REPLY TO QUERY: HOW OLD IS FIRST-ORDER LOGIC?

MOSES KLEIN

From: klein@math.wisc.edu (Moses Klein)
Subject: Re: How old is 1st order logic?

Kai Wehmeier <wehmeie@majestix.uni-muenster.de>wrote: "When and by whom was first order logic first explicitly isolated as a logical system? (My guess is Tarski, but I really do not know.)"

The earliest example that comes to my mind is Löwenheim, whose famous 1915(?) theorem on the existence of models of different transfinite cardinalities requires the assumption that the theory is first-order.

You might want to look at the last chapter of Stewart Shapiro's *Foundations without Foundationalism*, which includes a historical note on how first-order logic came to occupy center stage.

– Moses Klein (klein@math.wisc.edu)

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IRVING H. ANELLIS

Gregory Moore's articles "A House Divided Against Itself: The Emergence of First-order Logic as the Basis for Mathematics" (in Esther R. Phillips (editor), *Studies in the History of Mathematics* (Washington, D.C., Mathematical Association of America, 1987), 98–136) and "The Emergence of First-order Logic" (in William Aspray and Philip Kitcher (editors), *History and Philosophy of Modern Mathematics* (Minneapolis, University of Minnesota Press, 1988), 95–135) trace the history of the emergence of first-order logic. While finding first-order logics in Frege in 1879 and in Peirce in 1883, Moore states (pp. 129 and 128 respectively) that these were embedded within second-order logics and that "[f]irst-order logic emerged as a distinct subsystem of logic in Hilbert's lectures of 1917 and, in print, in [Hilbert and Ackermann 1928]."

Moore also points out that whereas Löwenheim's theorem as stated in 1915 is *about* first-order logic, the system in which he worked was Schröder's second-order logic, and indeed the *original statement* of the theorem by Löwenheim in 1915 is a formula of second-order logic (*q.v.* especially pp. 124–125, 122 respectively). The theorem (call it **LT**) states that for a infinite domain D (which is possibly denumerably infinite and possibly of higher cardinality), if a first-order proposition P (which Löwenheim called a "*Zählausdruck*") is valid in every finite domain but not in every infinite domain, then P is not valid in D. The proof of **LT** is carried out using second-order logic; as Löwenheim states (see p. 236 of the English translation of Löwenheim's 1915 paper in Jean van Heijenoort's *From Frege to Gödel*), Σ ranges over functions). In the next theorem in his famous paper of 1915, Löwenheim states that since Schröder's logic can express that a domain is finite or denumerable, then **LT** cannot be extended to Schröder's [second-order] logic.