for a real world view. It seems to me that, since Plotinus is at the border between the flowering of logic in Hellenism and Christian mysticism, he would be more intelligible and closer to me than the mysticism of the middle ages. I don't know the new "systems" of mystical philosophy at all.

Luzin begins this letter with a request to borrow Florensky's copy of *A System of Philosophy* by Hermann Cohen (1842–1918), head of the Marburg school of Neo-Kantianism. He is asking on behalf of someone else, not for himself. Florensky apparently owned a copy, since the editors note that he had referred to it on page 709 of his *Pillar and Foundation of Truth* [Demidov, Parshin, Polovinkin & Florensky *1989*, 159, n. 1].

Luzin is in the process of taking his exams in mathematics. He has passed two, is taking a third, and has two remaining: mechanics and differential equations.

> I long for the sun, the blue sky, whimsical things, stark clouds. I miss the water. I am tired of examinations, not so much of them as of hurrying to catch up on the material. The exams are made much easier by the professors giving us choice over their form. I am taking them with Byushgens. Byushgens is only sporadically studying mathematics, because, apparently, he is systematically studying philosophy. He is very attracted to "the philosophy of chaos."

Luzin plans over the summer to work on his own inaugural lectures. This is in preparation for his appointment as *privat-docent* at the Uni-

versity [Phillips *1978*, 283]. He also plans to work on two mathematical projects: "The possibility of a projective geometry of transcendental curves" and "The continuum problem." The latter was to become a major project, on which his lack of success would lead him to despair. This interest in, and despair over, "the continuum problem" is confirmed by the research of Roger Cooke on Luzin's diaries [Cooke *1993*]. By the end of the year he has changed direction again. He writes, on 24 December 1909 [Demidov, Parshin, Polovinkin & Florensky *1989*, 159], to say that he has "lost . . . faith in symbolic logic."

This letter shows, on the one hand, that his interests have turned back to mathematics, but on the other hand, that he has yet to find a mathematical direction He still has one examination to take, the one in mechanics.

> Beyond that, I don't know what to do. I have lost my interest and faith in symbolic logic. I am drawn to the theory of functions and the theory of electrons. But, meanwhile, I have not decided what to study. I am passively skimming over mechanics. I was tired of doing nothing. . . . I hope to cheer up after passing the last exam.

Luzin opens this letter with the remark that he has obtained an article of Minkowski that he had promised to get for Florensky, about the equation 1 sec. = 1 cm $\sqrt{-1}$. The editors think this may refer to a note about the metric in Minkowski space being transformed into imaginary time 1909 [Demidov, Parshin, Polovinkin & Florensky *1989*, 160, n. 1]. Florensky later made use of such ideas in his book *The Imaginary in Geometry*, 1922.

Trigonometric series

Luzin writes next on 22 September 1910 [Demidov, Parshin, Polovinkin & Florensky *1989*, 160–162] to announce that he is leaving to go abroad for two years. The editors note [Demidov, Parshin, Polovinkin & Florensky *1989*, 162, n. 1] that he had passed his examinations "brilliantly" and that he was confirmed as *privat-docent* at the beginning of 1911. His commission abroad would be extended until 1914. From the winter of 1911 until the late spring of 1914 he lives with his wife in Göttingen and Paris.

Luzin explains that he has been sick most of the summer of 1910 and was hospitalized for almost two months. This has prevented him from doing any mathematics. He has read only one book the whole summer [Demidov, Parshin, Polovinkin & Florensky *1989*, 161], *The Variety of Religious Experience* by William James, which had just appeared in Russian translation. "I am very pleased by its rich content and the versatile world view of its author."

Luzin then describes his current mathematical interest in trigonometric series, the work which was to lead to his famous dissertation *Integration and Trigonometric Series*. He also describes some of the negative consequences of his unsuccessful attempt to resolve the continuum hypothesis.

> I have just begun some work in the area of trigonometric series, which for me are just symbols satisfying some kind of general laws of sequences, which I can sense but have yet to describe. Trigonometric series, however, are very interesting in their own right. Thus, for example, the series
>
> $$\sum_{n=1}^{\infty} \frac{\sin nx}{\log n}$$
>
> converges everywhere and, consequently, represents a function $F(x)$. But it so happens that this is not a Fourier series. Nevertheless, until recently it was thought that if any $F(x)$ is decomposable as a trigonometric series, then it is just a Fourier series, that is its coefficients are given as follows [Luzin then gives the standard equations for the coefficients of a Fourier series in terms of the function].
> The cause of the error, of course, is that to express the coefficients of the trigonometric series by the formulas of Fourier implicitly assumes the validity of some operations with series that don't hold.
>
> There are other things in trigonometric series that are just as interesting.
>
> I have discovered the sad fact, that, while studying set theory, I lagged behind in other areas; in many of them I understand nothing at all, though some I understand, but, one could say, incorrectly. It turns out that, while studying science, I became unscientific! I think I will close the gap during the next two years. My only fear is not knowing languages (conversational).

This is to be a good prescription of his time abroad, as the correspondence will make clear. He writes to Florensky one more time, on 6 October 1910

[Demidov, Parshin, Polovinkin, & Florensky *1989*, 162–163], before going abroad. He thanks Florensky for sending a copy of his recently published book of folksongs and congratulates Florensky on his marriage the previous August.

His next letter is a short one from Paris, dated 23 December 1911 [Demidov, Parshin, Polovinkin & Florensky *1989*, 163], describing himself as "unbelievably depressed" by his situation in Paris. He writes a much longer and more informative letter on 6 April 1912 [Demidov, Parshin, Polovinkin, & Florensky *1989*, 164–170]. The current mathematical fashion in Paris, he says, is integral equations and he muses on their use in describing "the universe." He then ponders the relationship between science and truth, ending with this. "Thought is what shines amid the eternally long night. And that light is, for us people, *everything*."

Taking note of the political turmoil at Moscow University, Luzin expresses his wish to stay abroad as long as possible in order to continue to focus only on his mathematical work. This refers, as the editors note [Demidov & Ford *1966*, 170, n. 6], to the "Kassso affair." In 1911, as a protest against actions by the government minister L. A. Kasso, about 100 teachers left the University. Included among these were the mathematicians B. K. Mlodzeevsky, S. A. Chaplygin (1869–1942) a collaborator of Zhukovsky and future academician, I. I. Zhegalkin (1869–1947), and S. P. Finikov (1883–1964) a geometer and close collaborator of Egorov.

Luzin then turns to his work on trigonometric series, which is almost complete, and describes the beneficial effects of this achievement [Demidov, Parshin, Polovinkin & Florensky *1989*, 166–167].

> During the time of my commission to study abroad, I have been engaged in work on the trigonometric series of Fourier, which is almost complete. Perhaps it will be my first dissertation. I undertook this work primarily for myself, because after my failure with the Continuum problem, I simply wanted to test my strength and clarify the causes of my failure (their origins). Because this work has not been done by chance, by superficial analogies, and by "inspiration" (as I tried earlier), but by intense effort, and is almost completely finished (I am now finishing), I have received some compliments and feel better about myself (spiritually). If it is successfully completed, then I will no longer be nervous and can work peacefully. For a long time I have suffered from absolute insecurity. The absence of a mathematical school and simply the possibility to work have been great hindrances. For this reason, I am much obliged to the Germans (Hilbert, Landau).

> Taken as a whole, all that has happened to me (during the past 4 years) can now be viewed as an endless series of question marks and continual, difficult bewilderment. In addition to spiritual causes (I. e. the fact that I did not work as I should have), there are probably physical causes. . . .

He gives the idea of his work.

> The idea, the proof of which I have *almost* completed is to give hypotheses under which "every Fourier series is a series converging everywhere except at isolated points."
>
> These conditions completely eliminate the troublesome investigation, necessary up until now, about the convergence of these series.
>
> As for non-Fourier series, I was able to construct series *diverging* everywhere except at isolated points.

Divergence over infinity

After Luzin returned to Moscow from his commission abroad, he and Florensky were able once again to converse in person. Philosophical differences had arisen between them over over such issues as the nature of infinity. It is interesting to note how Luzin, who was so much influenced by Florensky both philosophically and mathematically, is now confident in his opposition not only to Florensky, but apparently to most of the Moscow University mathematics faculty as well.

This divergence has apparently led to tension in the relationship. Luzin writes not only to explain his position, but also to express his indebtedness to Florensky, and assure him that the divergence between them does not threaten their relationship. As we shall see, his reliance on Florensky for spiritual guidance would continue as a strong component in their relationship.

In his letter of 4 August 1915 [Demidov, Parshin, Polovinkin & Florensky *1989*, 177–179], Luzin vividly recounts his formative experiences with Florensky, going back to the days when he was active in the Student Circle of the Moscow Mathematical Society [Phillips *1978*, 282]. He is writing about the period between 1902, when Florensky founded the student circle, and 1904, when Florensky graduated and appointed Luzin as his successor as its secretary.

I am genuinely grateful for our relationship, for the attention you have shown me: believe that, in spite of the divergence in our opinions. Your relationship is precious to me, as it has always been ever since the time when I, as a student, timidly attended the meetings of the "Student Mathematical Society," of which you were the secretary.

Even then, I clearly felt your strength, wanted to get close to you, but was unable: I was afraid of you, afraid of your opinion of me, ashamed of my naivete and intellectual shortcomings.

Is it necessary after this to reiterate the joy that I experienced when you — *you* offered to me the position of your successor as secretary?

The memory of this, of this joy, at discovering your interest in me, personally, and . . . the glimmer of hope, hope that something will come of me — the memory of this will always remain with me, whatever the divergence. . . .

What you did for me at just that moment, giving preference to me, who was naive, knowing nothing, over S. S. Byushgens, *this* I will never be able to forget, because at that moment I experienced, for the first time, a joyful confidence in myself . . . .

A lot time has passed since then, but the memory of that is still very much alive. Nothing at all has changed. My attitude to you, to your strength, remains essentially the same.

Luzin then turns to the divergence of their opinions on infinity, describing it in a very emotional, almost religious, manner. It brings to mind the statement of Luzin in the letter of 7 April 1909 quoted above, giving his reaction to Florensky's lecture: "I am frightened by its deepness and the daring thoughts in some of its parts."

Concerning the "divergence" of our opinion — it is significant, this is true. But really, does *this* change anything, at least for me?

You are seeking the unwavering heart of indisputable Truth, the basis of everything, boldly stepping over everything, brushing aside theories like houses of cards, but I . . . I am not looking for the last "how" and "why" and am afraid of infinity, I sidestep it, I do not believe in it.

There is no actual infinity! and when we try to speak about it, we *in fact* always speak about the finite and the fact that after n comes $n + 1$ . . . that is all!

There is no indication of any break between them. On the contrary, Florensky wants Luzin to write an article for the journal of the Theological Academy, of which Florensky is the editor. In a letter of 24 October 1915 [Demidov, Parshin, Polovinkin & Florensky *1989*, 179], Florensky reminds Luzin that they had discussed such an article, and expresses his hope that Luzin would write one. Florensky seems quite interested in this possibility, and mentions it elsewhere [Trubachev *1987*, 301]. As far as we know, Luzin never did so, unless perhaps he wrote under a pseudonym.

Their close friendship continues as before. In his next letter, written 21 April 1916 [Demidov, Parshin, Polovinkin & Florensky *1989*, 182–183], Luzin regrets being too busy to visit Florensky. He misses the wide ranging "general conversations" he would like to have with Florensky, "which are always so interesting, and lead to new ideas." Nevertheless, he writes with a specific purpose — to give Florensky the date of the defense of his first thesis. This defense, like the thesis, was so brilliant that Luzin was, quite exceptionally, allowed to forgo the second thesis and was immediately awarded the doctorate in pure mathematics.

Florensky had specifically asked to know the date of the defense. Luzin invites him to attend, even though he anticipates a serious conflict between himself and the faculty — with Florensky on the opposing side!

> They intend an all-out attack on me, but I will not give up and intend to fight back. I expect there to be open fire on "the axiom of choice."
>
> One must say that my position is very difficult: because I attack Cantor. But he is defended by the whole faculty, so that I think there will be a general debate about the foundations of set theory, and there will be a very heated argument.
>
> If time allows you to be in Moscow at that time, I would be very pleased to see you — sincerely glad — though I know that my most dangerous opponent — is you.

After the Revolution

Luzin was appointed to the Mathematics faculty at Moscow University in 1917, just before the Revolution. By January 1918 he was commuting to the town of Ivanov-Vosnesensk, northeast of Moscow, to teach at the Ivanovsky Technical Institute in order to earn extra rations in the difficult

post-revolutionary conditions. Florensky was still living near the Moscow Theological Academy, 30 miles northeast of Moscow.

The Polish mathematician Waclaw Sierpinski spent the years of the First World War, 1914–1918, in Russia. There he became close to Egorov and Luzin. In fact there is a picture of the three of them together in Egorov's apartment taken in 1917. Early in 1918 Sierpinski left Moscow for Poland and later that year joined the faculty of Warsaw University. After Sierpinski's departure, Luzin writes a note to Florensky suggesting that he try to publish abroad 1916 [Demidov, Parshin, Polovinkin & Florensky *1989*, 186–187].

> . . . are you inclined to follow the wishes of some foreign friends, who are deeply interested in your books, and be willing to contact them or to send them something? Publishing abroad is possible. I would be willing to contact Professor Sierpinski and someone else besides.

This seems to clearly suggest that Sierpinski had been "deeply interested" in the work of Florensky. It is known that Sierpinski was religious, though Roman Catholic rather than Eastern Orthodox. It would be interesting to know whether any other evidence could be found of Sierpinski's interest in the work of Florensky. My attempts in Warsaw yielded neither documentation nor any knowledge of such interest. Apparently most materials related to Sierpinski from before the war have been lost.

Luzin's final letter to Florensky [Demidov, Parshin, Polovinkin & Florensky *1989*, 187–188], written 5 May 1919 (following the new calendar), is discussed in [Ford, *forthcoming*]. The Revolution had raised anew the need for "a source of spiritual life" and Luzin turns again to Florensky for spiritual guidance.

We know that Luzin attended Church in the 1920s an again after the war. He probably stopped in the 1930s, after the arrest of Egorov. He was not as stable a personality as either Egorov or Florensky.

Egorov became an open champion of a religious perspective during the 1920s. His arrest, in October 1930, was specifically connected to his involvement in Church life. Even after his arrest, and exile to Kazan the following year, he continued to resist by declaring a hunger strike, which led quickly to his death [Ford *1991*].

Florensky likewise openly defended the Church and a religious perspective, for example by openly dressing as a priest at meetings of scientific commissions on which he served. This led to his arrest in 1933. Under

interrogation, he was forced to implicate Luzin in a fictitious counter-revolutionary plot [Shentalinskii *1990*], [Demidov & Ford *1966*].

Luzin did not openly express his religious perspective after the Revolution and, despite attacks against him, escaped serious consequences. Although Luzin did not openly defend religion, he was a representative of the remarkable rebirth of religion in Russia in the face of the almost complete annihilation of public religious life.

I would like to add to my expression of gratitude to the anonymous referee already acknowledged, for his translation of the letter of 1 May 1906 and other helpful suggestions.

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