MINING OF RUSSELL

Russell and Analytic Philosophy, ed. A. D. Irvine and G. A. Wedeking. Toronto, University of Toronto Press, 1993, and *Introduction to Mathematical Philosophy*, by Bertrand Russell. New York, Dover, 1993, reprint of 1919/1920 edition, London, Allen & Unwin.

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This inexpensive ($6.95) reprinting of Russell's classic in the philosophy of mathematics and the recent anthology on his work are most welcome additions to Russellian studies. Like so much of his work, Russell's book merits regular revisitations by logicians and philosophers. The anthology features significant contributions by many of the major figures working on Russell today. They draw on newly accessible material by Russell either recently published in the emerging volumes of his *Collected Papers* or yet unpublished in the Russell archives at McMaster University. Irvine and Wedeking provide a helpful introduction. The sections numbers below correspond to those in the anthology.

Besides his well-known discussions on the definition of number and the logicist thesis that mathematics and logic only "differ as boy and man," (IMP, 194), Russell argues that mathematical induction is a defining feature of natural numbers, "not a principle" (IMP, 27) synthetic a priori, as Poincaré had claimed. He discusses relations, serial and cyclic order, and the similarity of relations, today called isomorphism.

Russell analyzes the notions of limit and continuity, and shows that, unlike what we may suspect from studying the differential calculus, the two notions apply not to numbers and functions only, but to any ordered series, such as points on a line, or perhaps moments in time. (IMP, 104) He analyzes Dedekind's and Cantor's concepts of continuity, in which each element "is what it is, quite definitely and uncompromisingly; it does not pass over by imperceptible degrees into another." (IMP, 105) Applauding Weierstrass, he shows that applying these notions to functions does not require infinitesimals, quantities that "involve . . . intervals that are not require infinitesimals, quantities that "involve . . . intervals that are not finite." (IMP, 116) Russell suggests further that his Weierstrassian concept of limit shows that infinitesimals cannot exist at all (IMP, 97). Of course, little could he know of Robinson's work on infinitesimals to occur some 45 years later. However, he was otherwise familiar with Peirce (IMP, 32), who argued, contrary to Russell's prevailing view, that the existence of infinitesimals (though different from Robinson's) could be shown and fruitfully developed.[1]

1. Descriptions. Russell interprets his theory of descriptions so that "it is only of descriptions — definite or indefinite — that existence can be significantly asserted." (IMP, 179) As a result, where *a* is a genuine name (not an "abbreviated description" like Homer), "*a* exists" is meaningless. Now, a name, he writes, directly designates an individual without being "a part of the fact asserted" (IMP, 175) But to know when something designates is not always easy. To assume the term "Socrates," for instance, is a genuine name is "very rash." (IMP, 175) And what shall we say about names on correspondence or today's computer login and email names, when we cannot always be sure that the person who sent us email is indeed the one whose "name" appears at the head of the message, or even that the "designatee" exists? In mathematics, it seems that number names (0, 1, 2, ...) are simply abbreviated descriptions, arguments satisfying one or more functions. (IMP, 164)

Several papers analyze facets of Russell's theory of descriptions and names. R. M. Sainsbury, in "Russell on Names and Communication," distinguishes several forms a descriptive theory of names might take based on a 4-place relation *R* "between a name, description, speaker, and an occasion." (3) One issue is whether a description(s) is associated with a name for all speakers, for each speaker irrespective of the occasion of utterance, or for each occasion each speaker makes an utterance. The other is whether the relation determines the public meaning (read "truth conditions"), public reference, or the speaker's own thought. Citing Russell, he shows him before 1920 to be concerned with the relation between the name and the description in the speaker's thoughts on a specific occasion, not, as many suppose, with public meaning or reference essential to interpersonal communication. (14) But there are perhaps other possibilities as well — for instance, that Russell is concerned with *private* reference to external (public?) objects. We could take his interest in fostering a "robust sense of reality" that distinguishes between Hamlet and Napoleon (IMP, 170) as concern with this merely private reference to the public Napoleon. Of

[1] See my "Peirce's Theory of Infinitesimals," *International Philosophical Quarterly*, 1991, pp. 127–140, where I analyzed his ideas and proved some theorems that follow from Peirce's axioms on infinitesimals.

course, Wittgenstein would object that a private reference entails a public one. In any case, Sainsbury's notion of Russell''s relation *R* is not synonymy; it is simply that the speaker has in mind some description or other when she uses the name. Citing Kripke, he shows how Russell's theory, despite its apparently different focus, answers some common problems about names — denials of existence and beliefs turning on identities (Quine's opaque contexts). Sainsbury's distinctions are important, and suggest further avenues of research.

Both Simon Blackburn and Alan Code's "The Power of Russell's Criticism of Frege" and Michael Pakaluk's "The Interpretation of Russell's 'Gray's Elegy' Argument" are concerned with a difficult passage in Russell's "On Denoting," his original paper on the theory. The passage turns on the problem of analyzing the relation, if any, between the sense and reference of a description (such as Russell's G) "The first line of Gray's Elegy"). Blackburn/Code argue that the passage is directed against Frege's view that the sense of a description (or name as well) is logically (not merely linguistically) connected to its reference yet distinct from it, whereas Pakaluk argues that it is directed against Russell's own view in the *Principles of Mathematics* that descriptions must be analyzed into meanings and denotations (41). Blackburn/Code interpret Russell as saying that to grasp the sense of phrase G), we cannot *use* G and speak of "the sense of the first line of Gray's Elegy" because that focuses us on (the sense of) the first line of the poem itself. Instead, we might *mention* the phrase and speak of "the sense of 'The first line . . .'," but that does not show any logical connection between the postulated sense and reference of G (29), as Frege claimed. The two different interpretations turn in part on whether Russell's mature theory is seen as rejecting the notion of the sense of a description altogether or maintaining it in some way. Regardless of whose interpretation is best, both illuminate these dark issues considerably.

The thrust of the theory of descriptions is that they must be analyzed in context, not in isolation. The details of the theory blossomed over several years. Francisco Rodríguez-Consuegra, referring to several unpublished manuscripts in the Russell archives, in "The Origins of Russell's Theory of Descriptions," traces the history of Russell's thought on the topic and provides much detail and insight into Russell's ideas on names, descriptions, propositions, and their interactions. He notes the influence of Bradley, Moore, and Whitehead, as well as Russell's view that his analyses of number and descriptions "though formally they are merely nominal definitions, in fact embody new knowledge . . ." (81–82, quoting Russell's "Reply to Criticisms", in Schilpp's volume, 690–691) because they reveal conceptual connections that were previously unrecognized.

In his 1990 work, *Descriptions*, Stephen Neale cogently argued that Russell's theory of descriptions is applicable to the analysis of a wide range of sentences in natural language.[2] In "Grammatical Form, Logical Form, and Incomplete Symbols," he continues his Russellian analysis of natural language with the intent of revealing their logical form, "the structure imposed [on a sentence] in . . . providing a systematic semantics based on a systematic syntax."(129) His purpose, like Russell's he says, is to avoid the bad philosophy that arises from bad grammar (98). He champions the use of Restricted Quantification (RQ) to elucidate the logical form of sentences. RQ is the use of quantifier phrases — like [most farmers *x*] or [some cows *y*] — instead of the traditional universal and existential quantifiers with predicates. With many examples, Neale argues that RQ promotes Russellian analyses of complex sentences involving anaphoric pronouns (roughly, those referring to antecedent nouns), like 'them' in "Russell bought some hens, and Whitehead vaccinated them." (122) He investigates Chomsky's notion of LF (logical form), which reveals quantifier scope, and argues that its "mapping . . . to . . . RQ looks to be straightforward" (113). Against G. Evans, he argues that RQ has "the same expressive power" as Binary quantification (BQ), another method favored by Evans. That may be so, but if so, it's not clear why either should be thought to reveal "the logical form" of sentences better than the other, or LF, or even traditional quantification (supplemented by set theory, when necessary as with "most"). Each method has its appeal and each may promote an otherwise identical "systematic semantics" that exhibits relevant truth conditions. So logical form may be something any of several methods, conjoined with the right analysis, may reveal. Or perhaps there is more than one logical form: different methods may reveal slightly different logical forms of natural language sentences "equally" adequate by somewhat different criteria or for different purposes or tastes.

In "Russell's Strange Claim that '*a* exists' is Meaningless . . .", William Lycan provides careful analyses of a series of arguments Russell seems to, or may, have had in mind to support this claim. He rejects some of the arguments as Russell's because he finds few passages that lend themselves to such interpretations. As for the suggestion that $(\exists x)(x = a)$ (a theorem in Russell's logic if *a* is a genuine name) means "*a* exists" on standard contemporary accounts, he notes that Russell "would have to deny" it. (146) He pursues Russell's thought (IMP, 164–165) that the basic notion of existence consists in a propositional function's being "sometimes true," (147) and concludes that Russell's type theory applied to individuals is critical in defending Russell's claim.

2. Logic and Mind. The private/public dichotomy raised by Sainsbury's paper above is a theme in other papers as well. It is connected

[2] For an analysis of Neale's *Descriptions*, see my article in THIS JOURNAL, vol.4, no.3, July, 1994, pp. 331–340.

with Russell's logical atomism, which sees, at a subsurface logical level, a 1–1 correspondence between language and the world, where the one side comprises, perhaps, language's logical forms. In "Regarding Privacy," R.E. Tully analyses Wittgenstein's attack on Russell's (and his own earlier) logical atomism and his argument against the possibility of a private language. He argues that Wittgenstein's argument is directed against a sceptical or solipsist view that Russell himself attacked. Russell's "private language," he suggests, is also public — private cells of each person's sense data coming together publicly in science.

In "Terms, Relations, Complexes," Nicholas Griffin seeks something that holds the remarkable variety of Russell's philosophy together. He finds it not in any specific doctrines or principles, but rather in Russell's persistent search for unities among individuals and complexes. Most of his paper focuses on this problem in Russell's theory of relations and propositions before 1914, with some analysis of Russell's later attempt to construct the world out of sense data. But Griffin promises us more in another paper.

In "Why Russell Abandoned Russellian Propositions," Bernard Linsky argues that Russell embraced them early on as bearers of truth values, but by 1918 in his lectures on logical atomism, he rejected them because ultimately "there is nothing to hold them together." (200) If propositions were anything more than symbols (IMP, 155), they would have to be complex, but only facts can be complex. Linsky suggests that perhaps Russell was premature in banishing propositions from his ontology.

In "Wittgenstein versus Russell on the Analysis of Mind," Stuart Shanker argues that Wittgenstein develops much of his later views on psychology in response to Russell's *Analysis of Mind*. He argues that Russell largely presented a longstanding causal theory of human desires, sensations, and actions as events to be explained in a way similar to, if not based on, a mechanistic physics. Referring to Wittgenstein's work that's become available in English since the '80s as well as the more familiar *Investigations*, Shanker analyzes Wittgenstein's contrary arguments that mental events are neither explicable in terms of brain events nor even in need of traditional causal explanations at all. He shows how Russell draws on predecessors like James and Watson, and points out the tension in Russell's views: we know our desires privately or subjectively through "introspection," yet we know our own motives in the same way "we discover other peoples's" (231), by observing them. Shanker shows Wittgenstein pursuing this second strand of thought like Russell through analyzing our mental language. But whereas Russell sought different forms of sentences like "I think" to reveal a deeper metaphysical truth, Wittgenstein argued that different forms do not reveal anything more because the important thing is the content. (236) And it is the content that makes it a grammatical, or if you will, logical truth that, for instance, intentions "determine those actions which fulfil them." (230) So for both Russell and Wittgenstein, the logical notions of form and content become crucial to questions of philosophical psychology.

3. Logicism. Russell is plainly troubled by the apparent necessity of the axioms of choice (which he calls the multiplicative axiom because of its importance for defining multiplication of an infinite number of terms), infinity, and reducibility for generating mathematics (in particular set theory as known at the time), but he does not let his concern weaken his logicist convictions. He suggests that at least the axiom of choice may eventually be proved (perhaps through one of its equivalent principles), but if not (as Gödel's and Cohen's independence results later showed), we can simply assume it or the others when necessary to prove a desired result. But he does not acknowledge that this step toward a hypothetical or "if . . . then" view of mathematics *à la* Peirce or Putnam is a move away from logicism.

Whether or not mathematics is logic, there is a certain tension in Russell's views on the truth conditions or ontological (if you will) foundations of logic. On the one hand, he argues that we do not know that the axiom of infinity is true, because we cannot know that there are in fact an infinite number of individuals in the universe. This stems from the difficulty in defining what an individual is, and from the finitude of our experience (IMP, 132–134). On the other hand, he argues that logicians should keep a "certain lordliness . . . [and] not condescend to derive arguments from the things he sees about him" (IMP, 192) because they are not concerned with the details of this world. But that very concern is what he suggests is necessary to establish that the axiom of infinity is true. So he wavers from his claim that logic is strictly *a priori*.

Michael Detlefsen in "Logicism and the Nature of Mathematical Reasoning" insightfully contrasts Russell's views with Poincaré's on the topic. The Kantian Poincaré, he notes, holds that even if it were successful in axiomatizing and proving all of mathematics, the logicist program would still miss the *epistemological* foundations of mathematics, which relies not on the detailed logical steps the logicist offers, but on an intuition of, in Poincaré's words, the entire "unity of the demonstration." (271) For Poincaré, this intuition also apprehends details of specific subject matter, whereas Russell emphasizes the universal character of mathematical or logical reasoning. Detlefsen also observes the distinct notions of synthetic reasoning: Russell's leading us from one theorem to another having the same truth conditions, while Poincaré's leading us from theorem to theorem "even though the truth-conditions of the former may not cover those of the latter." (281)[3] A Russellian could then question the validity or (logical!?)

[3] For an alternative analysis of the analytic/synthetic distinction in mathematics based on some of Peirce's ideas, see my "Peirce's Theoremic/Corollarial Distinction and the Interconnections between Logic and Mathematics," to

certainty of the purported theorem. But a Kantian could reply that intuition is needed to see even the truth of *modus ponens*, and so is just as certain (or dubious) in the one realm as the other.

Judy Pelham's "Russell's Early Philosophy of Logic," provides insight into Russell's interactions with Bradley and Moore as he first developed his logic in the *Principles of Mathematics*. There are, of course, many differences between Russell's views at this point and those he developed a few years later while writing *Principia Mathematica* with Whitehead. Among these is the claim that implication is a primitive relation (339) not to be analyzed in terms of truth functions, as he would later hold (IMP, 154). The question arises as to how we know this primitive, unanalyzable relation. Had Russell been content to accept the answer "intuition", he might not later have presented such an instructive contrast with Poincaré on mathematical intuition.

In "Russell's Logicism and Categorical Logicisms," Jean-Pierre Marquis offers an interesting reinterpretation of logicism, one quite different from Russell's. Marquis argues that the logical concepts in terms of which (according to logicism) mathematical concepts are to be defined can themselves be reinterpreted as concepts in category theory, and that mathematical theorems can then be derived from axioms of category theory. He points out that the increasing levels of abstraction that characterize the pursuit of logic as described by Russell may go in other equally valid directions than that of set theory and first order logic. (300) What makes one way "logical", he says, is that it is "universal." (He also seems to take this feature as necessarily (?) characterizing "the foundation" of mathematics.) So Marquis introduces some basic ideas of category theory and eventually defines a limit for a diagram (different from a limit in the differential calculus). It is this notion of limit (or perhaps a more general notion of adjoint functor) that he takes to be universal and "logical" because with it we can define truth functions and quantifiers, among many other things. (315) Marquis cites references to recent work showing how results in category theory (or its branch topos theory) imply or are equivalent to results in logic. (Similar connections have long been known between algebra and model theory.) Marquis shares with Russell and many of his critics the view that *some* branch of mathematics or logic must be "*the* foundation" (ontological or epistemological) of the subject(s). However, it is questionable whether any of these approaches provides more of a foundation for mathematics than any other. Showing that one or many branches of mathematics can be defined in, and derived from, another does *not* show that that same corpus (or even a larger one) cannot be defined in,

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appear in Nathan Houser, Don D. Roberts and James W. Van Evra (editors), *Studies in the Logic of Charles S. Peirce*, Indiana University Press, 1997.

and derived from, yet another branch. (And in light of the vastness of current mathematics, and its continually evolving nature, it seems presumptuous of Russell and Marquis (in different ways) to say that their foundational branch applies to all of mathematics, "pervades" it, and is "universal." (316)) It is probably more fruitful to view each derivation as providing a different perspective on the same or similar mathematical structures or objects. Perhaps some approaches will have certain ontological, epistemological, mathematical, computational, or "logical" features that others lack, and these may serve to distinguish different notions of foundations. But an "equal opportunity perspective," if you will, is what's needed at this point. In any event, Marquis's approach through category theory is worth pursuing. I look forward to more.

Russell argues that classes, being "logical fictions," should ultimately be eliminated from a proper metaphysical foundation for logic, apparently because they require complex type restrictions to avoid contradictions (IMP, 137) and in a perfect symbolic language, no undefined symbols would represent classes at all (IMP, 182). In IMP, he indicates his dissatisfaction with the detail of his and Whitehead's solution in PM, but reaffirms his belief that *some* type theory will do the job. Peter Hylton's "Functions and Propositional Functions in *Principia Mathematica*"and Gregory Landini's "Reconciling PM's Ramified Type Theory with the Doctrine of Unrestricted Variable of the *Principles*" fruitfully explore the type theories which emanated from Russell's concern with classes.

4. Analytic History. In "Russell Making History: The Leibniz Book", Graeme Hunter argues that with his book on Leibniz, Russell initiated"the analytic approach to the history of philosophy." (397) He notes the book's influence on Joseph, Broad, Sellars and others, and analyzes Russell's method while showing connections with his logic. He argues that Russell's advance was not, as popular myth has it, that he was the first to criticize the Hegelian influence on British philosophy and its history. Nor was he the first to reawaken interest in Leibniz. Others had done both. No, what was original was Russell's "anti-historicist interest in what he calls 'philosophic truth' with the historiographical principle that philosophers inevitably fall into one or other of a few great types." (400) The issue then becomes, "What is a philosophic type?" It is not a logical type which Russell invoked to avoid the paradoxes, though the one may apply to the other. Nor is it simply one of the elements of the familiar dichotomies realist/idealist or rationalist/empiricist, though that's part of it. It is, only somewhat more precisely, the set of "main doctrines" (401) of a philosopher. These form a set of axioms from which Russell presumes the philosopher's other doctrines follow. In Leibniz, Russell distinguished five such axioms. Whatever is inconsistent with them is not part of the type from which we ascertain philosophic truth. Hunter cites Cassirer's criticism that in seeking the "main doctrines" in a timeless philosophic type, it's too easy to emphasize traditional doctrines (as Russell did with Leibniz''s views on substance) and discount new insights (or "main doctrines"!) "as contradictions." (409) A related question of logical interest is, "What set of main (basic?) axioms or doctrines would, barring inconsistency, generate a richer or more comprehensive corpus of a philosopher's system or a mathematical/ logical theory than a given set, such as Russell's on Leibniz' system?" It would also be instructive to seek other illustrations besides Leibniz of the influence of Russell's notion of a philosophic type on others doing an "analytic history of philosophy."

Finally, like the anthology itself, the references at the end of each essay provide a splendid set of avenues to pursue Russellian studies.