MATHEMATICAL LOGIC IN POLAND 1900 — 1939: PEOPLE, CIRCLES, INSTITUTIONS, IDEAS

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1. INTRODUCTION

Assume that someone would try to predict the development of mathematical logic circa 1900. Probably, he would point out Germany, England, and perhaps France as the central countries. Certainly, this person would not mention Poland, and not only because there was no such country at that time. Thirty year later, Heinrich Scholz, the first modern historian of logic, called Warsaw one of the capitals of mathematical logic (see [Scholz *1930*]). How did a country without special traditions in logic so quickly arrive at the top of this field? What happened that permitted Fraenkel and Bar-Hillel to write: "There is probably no country which has contributed, relative to the size of its population, so much to mathematical logic and set theory as Poland"? ([Fraenkel & Bar-Hillel *1958*, 200]).[1] This paper tries to explain the phenomenon called "Polish logic" by pointing out the wider context in which logic in Poland was done.[2]

I. PEOPLE, CIRCLES, INSTITUTIONS

2. LVOV 1900 — 1939

In 1895, Kazimierz Twardowski was appointed professor of philosophy at Lvov University. Twardowski was a student of Brentano and he inherited some general metaphilosophical views of his teacher, in particular rationalism, the demand for clarity of language and thought, hostility to speculation, and the belief that philosophy is a science. He wanted to implement these ideas in Polish philosophy. And he intended to create a school of scientific philosophy. He succeeded and established the analytic movement, commonly known as the Lvov-Warsaw School. Twardowski himself was not a logician. Contrary to the assertions of his students, he never maintained that mathematical logic would have importance for philosophy. He even feared that mathematical logic, improperly applied, could result in symbolomania; see [Twardowski *1921*]. However, he certainly stressed the real significance of logical culture for philosophy. Moreover, his metaphilosophical views were a natural environment for doing semiotics, formal logic and methodology of science. In the academic year 1899-1900 Twardowski delivered a course on logic. This course was basically devoted to Brentano's logical innovations, but Twardowski also informed his students about the recent trends in logic. It was the first academic contact of Poles with mathematical devices in logic. Twardowski himself was fully conscious that his course opened a new period in the history of logic in Poland. In 1932 he introduced Heinrich Scholz to the Polish Philosophical Society in Lvov with the words (*Ruch Filozoficzny* **XIII** (1935), 41-42):

> Dear Professor Scholz! It is my special pleasure that we can welcome you into the Polish Philosophical Society in Lvov. We are very grateful that you decided to accept our invitation and come back to your place from Warsaw *via* Lvov and tell us about your results. In traveling from Warsaw to Lvov, you moved in the direction opposite to that in which interest in logistics and the research in it moved in Poland. This does not mean that this interest and research left Lvov, because it is not so; but the point is that logistics in Poland had its beginning in Lvov. Here in Lvov the first Polish work devoted to logistics, that is algebraic or mathematical logic, as it was called at that time, appeared in 1888. It is an essay "Algebra in Logic" published by [...] Stanisław Piątkiewicz. Eleven years later, in the academic year 1899-1900, I lectured in Lvov "On reforming tendencies in formal logic" and I informed the youth about those efforts, including that of George Boole, which prepared the present logistic. Jan Łukasiewicz was then among the participants of this course. Since that time, he has been faithful to mathematical logic and took it as one of the main fields of his research. When he later became a docent in Lvov, he was able to infuse his own interest into several of his own, as well as of my, students [...]

Of course, Twardowski's course was not sufficient as a start for serious work in mathematical logic. Łukasiewicz very soon began to study works of Frege and Russell. His lectures in logic, which started in 1906, attracted, among others, Kazimierz Ajdukiewicz, Tadeusz

Czeżowski, Tadeusz Kotarbiński, and Zygmunt Zawirski and induced them to complete their doctoral dissertations in logic. Łukasiewicz reports on Leśniewski's appearance in this way (*Memoirs*, unpublished, p. 2).[3]

> One afternoon, someone knocked at my flat. I opened the door and saw a young man with a small bright pointed beard, in a hat with a wide brim, and with a big black knot instead of a tie. The young man greeted me and kindly asked me: "Does Professor Łukasiewicz live here?" Yes — I answered. "Perhaps you are Professor Łukasiewicz" said the unknown man. Yes — I answered. "I am Leśniewski and I came to show you proofs of a paper which I wrote against you" — he said. I invited him to the room. It appeared that Leśniewski's submitted to "Przegląd Filozoficzny" a paper in which he criticized some of my views contained in my book "On the Principle of Contradiction in Aristotle". This criticism was written with such scientific rigour that nothing could be objected against. I remember that when, after the discussion, which persisted for one hour, Leśniewski said "good bye", and I went, as usual, to the Scottish Cafe, I told my friends waiting there: I liquidate my logical business, because there arose a factory which is beyond competition.[4]

Thus, a group of young people strongly interested in logic arose in Lvov. It was the first logical circle in Poland in the 20^{th} century. The Lvov logicians were influenced not only by Twardowski and Łukasiewicz. We must also mention that Wacław Sierpiński was professor of mathematics in Lvov at that time and that young philosophers interested in logic participated in his classes on set theory. In particular, Sierpiński trained Zygmunt Janiszewski, who played an enormous role in the subsequent development of logic and the foundations of mathematics in Poland.

In 1914–1918, Lvov University was restricted in its activities, due to the fact that the town was occupied by the Russians. Normal activity was reestablished after the World War I, in a newly independent Poland. The university staff changed considerably, because several scholars went to other universities. Of the people already mentioned, Łukasiewicz (in 1915), Leśniewski, Czeżwski, Kotarbiński and Zawirski (in 1924) left the Lvov logical circle. Only Twardowski and Ajdukiewicz remained in Lvov. Twardowski did not change his attitude toward mathematical logic, but Ajdukiewicz began, at first as docent and then (since 1928) as professor, teaching logic to philosophers and mathematicians.[5] His lectures and seminars gave a good logical background to several young philosophers, including, among others Izydora Dąmbska, Maria

Kokoszyńska, Seweryna Łuszczewska-Romahnowa and Henryk Mehlberg who graduated in twenties. The Lvov logical community included also some of Twardowski's old students, such as Stanisław Kaczorowski, Stanisław Sośnicki and Stanisław Smolka.

Mathematicians in Lvov were interested to some extent in logic and the foundations of mathematics. Eustachy Żyliński, Stefan Banach, Stanisław Mazur and Hugo Steinhaus even published some papers, Żyliński in logic, Banach, Mazur and Steinhaus in foundations. In 1928, the university decided to establish a professorship in mathematical logic; the position was located in the Faculty of Mathematics and Natural Sciences. Two persons applied: Leon Chwistek and Alfred Tarski. Chwistek was supported by mathematicians, particularly by Banach (his position in this competition was not easy, because he published together with Tarski a famous paper on the Banach-Tarski paradox) and Hugo Steinhaus (whose sister was Chwistek's wife), but Twardowski and Ajdukiewicz acted for Tarski. The Council of Faculty asked Twardowski, Hilbert, Russell and Whitehead to serve as referees. Of the foreigners, only Russell replied. He wrote to the Dean of the Faculty:[6]

29 December 1929

Dear Sir,

I much regret that owing to my absence in America, your letter on the 31th October has remained hitherto to unanswered. I know the work of Dr. Chwistek and think very highly on it. The work of Mr. Tarski I do not at the moment remember, to which I do not have access at present. In these circumstances, I can only say that in choosing Dr. Chwistek you will be choosing a man who will do you credit, but I am not in position to compare his merits with those of Mr. Tarski.

Believe me with highest respect. Yours faithfully,

Bertrand Russell

Russell's opinion was probably decisive, and Chwistek won this competition.[7] One thing should be explained in this context. Sometimes one can hear the opinion that Tarski lost in Lvov because of his Jewish origin. Certainly, being a Jew did not help in the academic promotion and Tarski's hopes connected with his Polonization were not quite reasonable. However, the antisemitic attitude in Poland was not particularly strong in the time of the competition, *i.e.*, in 1928-1930; it became much stronger later. Chwistek was older, what was an important factor in Polish universities. Moreover, he was better known in scientific circles for his writings. These reasons together with the support by influential mathematicians and Russell's letter sufficiently explain why the ultimate decision was in Chwistek's favour. He was appointed in 1930.[8]

The controversy over the position in mathematical logic divided philosophers and mathematicians in Lvov. According to the recollections of several people, Twardowski felt offended that his preference for Tarski was not followed.[9] This situation did not help the cooperation between mathematician and philosophers in Lvov. Each group acted separately. Thus, no unified logical circle arose in Lvov in the interwar period. Chwistek established his own school with Władisław Hetper and Jan Herzberg as the leading persons. This group worked on Chwistek's project of the foundations of mathematics based on the so-called rational semantics. Józef Pepis, an author of important papers on the decision problem, also belonged to Chwistek's circle. Ajdukiewicz continued to teach philosophers. However, the sympathies of young philosophers in Lvov were divided between him and Roman Ingarden, an eminent phenomenologist who was appointed professor in Lvov in 1933. Anyway, Ajdukiewicz, in the 1930s, trained only one logician, namely Zygmunt Schmierer.

The Polish Philosophical Society in Lvov organized several meetings with talks in logic. Some of them were of the utmost historical significance. Let me mention two examples. In 1920, Łukasiewicz delivered in Lvov two papers in which he outlined his formal system of three-valued logic. In 1930, Tarski chose Lvov as the place for his public lecture on the semantic definition of truth.

3. CRACOW 1900-1939

Stanisław Zaremba was the pioneer of modern logical interests in Cracow. Strictly speaking, Zaremba was interested rather in the foundation of mathematics than in mathematical logic. This attitude was influenced by his study at the Sorbonne where he obtained his doctoral degree (1899). Zaremba wrote several books, among others [Zaremba *1915*, *1926*] in which he described the logical structure of mathematics. It was Jan Śleszyński who introduced mathematical logic as an academic subject. He studied in Odessa and Berlin. In 1893, he was and he was appointed professor in Odessa. In 1911, he became extraordinary professor of mathematics at Jagiellonian University and began his lectures in mathematical logic and the foundations in mathematics. In 1919, he became professor of logic and mathematics. Śleszyński did not publish very much, but his extensive lecture notes in logic were edited by his students (see [Śleszyński *1925-1929*]). Antoni Hoborski, Otto Nikodym, Edward Stamm and Witold Wilkosz were other mathematicians in Cracow who studied after World War I and worked in mathematical logic and the foundations of mathematics. Hoborski and Nikodym did this only occasionally. Stamm wrote papers on algebra of logic and its applications to numners. He never attained any university position. Hoborski was professor at the Academy of Mining, Nikodym was docent at the University. Wilkosz was appointed professor at the Jagiellonian University in 1921. He lectured on set theory and the logical foundations of mathematics, and published several books, among others [Wilkosz *1925*, *1932*]. In the thirties he attracted some young mathematicians to logic, among them Jerzy Kuczyński, known as an unsuccessful critic of Gödel's theorems (see [Kuczyński *1938*]). In general, the Cracow logicians, other than Chwistek, can be compared with the followers of Peano in their primary focus rather on the logic of mathematics than on mathematical logic.

Śleszyński's position in logic was not filled after his retirement (1924). In 1937, the University created a new professorship in logic and Zawirski was appointed professor; this position was connected rather with philosophy than mathematics. Zawirski's activity was too short (only two years before the World War II) to bring results in the training new logicians.

4. WARSAW 1900-1939

German troops very soon took Warsaw in World War I. The superpowers which partitioned Poland in the eighteenth century made several efforts to gain the sympathies of Poles in 1914-1915. In particular, the German authorities agreed to reopen the Polish Warsaw University. It happened in 1915. Łukasiewicz was appointed as professor of philosophy. He began his lectures on logic for mathematicians and philosophers. Kazimierz Kuratowski, one of the most distinguished Polish mathematicians, reports on these lectures in the following way ([Kuratowski *1980*, 23-24]):

> Jan Łukasiewicz was another professor who greatly influenced the interests of young mathematicians. Besides lectures on logic and the history of philosophy, Professor Łukasiewicz conducted more specialized lectures which shed new light on the methodology of the deductive sciences and the foundations of mathematicial logic. Although Łukasiewicz was not a mathematician, he had an exceptionally good sense of mathematics and therefore his lectures found a particularly strong response among mathematicians [...] I remember a lecture of his on the methodology of the deductive sciences in which he analyzed, among other things, the principles which any system of axioms should satisfy (such as consistency and independence of axioms). The independence of axioms in particular was not always observed by writers and even in those days was not always exactly formulated. Łukasiewicz submitted to detailed analysis Stanisław Zaremba's *Theoretical Arithmetic* (1912) which was well known at that time, questioning a very complicated principle formulated in that work, which was supposed to replace the rule of the independence of axioms. The criticism was crushing. Nevertheless, it brought about a polemical debate in which a number of mathematicians and logicians took part in the pages of the *Philosophical Review* (1916-1918). I mention this because a byproduct of Łukasiewicz's idea in our country was the exact formulation of such notions as those of quantity, the ordered set, and the ordered pair (the definition of the ordered pair which I proposed during the discussion was to find a place in world literature on the subject). This illustrates the influence brought by Jan Łukasiewicz, philosopher and logician, on the development of mathematical concepts.

This quotation gives evidence that Łukasiewicz propagated logic among students of mathematics in Warsaw very successfully. However, his enthusiasm for logic, together with his great skills as a teacher, would certainly not sufficient to alone account for the later enormous success of mathematical logic at Warsaw University. The decisive point for the rise and development of the Warsaw Logic School was the place of logic in the program of mathematics, elaborated by the founders of the Polish Mathematical School, Janiszewski, Sierpiński, and Stefan Mazurkiewicz; this project is called the Janiszewski program for Janiszewski's role in its final version. For them, the future of Polish Mathematics is in research connected with the new branches of mathematics, namely set theory and topology as well as their applications to the classical parts of mathematics, like algebra, geometry, or analysis. This project also attributed an important role to mathematical logic and the foundations of mathematics. Thus, according to the program of the development of mathematics in Poland (usually called the Janiszewski program), both fields found their place at the very center of mathematics.

The Janiszewski program also led to some practical solutions in the sphere of the organization of scientific life and the university. The

Department of Philosophy of Mathematics at the Faculty of Mathematical and Natural Sciences was founded very soon. Leśniewski was appointed as professor; he was recommended by Sierpiński. Why Leśniewski, not Łukasiewicz? The latter left the University for a position in the government under Paderewski; Łukasiewicz acted as the Ministry of Religions and Education. However, Paderewski's government fell after one year and Łukasiewicz wanted to come back to the University. The special position in philosophy was established for him at the Faculty of Mathematical and Natural Sciences, but, in fact, it was a position in mathematical logic. Thus, the University of Warsaw had two professors of logic. Another sign of the role of mathematical logic in the Warsaw Mathematical Circle is connected with the history of *Fundamenta Mathematicae*, a famous Polish journal. The first idea was to publish it in two series of which one should be entirely devoted to logic and the foundations of mathematics. Ultimately, the journal was published without being divided into series, but logic was heavily present in it. Mazurkiewicz, Sierpiński, Leśniewski and Łukasiewicz constituted the editorial board: two mathematician and two logicians.

These facts are remarkable from the sociological point of view. The appointment of two non-mathematicians as professors of logic at the Faculty of Mathematical and Natural Sciences was a brave sociological experiment, if we remember that when Hilbert passed to logic and the foundations of mathematics, many of his colleagues said that he went crazy or lost his creative powers for doing mathematics. Thus, the insertion of nonmathematicians among mathematicians was always a risky move. However, this move profited Warsaw greatly, at least from the point of view the subsequent development of logic.

The situation of logic in the Warsaw mathematical community explains why mathematical logic developed in Warsaw much better than in any place else in Poland. A surprising fact is that up to 1918 Cracow was the strongest logical circle in Poland. In order to see this, we can *e.g.*, compare two books: [Łukasiewicz *1910*] and [Chwistek *1912*]. Both books concern very similar topics and were published approximately at the same time. Łukasiewicz's book is beautiful from the philosophical point of view, but based on a very poor logical skeleton. While Łukasiewicz employed tools accessible from [Couturat *1905*], Chwistek used ideas of *Principia Mathematica* extensively. Thus, from the point of view of logic, the very epoch in the development of logic divides both books. Śleszyński's [*1925-1929*] *Proof Theory* probably covers the material of his lectures since 1914. This book gives evidence that Śleszyński's courses in mathematical logic were quite advanced and related the structure of logic given by *Principia Mathematica*. On the other hand, we have reasons to think that Łukasiewicz's lectures in Warsaw before 1918 did not exceed very much the level of Couturat's book. As a matter of fact, Couturat's [*1905*] book was translated into Polish as the first volume in the series of textbooks in mathematics, and was published in 1918. As we read in its preface, Łukasiewicz initiated this translation. This is indirect evidence that he used this book as a textbook for students. In order to be fair, one must also note that people in Warsaw were quite conscious of the defects of Couturat's book. Bronisław Knaster, the translator, writes in the preface to the Polish edition ([Knaster *1918*, p. III]):

> As a deductive theory Couturat's work — when seen in the light of recent requirements — is not free from certain defects of composition, incorrect formulations, and inexact arrangement.

Łukasiewicz probably pointed out himself in his lectures what was wrong with Couturat, but we do not know whether his criticism was made from the point of view of the *Principia* as a point of reference for logical novelties. However, it is rather certain that the Fregean-Russellian model of logic, according to which logic starts with propositional calculus and proceeds to quantificational theory, was adopted in Cracow earlier than in other places in Poland. Why then did Warsaw become the capital of logic in Poland?

The explanation rests on the attitude to logic among mathematicians. In Cracow, logic was respected, but regarded as something introductory for mathematics, as its servant. This attitude led the mathematicians in Cracow to place logic on peripheries of mathematics, whereas the people in Warsaw were convinced of its crucial significance. This difference was central. In the twenties and thirties, there was a great controversy among Polish mathematicians as to how mathematics should be understood. The mathematicians from Warsaw saw mathematics as based on logic and set theory, but the scholars from Cracow were inclined to think that logic and set theory were not important for mathematics. In this way, Cracow continued Poincaré's style of thinking, probably connected with his influence on Zaremba. This ideology considerably restricted the development of mathematical logic at the Jagiellonian University. On the other hand, no such limitations were present in Warsaw. Lvov was more neutral in this controversy. The mathematicians of Lvov were not hostile toward logic, but, on the other hand, they did not particularly support it. When Chwistek became professor at Lvov, he was kindly welcomed, but did not place his field as central. This explains why logic in Lvov developed better than it did in Cracow, but worse than it did in Warsaw, and why the difference between Lvov and Warsaw occurred not simply because Chwistek's ideas were somehow esoteric.

Leśniewski and Łukasiewicz, the new professors of logic with a philosophical past, had to change their scientific profile. The change was really very considerable. This is reported by Leśniewski *1927*, 181-182; 197-198]:

> In the year 1911 (still in my student years) I came across a book by Jan Łukasiewicz about the principle of contradiction in Aristotle [...] This book, which in its time had a considerable influence upon the intellectual development of a number of 'philosophers' and the 'philosophizing' scholars of my generation, became a revelation to me in many respects and for the first time I learned of the existence of the 'symbolic logic' [...] of Bertrand Russell as well as his 'antinomy' regarding the 'class of classes which are elements of themselves' [...] The first encounter with 'symbolic logic' created within me a strong aversion to that discipline for a number of years to come. Even the expositions of 'symbolic logic' which I attempted in succession, wishing somehow to absorb the results reached by the exponents of this science, were incomprehensible to me, and not because of my own fault, as I once thought. Steeped in the influence of John Stuart Mill on which I mainly grew up, and 'conditioned' by the problems of 'universal grammar' and of logico-semantics in the style of Edward [should be of course: Edmund; it is a mistake in the English edition — J. W.] Husserl and by exponents of the so-called Austrian School, I ineffectually attacked the foundations of 'logistic' from this point of view. Not possessing the faculty of entering into the spirit of other people's ideas, I was estranged from the science itself by the considerable effect of the obscure and ambiguous comments provided by its exponents. The decidedly skeptical dominant note of the position I occupied for a number of years in relation to 'symbolic logic' stemmed from the fact that I was not able to become conscious of the real 'sense' of the axioms and theses of that theory, 'of what' and 'what', respectively it was desired to 'assert' by means of axioms and theorems. (Leśniewski *1927*, 181–182) [...]
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> [...] Living intellectually beyond the sphere of the valuable achievements of the exponents of 'Mathematical Logic', and yielding to many destructive habits resulting from the one-sided, 'philosophical' grammatical culture, I struggled in the works mentioned [*i.e.*, works written in 1911–1915 — J. W.] with a number of problems which were beyond my powers at that time, discovering already-discovered Americas on the way. I have mentioned those works

desiring to point out that I regret that they have appeared in print, and formally 'repudiate' them herewith, though I have already done this within the university faculty, affirming the bankruptcy of the 'philosophical'-grammatical work of the initial period of my work [...]

We have also an interesting report by Łukasiewicz [*1936*, 227-228]:

My critical appraisal of philosophy as it has existed so far is the reaction of a man who, having studied philosophy and read various philosophical books to the full, finally came into contact with scientific method not only in the theory, but also in the direct practice of his own creative work. This is the reaction of a man who experienced that specific joy which is a result of a correct solution of a uniquely formulated scientific problem, a solution which at any moment can be checked by a strictly defined method and about which one simply knows that it must be that and no other and that it will remain in science once for all as a permanent result of methodical research. This is, it seems to me, a normal reaction of every scientist to philosophical speculation. Only a mathematician or a physicist who is not versed in philosophy and comes into casual contact with it usually lacks the courage to express aloud his opinion of philosophy. But he who has been a philosopher and has become a logician and has come to know the most precise methods of reasoning which we have at our disposal today, has no such scruples.

The conversion of both was different. While Łukasiewicz came to mathematical logic *via* his success as a teacher, Leśniewski changed his primary attitude through his own personal reflections. Independently of motives, both reports are interesting from the "ideological" point of view. They show that Leśniewski and Łukasiewicz worked in logic not only in order to fulfill their teaching duties, but also show that they accepted the very way of thinking in logic. Both stress that logic is at the pinacle of the scientific methods, at least so far as the matter concerns precision. Thus, Twardowski's ideal of clarity found its new expression in mathematical logic. It was very important. Of course, Łukasiewicz and Leśniewski could not be mathematicians in the normal sense, as they were not able to do mathematics, even as theory. They had to do mathematical logic. And it was accepted by Sierpiński and other mathematicians in Warsaw: they neither expected nor demanded that Łukasiewicz and Leśniewski would be working mathematicians. And they did not protest when gifted students of mathematics decided to concentrate on mathematical logic. To be fair to the history, this harmony came to an end in the thirties. In 1929, Leśniewski published in *Fundamenta Mathematicae* a long essay on the foundations of mathematics (see [Leśniewski *1929*]). Sierpiński made some very critical and sarcastic comments on this paper, perhaps also because Leśniewski had previously criticized standard set theory.[10] In any case, Leśniewski felt offended, withdrew the second part of the paper from *Fundamenta*, and left its editorial board. Łukasiewicz felt obligated to show solidarity with his colleague (and a very close friend) and he also resigned from his post with *Fundamenta*. Although this incident ended the almost ideal cooperation of mathematicians and logicians in Warsaw, the position of logic at the Faculty of Mathematical and Natural Science was sufficiently strong to survive this affair.

The climate around logic in Warsaw was also created by philosophers. In 1919, Kotarbiński became professor of philosophy at Warsaw University. He was sympathetic to logic due to his general metaphilosophical principles. He lectured on logic for philosophers and students of other fields. He influenced many young people not only for his magnificence as a teacher, but also for his unusual personality.[11] Logic also had friends at the Faculty of Theology. Father Stanisław Kobyłecki recommended that his students attend courses of logic. These facts created a unique environment for logic in Warsaw.

Łukasiewicz and Leśniewski consciously aimed at creating a logical school in Warsaw. They succeeded for various reasons, among which were their teaching skills, the protection they received from mathematicians, and by having Twardowski's techniques of teaching, known by both from their students days in Lvov. It was also important that the founders of the Warsaw Logical School were different scientific personalities. This is documented by Bolesław Sobociński [*1956*, 42-43]:

> There is an interesting contrast [...] between the two great figures of the Warsaw School of Logic, Łukasiewicz and Leśniewski. The latter was also a philosopher by training; he too moved away from philosophy and avoided even philosophical "asides" in his published work. But, unlike Łukasiewicz, he held that one could find the "true" system in logic and in mathematics. His systematization of the foundations of mathematics was not meant to be merely postulational; he wished to give, in deductive form, the most general laws according to which reality is built. For this reason, he had little use for any mathematical or logical theory which, even though consistent, he did not consider to be in accord with the fundamental structural view of reality [...] Thus, in a sense, though he never mentions philosophy, Leśniewski may be regarded as a philosopher of logic, one of the greatest in this small group. Łukasiewicz had no such preoccupation. He did not try to construct a definite system of the foundations of the deductive sciences. His aims were, on the one hand, to provide exact and elegant structures for many domains of our thinking where such had either been wanting or insufficient; and on the other, to restore the vital historical dimension to logic.

This contrast, quite accidental from the objective point of view, gave to student seriously interested in logic an opportunity to find a variety of insights in teaching presented by both professors. Leśniewski and Łukasiewicz very soon found great students. Alfred Tarski was the first of them, and his extraordinary scientific activity became one of the main pillars of the Warsaw Logical School. Tarski began his studies with Leśniewski and prepared his doctoral thesis under his supervision (1923).[12] Then, he moved to set theory. After habilitation (1924), Tarski (he is the youngest docent in the history of Polish mathematics) began his metamathematical studies and already in the middle of the twenties he became the third leader of the School. Other members of this group included (in the alphabetical order): Stanisław Jaśkowski, Adolf Lindenbaum, Andrzej Mostowski, Moses Presburger, Jerzy Słupecki, Bolesław Sobociński, and Mordechaj Wajsberg; Jaśkowski, Lindenbaum, Presburger, Sobociński,, and Wajsberg graduated in the twenties, Mostowski and Słupecki in the thirties (Sobociński in philosophy, the rest in mathematics). We must also mention a number of less well-known people, such as Jerzy Billig, Jerachmiel Brykman, Zygmunt Kobrzyński, and Zygmunt Kruszewski, participated in logical research in Warsaw, and such as Kazimierz Pasenkiewicz, who began his academic career after 1945. In the last years before World War II, Czesław Lejewski graduated (1939), and Henryk Hiż began his studies. Was it a big or a small group? Of course, everything depends on a point of reference. Judging from the present point of view, a dozen or so persons working together on logic is perhaps not so many. However, if we look at the Warsaw group from the perspective of the interwar period, we must remember that no other place at which logic was done at that time had even one third this amount. Thus, Warsaw was certainly the most "logically" populated place in the world. However, the Warsaw logical community was not limited to the Warsaw School of Logic. We must remember that several philosophers (for example, Janina Hosiasson, later Mrs. Lindenbaum, Dina Sztejnbarg, later Mrs. Kotarbiński, Jan Drewnowski, and Edward Poznański) and mathematicians (for example, Kuratowski) were well trained and deeply interested in logic. Still other persons, for example Father Jan Salamucha, came from the Faculty of Theology.

Although the Warsaw Logical School was strongly unified by a common scientific concern, it was quite complex from the sociological point of view. Some of its members, like Lindenbaum, were rich, but others, like Tarski, rather poor; some, like Lindenbaum, were leftists, others, like Łukasiewicz, shared conservative opinions; some, like

Łukasiewicz, twice elected as the Rector Magnificus, were VIP's of the academic life, others, like Tarski, taught in secondary schools, or, like Presburger, worked as modest clerks; some (Tarski, Lindenbaum, Presburger, Wajsberg) were Jews, others (Leśniewski, Sobociński radical antisemites. These differences created various social problems and tensions of which perhaps the Jewish question was the most important. It is well documented by Leśniewski's comments concerning the professorship for Tarski (a letter to Twardowski from September 8, 1935). At first, Leśniewski recognized that Tarski deserved to be appointed professor, because his research is specialized, but then he writes:

> [...] according to several facts from the last years [...] I have several aversions to Tarski. Although I tend [to dislike him] and, according to the reason given above, I will still do everything which is dependent of me in order that he may obtain the professorship in Warsaw, I admit however (because I can be responsible only for my deeds, but not for feelings), that I would be quite pleased by reading someday in newspapers, that the ordinary professorship had been offered to him, for example in Jerusalem, from which place he could send us, with a great profit, offprints of his valuable papers.[13]

However, it seems that the common logical enterprise made these differences easier to tolerate. Tarski once said (personal communication by Hiż: religion divides people, logic brings them together. We can replace "religion" by "ideology" in this context. The history of the Warsaw Logical School documents that this opinion is correct, at least to some extent.[14] In particular, there was a considerable cooperation in the School what is well-documented by the following quotation (see [Łukasewicz & Tarski *1930*, 219]):

> In the course of the years 1920-1930 investigations were carried out in Warsaw belonging to that part of metamathematics — or better metalogic — which has as its field of study the simplest deductive discipline, namely the sentential calculus. These investigations were initiated by Łukasiewicz, the first results originated both with him and with Tarski. At the seminar for mathematical logic which were conducted by Łukasiewicz in the University of Warsaw from 1926, most of the results stated below of Lindenbaum, Sobociński and Wajsberg were stated and discussed. The systematization of the concept was the work of Tarski.

One can find a similar record in Łukasiewicz's preface to his textbook of logic [Łukasiewicz *1929*, 9]:

> I owe most, however, to the scientific atmosphere which had developed in Warsaw University in the field of mathematical logic. In discussions with my colleagues, especially Professor S. Leśniewski and Dr. A. Tarski, and often in discussions with their and my own students, I have made clear to myself many concepts, I have assimilated many ways of formulating ideas and I have learned about many new results, about which I am today not in position to say to whom the credit of authorship goes.

Łukasiewicz's and Leśniewski's seminars were natural places for giving papers in logic. However, there were also other organizational forums available. The Warsaw Philosophical Society had a special section of logic. Logical works were presented to the Warsaw Scientific Society. Łukasiewicz, who was the main *spiritus movens* of the Warsaw Logical School, undertook efforts to organize purely logical institutions, completely independent of any other scientific fields. In 1936, the Polish Logical Society was established (April 22, 1934). Łukasiewicz also tended to favor establishing a professional logic journal, although Polish logicians had no problems with finding places to publish their writings. They published in philosophical journals (*Przegląd Filozoficzny*, *Kwartalnik Filozoficzny*, *Ruch Filozoficzny*, *Studia Philosophica*, mathematics journals (*Fundamenta Mathematicae*, *Wiadomości Matematyczne)*, and transactions of various scientific societies. In 1934, Łukasiewicz initiated *Studia Logica*, a series of publications devoted to logic. However, only one work was published in it, namely Jaśkowski 1935. Ultimately, Łukasiewicz succeeded in his efforts and the two first volumes of *Collectanea Logica* were prepared in 1939; unfortunately, all copies were destroyed in September of that year.

These deeds of Łukasiewicz resulted from a special ideology concerning logic and its place in science. According to Łukasiewicz, logic is an autonomous subject which is subordinated neither to philosophy nor to mathematics. On this view, logic is no servant of any other science. A subject having this position deserves its own societies and journals, because not all needs of logic can be fulfilled by other fields, even those very close to logic.

Although this ideology was developed by Łukasiewicz, we find its roots in some views of Janiszewski, who defended logic against Poincaré's criticisms even before 1918, *i.e*, before the beginnings of the Warsaw Logic School. In particular, Janiszewski defended the value of logic as a theoretical science, independently of its practical applications, for example in mathematics. Probably Janiszewski's views on this question were accepted by other mathematicians in Warsaw. Although

Sierpiński was irritated by some peculiarities and, according to him, oddities, of Leśniewski's way of doing logic, he certainly had nothing against Łukasiewicz's style, which was much less philosophical and more concurrent with ordinary mathematical practice. In any case, this ideology, though perhaps somehow strange from the present point of view, was of the utmost significance for the development and scientific success of logic in Warsaw. Now, we see that this success was a result of a quite conscious and systematically realized enterprise. Certainly, the fathers of the Warsaw Logic School had a bit of luck. For example, it was a lucky event that Alfred Tarski appeared as a student about 1920 and that he decided to work in logic, or that Adolf Lindenbaum moved from topology to logic. However, any important achievement in science is connected with lucky opportunities, but these usually act in favour of those who are better.

5. ADDITIONAL REMARKS

Besides Cracow, Lvov and Warsaw, Poland had three other universities in the interwar period, namely in Wilno, Poznan and Lublin; the last was a Catholic university. The professorship for logic and methodology of science was established in Poznan in 1919. Władyslaw Kozlowski was appointed professor. However, he was not a logician in the strict sense; he was rather interested in methodology of science than in logic. After his retirement in 1928, this position was given to Zawirski. He occupied this post up to 1937, but he did not train many people; only Zbigniew Jordan should be mentioned in this context. In 1937, Zawirski moved to Cracow (see section 3 above). The position in Poznan was not occupied in 1937-1939.[15] Neither Wilno nor Lublin had positions for logic. However, Czeżowski, professor of philosophy at Wilno, lectured on logic there and influenced some people, among them Father Antoni Korcik. There is nothing in particular to say about logic at Lublin Catholic University during this time.

Thus, Poland had five professorships in mathematical logic in 1939, two in Warsaw, one in Cracow, one in Lvov, and one in Poznan. It is surprising that outside Poland there was only one, namely in Münster in Germany, where Heinrich Scholz was professor. This makes a point about the status of logic in the interwar Poland. But not only this.

The subject was extensively taught in universities and secondary schools. The textbooks of logic show that the level of logical training was high. Even in secondary schools, the teaching far exceeded the traditional pattern which usually was restricted to syllogisms. Polish students were trained also in propositional calculus and predicate logic. At universities, logic was included in courses on the main problems of philosophy. The scope of logic was dependent on particular professors. For example, the course conducted by Kotarbiński (see [Kotarbiński *1929*]) covers propositional calculus, calculus of names (Leśniewski's ontology), traditional logic, and selected problems of the methodology of deductive sciences. Zawirski's textbook (see [Zawirski *1938*]) provides another example. Its table of contents is the following (theory of propositional variables = propositional calculus, theory of names variables = predicate calculus): I. Introduction 1. A general characterization of logic; 2. Semantic categories. Names and sentences. Truth-function and truth-functors; 3. The division of logic. Remarks on the relation of traditional logic to modern logic; II. The Theory of Propositional Variables. 1. Possibilities of different systems of propositional variables; 2. The theory of deduction based on disjunction and negation; 3. The methodology of the theory of deduction; 4. Remarks on the theory of deduction with quantifiers; III. The theory of names variables. 1. Propositional functions with one name variable; 2. Traditional logic; 3. On antinomies.

Of course, the lectures of logic for students of mathematics had a special character. Every student of mathematics had to complete a general course in logic. The level in Warsaw is well illustrated by [Łukasiewicz *1929*] or even by the more elementary [Ajdukiewicz 1928];[16] it is perhaps interesting that a general course of logic for mathematicians also covered some philosophy of science. The students especially interested in logic could choose more specialized courses. For example, Leśniewski, Łukasiewicz, and Tarski conducted separate courses and seminars in Warsaw. Thus, students in Warsaw could participate in different logic classes in any year of their studies. Thus it is hardly strange that those students of mathematics in Warsaw who decided to specialize in logic graduated as competent logicians and were immediately prepared to undertake creative scientific activity. Very often masters theses contained important results. For example, Słupecki solved the problem of the functional completeness of three-valued logic in his thesis. The demands for doctoral theses were very high, which sometimes resulted in strange (for us) verdicts. For example, Tarski recognized that Presburger's famous results on the completeness of arithmetic with addition as the sole operation was insufficient for earning the doctorate.

Polish logicians had numerous international contacts. Scholz, Quine, Carnap, Menger and Zermelo visited Poland and were greatly impressed by the work of Polish logicians. Poles revisited Münster (Łukasiewicz, Leśniewski) and Vienna (Tarski) as well as participated in many congresses, particularly in Prague (*1934*) and Paris (*1935*). Tarski's visits in Vienna were recognized as very important events (see [Menger *1994*, Chapter XIII]). And his lectures on semantics in Paris in 1935 are described (see [Ayer *1977*, 116]) as the philosophical highlight of the Congress. The novelties in logic from around the world were well known and quickly assimilated in Poland. In particular, Łukasiewicz and Leśniewski very early appreciated fully the greatness of Frege. Łukasiewicz writes in his *Memoirs* (pp. 57-58):

> The first volume of Husserl's logical investigations made a great impression in Lvov [...] However, the second volume disappointed me. It contains obscure philosophical talk, which repels me from all German philosophers. I wondered that such a difference can hold between two volumes of the same work. Later, I became convinced that it was not Husserl who spoke in the first volume, but someone much greater used by him, namely Gottlob Frege.

Russell [*1959*, 86] once considered how many readers had read everything in *Principia Mathematica*:

> I used to know of only six people who read the later parts of the book. Three of these Poles, subsequently (I believe) liquidated by Hitler.

Although Russell gave no names, we may guess that Chwistek, Leśniewski and Tarski were the Polish readers of the entire *Principia*;[17] Leśniewski even considered translating the *Principia* into Polish.

Gödel discovered his famous incompleteness theorems in 1930. These results were published in 1931. On April 15, 1931, Tarski delivered in Warsaw a talk "On a Certain System of Mathematical Logic, and the Methodological and Semantic Problems Following from It", in which he informed his listeners about the first Gödel incompleteness theorem. It was probably the first foreign report on this celebrated result.

The status of logic in Poland at that time can be also be testified to by inspecting the programs of Polish philosophical congresses (Lvov *1923*, Warsaw *1927*, Cracow *1936*), which had separate section of logic. Logic also considerably influenced Polish philosophy and contributed to the triumph of analytic philosophy in this country. This influence also touched catholic philosophy. In 1936, the so called Cracow Circle was organized. It consisted of Father Józef Bocheński, Drewnowski, Father Salamucha and Sobociński. The Circle proposed a renovation of catholic philosophy through mathematical logic. Two great ideas of Polish analytic philosophy, Kotarbiński's reism and Ajdukiewicz's radical conventionalism were strongly motivated by logic. This shows that mathematical logic in interwar Poland far exceeded its strictly professional scope.

4. AFTER 1939[18]

World War II was disasterous for Polish logicians. The Lindenbaums, Pepis, Presburger, Salamucha, Schmierer and Wajsberg were killed by the Germans, Hetper and Herzberg perished in the Soviet Union, and Chwistek died in 1944 in Moscow, Leśniewski died just before the war, Smolka in 1947, and Zawirski in 1948. Bocheński, Hiż, Jordan, Lejewski, Łukasiewicz, Mehlberg, Poznański, Sobociński and Tarski left Poland in 1939-1948. The war also stopped normal education. Several completed writings were destroyed or lost.

I already mentioned the fate of *Collectanea Logica*. Of other losses, Leśniewski manuscripts, including his book on antinomies, are perhaps the greatest loss. Very often people had to choose between keeping their scientific notes and food. In his recollections, Mostowski (see [Crossley *1975*, 33]) says that he decided to take bread when he had to leave Warsaw after the Warsaw Uprising in 1944; he left notes concerning the independence of the continuum hypothesis. However, this time was not entirely lost. Poles organized an enormous system of clandestine education comprising high schools as well as universities. Logic was taught at both levels, with some success in training logicians; for example Andrzej Grzegorczyk and Jan Kalicki (he later left Poland) graduated at that time.

After 1945, Poland quickly restored its academic and scientific life. It was not an easy task, partly as a result of geopolitical changes: the country lost its former eastern parts, but extended to the west. Lvov and Wilno were now outside Poland. New universities were organized in Lodz, Lublin (the state university), Torun and Wroclaw. The positions in logic were reestablished in Warsaw, Cracow and Poznan, and established in other universities, including the Lublin Catholic University. Some departments of logic were located in faculties of mathematics and natural sciences, others in faculties of humanities or philosophy and history, depending on the organization of the faculties of humanities or philosophy and history, in turn depending on the organization of a given university. The situation was not stable and locations of the logical department changed, for example in Lublin (from mathematics to the humanities). The losses caused by deaths and emigration were difficult to fill. The four greatest logical masters of prewar Poland passed away (Leśniewski, Chwistek) or decided to live abroad (Łukasiewicz, Tarski).[19] Only three members of the Warsaw School of Logic remained in Poland. Mostowski took the leading position after Leśniewski; Jaśkowski went to Torun; and Słupecki went at first to Lublin (the state university), then to Wroclaw. Ajdukiewicz moved to Poznan, Czeżowski to Torun (as professor of philosophy), Father Korcik to Lublin (as professor of logic at the Catholic university), the Kotarbiński's worked in Warsaw and Lodz. Zawirski taught until his death in Cracow, then his position was left unfilled for three years, and later was taken by Pasenkiewicz. Warsaw remained the most important logic center in Poland, and Wroclaw became another place in which logic and the foundations of mathematics flourished. The Wroclaw mathematical community was formed by mathematicians from Lvov with Steinhaus as the dominant figure. Edward Marczewski was another mathematician who influenced the development of foundations.

We should also note remarkable the textbooks of logic, above all [Mostowski *1948*], which certainly had nothing equal in the world at that time. It was the first general textbook of mathematical logic which covered Gödel's incompleteness results and Tarski's undefinability theorem.[20] An interesting book was written by Czeżowski (Czeżowski *1949*]). Both books, although published after the war, were clearly based on prewar standards and show how advanced teaching of logic in Poland was. This was the situation about 1950.

Until 1948, the academic life in Poland followed the pattern that existed before 1939. Later, the communist political power began to tend toward changes modeled on the system of the Soviet Union. The most difficult period took place in 1950-1954. Surprisingly, this time was not particularly bad for logic. The number of positions even increased, because several departments of philosophy were transformed into departments of logic. At some universities, for example in Torun and Wroclaw, the number of departments of logic doubled. Kotarbiński, Czeżowski and Kokoszyńska (Wroclaw), formerly professors of philosophy, became professors of logic. When Ajdukiewicz moved from Poznan to Warsaw (1954), a new logic department was established for him at the university. New logical departments arose in the Polish Academy of Sciences: one in the Institute of Philosophy and Sociology and the second in the Institute of Mathematical Sciences. Departments of logic also acted outside universities. A particularly good circle was organized by Słupecki at the Pedagogical College in Opole. Teaching of logic was extended, for example for students of legal faculties. Ajdukiewicz initiated *Studia Logica*, a specialized logic journal; the first volume appeared in 1953. Although orthodox Marxists wanted to subordinate logic to dialectics, the former defended itself. Moreover, some Polish Marxists, for example Adam Schaff, began to stress the autonomy of logic and even argued that dialectic must preserve logical rules. These facts well tested the strength of logic in Poland. It was the real inheritance of the prewar period. However, some bad things also happened. The field of logic was sharply divided into two parts: logic and the foundations of mathematics. It was partially concurrent with tendencies internationally; but it also resulted from the political situation. Simply speaking, mathematicians specializing in logic were afraid of being controlled by Marxism and preferred to be rather pure mathematicians than logicians or philosophers. This was justified by the fact that logic was officially regarded as one of the so-called ideological disciplines. Thus, the close cooperation between logicians with mathematical origins and logicians with philosophical origins, formerly one of the cornerstones of Polish logic and its power, was over, and came to be replaced by the more or less peaceful coexistence between logic and the foundations of mathematics. The research tradition of the golden period was continued by the "old" people, like Jaśkowski, Mostowski, and Słupecki, as well as the new generation, including among others, Ludwik Borkowski, Andrzej Grzegorczyk, Tadeusz Kubiński, Jerzy Łoś, Jan Mycielski, Helena Rasiowa, Czesław Ryll-Nardzewski, Roman Sikorski, Wanda Szmielew, and Roman Suszko.

In order to complete this report, let me say a word about logicians who left Poland after 1939. Łukasiewicz was invited to Ireland and appointed professor of logic at the Royal Academy in Dublin. Tarski after some difficult years in the USA, became a professor at the University of Berkeley in 1946, where he very successfully continued his teaching and research. In particular, he created the California School of Logic, carrying out to some extent the Warsaw pattern. He also propagated the idea of the autonomy of logic. For example, Tarski declared (in a letter to Alonzo Church, October 7, 1946):[21]

> I cannot deny, however, that personally I should be happy if also another type of article appeared in the *Journal* [*of Symbolic Logic* — J. W.] in a larger amount than they appear so far; in fact articles which could be regarded as belonging not to logic in the strict sense but to philosophy, to mathematics, or to other disciplines — under the condition, however, that these articles either apply methods of modern

> logic in an essential way or have implications which are essentially relevant to logic.

Kalicki also taught at Berkeley. Bocheński became a professor at the University of Freibourg, Hiż at the University of Philadelphia, Jordan at the University of Ottawa, Lejewski at the University of Manchester, Mehlberg at the University of Chicago, Poznański at the University of Jerusalem, and Sobociński at Notre Dame University. Thus, all the emigrants found academic posts — which gives strong testimony to the quality of logic and analytic philosophy in prewar Poland.

II. IDEAS AND RESULTS

7. LOGIC, PHILOSOPHY, MATHEMATICS

Mathematical logic is a specialized field, more similar to mathematics than to anything else. However, its development was closely connected with various philosophical backgrounds and controversies. In particular, formal research was surrounded by some philosophical insights in the sense that logical systems should serve as a justification of general questions concerning mathematics, for example its relation to logic. Mathematical logic from the beginning was closely connected with great schools in the foundations of mathematics, namely logicism, formalism and intuitionism. Logicians belonging to particular schools, especially in the early stage of the development of mathematical logic (1900-1930) were often interested in different logical problems and even in systems than were their colleagues from other camps. Certainly the Hilbertians stressed different points than the Russellians did, and the Brouwerians looked at different aspects than the others. How was it in Poland?

We can divide Polish logicians into two groups. Leśniewski and Chwistek based their research on explicit philosophical presuppositions, and in this respect, they intended to give the foundational schemes similar to logicism. As a matter of fact, Chwistek in his earlier stage (before moving to Lvov) tried to improve the Russellian ramified theory of types by deleting existence axioms, in particular the axiom reducibility. He elaborated a version of the simple theory of types. This effort was evaluated very highly by Russell himself [Russell & Whitehead *1925*, vol. 1, p. XIII]:

> [...] Dr. Leon Chwistek [...] took the heroic course of dispensing with the axiom [of reducibility — J. W.] without adopting any substitute [...]

Then Chwistek passed to the construction of own logical system (see section 8.f2 below). Leśniewski also produced a foundational *logica magna*, but he went his own way from the beginning (see section 8.f1 below).

However, the rest of the Warsaw Logical School was not bound by any philosophical ideology. Łukasiewicz and Tarski were typical examples here. Both were ready to investigate any logical problem, independently of whether it originated in logicism, intuitionism, or formalism. Tarski stressed several times that his formal research did not assume any general foundational view. This attitude was genetically connected with the ideology of the Polish Mathematical School. Perhaps the clearest expression of this view can be found in Sierpiński [Sierpiński *1964*, 25]:

> Still, apart from our personal inclination to accept the axiom of choice, we must take into consideration, in any case, its role in the Set Theory and in the Calculus. On the other hand, since the axiom of choice has been questioned by some mathematicians, it is important to know which theorems are proved with its aid, and to realize the exact point at which the proof has been based on the axiom of choice; for it has frequently happened that various authors have made use of the axiom of choice in their proofs without being aware of it. And after all, even if no one questioned the axiom of choice, it would not be without interest to investigate which proofs are based on it and which theorems are proved with its aid — this, as we know, is also done with regard to other axioms.

A very similar opinion was voiced by Tarski:

> We would of course fully dispose of all problems involved [i.e., concerning inaccessible cardinals — J. W.], if we decide to enrich the axiom system of set theory by including (so to speak on a permanent basis) a statement which precludes the existence of "very large" cardinals, e.g., by a statement to the effect that every cardinal $> \omega$ is strongly incompact. Such a decision, however, would be contrary to what is regarded by many as one of the main aims of research in the foundations of set theory, namely, the axiomatization of increasingly large segments of "Cantor's absolute". Those who share this attitude are always ready to accept new "construction principles", new axioms securing the existence of new classes of "large" cardinals (provided they appear to be consistent with old axioms), but are not prepared to

> accept any axioms precluding the existence of such cardinals — unless this is done on a strictly temporary basis, for the restricted purpose of facilitating the metamathematical discussion of some axiomatic systems of set theory.

Tarski himself summarized very clearly the general position of the Warsaw logicians (see [Tarski *1986*, vol. 4, 713]):

> As an essential contribution of the Polish school to the development of metamathematics one can regard the fact that from the very beginning it admitted into metamathematical research all fruitful methods, whether finitary or not.[22]

It is also the second source of this philosophically-neutral attitude to formal investigations in logic. I mean here the general metaphilosophical view of the Lvov-Warsaw School. It was like this (see [Dąmbska *1948*, 17]):

> The philosophers of the Lvov group were not united by any common doctrine, by a uniform world-view. Not the content of philosophy but rather the method of philosophizing and the comment language were the factors which formed the foundation of the intellectual community of those people. This is why the school could produce spiritualists and materialists, nominalists and realists, logicians and psychologists, philosophers of nature and art theorists.

It is an interesting fact that views of Twardowski and Sierpiński arose in the same city, namely Lvov. We have no data about any possible influence between them, but perhaps this fact is remarkable.

However, it does not mean that "aphilosophical" logicians had no philosophical views of their own connected with logic. They had; and sometimes it led to a sort of a cognitive tension as in the case of Tarski (see [Mostoweski *1967*, 81):

> Tarski, in oral discussion, has often indicated his sympathies with nominalism. While he never accepted the 'reism' of Tadeusz Kotarbinski, he was certainly attracted to it in the early phase of his work. However, the set-theoretical methods that form the basis of his logical and mathematical studies compel him constantly to use the abstract and general notions that a nominalist seeks to avoid. In the absence of more extensive publications by Tarski on philosophical subjects, the conflict appears to have remained unresolved.[23]

Mostowski himself was close to constructivism. However, reviewing the difficulties in producing a textbook of mathematical logic, he says [Mostoweski *1948*, p. VI]):

> As far as the matter concerns the third difficulty connected with acceptance of a definite philosophical standpoint in the foundations of mathematics, I intentionally avoided touching those questions in the text, because they obviously exceed the scope of formal logic. I treat a logical system as a language in which one speaks about set and relations. I adopted the axiom of extensionality for these entities and I recognized that they obey the principles of the simple theory of types. This standpoint is a convenient base for developing formal problems and concurs with the more or less conscious view of most mathematicians, which does not mean at all it would have to be accepted by philosophers without any reservation [...] I am inclined to think that a satisfactory solution of the problem of the foundations of mathematics will follow the path pointed out by constructivism or in a direction close to it. However, it would be impossible to write a textbook of logic on this basis at the moment.

It is a very instructive fragment. Firstly, we have a clear distinction of the "official" science, mathematical logic in this case, and a "private" philosophy. Secondly, we have an equally clear preference for the needs of the "official" science. Thirdly, we have here a good summary of the set-theoretical ideology in doing mathematical logic and the foundations of mathematics which perhaps could be regarded as a continuation of logicism, although without its principal claim that mathematics is reducible to logic.

In this situation it is difficult to describe the philosophy of logic and mathematics of the Warsaw School of Logic. However, this task is not entirely hopeless, because we can find views more typical or more important than others, even if not shared by everybody in the School. Certainly, all logicians in the Warsaw School (or even in Poland) agreed (I mean here during the golden period) that logic is extensional. Thus, there is no logic of intensional contexts. This explains, for example, why Polish logicians were not particularly interested in modal logic as an extension of classical logic, because it leads to intensional modal logic, as in the case of the Lewis systems. This is why Łukasiewicz took many-valued logic as the base for his modal systems. Although Łukasiewicz's discovery of many-valued logic is certainly one of the most remarkable achievements of Polish logicians, most of them recognized the priority of the two-valued pattern. Practically, only Łukasiewicz and Zawirski voted for the superiority of many-valuedness for philosophical reasons; the others regarded many-valued logic as purely formal constructions, deserving attention, for example, for algebraic reasons, but not as the very rivals of classical logic.

Traditionally, the problems of how logic is related to reality, and what the epistemological status of logical theorems is, are among the most important ones in the philosophy of logic. Łukasiewicz discussed this question ([Łukasewicz *1936*, 233]) in connection with a choice between two-valued and many-valued logic. At first he believed that experience would decide this question:

> I think that in Carnap the attempt to reduce certain objective problems to linguistic ones results from his erroneous interpretation of the *a priori* sciences and their role in the study of reality. That erroneous opinion was taken over by Carnap from Wittgenstein, who considers all *a priori* propositions, that is, those belonging to logic and mathematics, to be tautologies. Carnap calls all such proposition analytic. I have also opposed that terminology, since the associations it evokes many make it misleading. Moreover, Carnap believes, together with Wittgenstein, that *a priori* propositions do not convey anything about reality. For them the *a priori* disciplines are only instruments which facilitate the cognition of reality, but a scientific interpretation of the world, could, if necessary, do without those *a priori* elements. Now my opinion of the *a priori* disciplines and their role in the study of reality is entirely different. We know today that not only do different systems of geometry exist, but different systems of logic as well, and they have, moreover, the property that one cannot be translated into an other. I am convinced that one and only one of these systems is valid in the real world, that is, real, in the same way as one and only one system of geometry is real. Today, it is true, we do not yet know which system that is, but I do not doubt that empirical research will someday demonstrate whether the space of the universe is Euclidean or non-Euclidean, and whether relations between facts correspond to two-valued logic or to one of the many-valued logics. All *a priori* systems, as soon as they are applied to reality, become natural-science hypotheses which have to be verified by facts in a way similar to the way in which is done with physical hypotheses.[24]

However, Łukasiewicz later changed his mind and accepted a position close to that of Carnap (see [Łukasiewicz *1952*, 333]):

> We have no means to decide which of the n-valued systems of logic is true [...] Logic is not a science of the laws of thought or of any real object; it is, in my opinion, only an instrument which enables us to draw asserted conclusions from asserted premises [...] The more useful and richer a logical system is, the more valuable it is.

However, most Polish logicians did not share adherence to conventionalism and pragmatism in logic. They rather agreed that logic refers to very general features of reality. Kotarbiński [Kotarbiński *1929*, 210-211] gave the following characterization of Leśniewski's calculus of names:

> We will add that Lesńiewski calls his system 'ontology' in harmony with certain terms used earlier (as in the 'ontological principle of contradiction', the thesis that no object may possess and not possess the same feature) as it was presented in Łukasiewicz's book *On the Principle of Contradiction in Aristotle* [...]) [...] It must, however, be admitted that if the Aristotelian definition of the supreme theory [...] be interpreted in the spirit of a "general theory of objects", then both the word and its meaning are applicable to the calculus of terms as expounded by Lesńiewski.

This interpretation was fully confirmed by Leśniewski himself (Leśniewski *1931*, 374]:

> *Generalizing the terminological preference of* Łukasiewicz *as cited* by Kotarbiński to which I became accustomed over a period of some years, and taking into consideration the relation existing between the single characteristic primitive term of my theory and the Greek participle [i.e., *on* — J. W.] explained by Kotarbiński, I used the name 'ontology' [...] to characterize the theory I was developing, without offence to my 'linguistic instincts' because I was formulating in that theory a certain kind of 'general principle of existence'.[25]

This realistic view of logic resulted in rejection of consideration of logical theorems as tautologies which are devoid of empirical content. Tarski criticized Carnap's sharp distinction of analytic and synthetic propositions. This is clearly expressed in his letter to Morton White (see [Tarski *1987*, 31]):

> I would like to be inclined (following J. S. Mill) to hold that logical and mathematical truth don't differ in their origin from empirical truth — both are results of accumulated experience [...] I think that I am ready to reject certain logical premises (axioms) of our science in exactly the same circumstances in which I am ready to reject empirical premises (e.g., physical hypotheses) [...] Axioms of logic are of so general a nature that they are rarely affected by [...] experiences in special domains. However, I don't see here any difference 'of principle'.

Thus, the prevailing view of Polish logicians was that formal sciences are basically empirical, although they are farther from experience than other elements of our knowledge.

Leśniewski [*1929*, 487-488] summarized his view on the nature of logic, called by him 'the intuitionistic (better 'intuitive') formalism', in the following way (see [Leśniewski *1929*, 487-488]):

> Having no predilection for various 'mathematical games' that consist in writing out according to one or another conventional rule various more or less picturesque formulae which need not be meaningful, or even — as some of the 'mathematical gamers' might prefer — which should necessarily be meaningless, I would not have taken the trouble to systematize and to often check quite scrupulously the directives of my system, had I not imputed to its theses a certain specific and completely determined sense, in virtue of which its axioms, definitions, and final directives [...] have for me an irresistible intuitive validity. I see no contradiction, therefore, in saying that I advocate a rather radical 'formalism' in the construction of my system even though I am an obdurate 'intuitionist'. Having endeavored to express my thoughts on various particular topics by representing them as a series of propositions meaningful in various deductive theories, and to derive one proposition from others in a way that would harmonize with the way I finally considered intuitively binding, I know no method more effective for acquainting the reader with my logical intuitions than the method of formalizing any deductive theory to be set forth. By no means do theories under the influence of such a formalization cease to consist of genuinely meaningful propositions which for me are intuitively valid. But I always view the method of carrying out mathematical deductions on an 'intuitionistic' basis of various logical secrets as a considerably less expedient method.

This passage is of the utmost importance for everyone who wants to understand the spirit of logical research in the Warsaw School. If we skip personal or autobiographical statements (indicated by indexicals) from the last quotation, we obtain a very good summary of the general attitude toward logic according to which it is not a meaningless activity but something which essentially deals with sense. Leśniewski once said (personal communication of Hiż: "logic is a formal exposition of intuition". Perhaps this is the best short summary of the philosophy of logic proposed by most Polish logicians.

Finally, let me mention some conditions of the "quality" of good formal systems. Apart from the obvious demand of consistency, Polish logicians demanded that logical systems should be as simple and economical as possible. They should be based on the minimal number of primitive notions, axioms and rules of inference. Hence, systems based on one primitive term and the one shortest axiom were sought. These efforts were sometimes strongly criticized, for example by Grzegorczyk (see [Grzegorczyk *1962*, 197-198]):

> They [Warsaw logicians — J. W.] were involved in a specific intellectual sport which consisted in shortening axiom systems. Many records in that field were established by that School. There are the single axioms, the shortest among those which are known, and even the shortest possible axioms for the various systems of propositional calculus. Usually those shortest axioms have no application in practice and are difficult to remember. There are axiom systems which are longer but much more intuitive, and which are in common use. Łukasiewicz was even a practioner of that sport; he claimed that the intuitive content of an axiom system is not important, since it is only the set of those theorems which are its consequences which counts. If an axiom system yields all the theorems which we want to have in a given theory, then that axiom system is good. In this sense, in Łukasiewicz's opinion, there are very good axiom systems, and the choice one makes depends upon one's personal preferences. This fully justifies the predilection for brevity, manifested by Łukasiewicz himself.[26]

However, the length of an axiom system is an objective feature which is perhaps fairly interesting to investigate independently of any application of logic to mathematics or any other special subject. The point is that the Warsaw Logical School considered logic as fully autonomous, as I have already stressed. Let me quote one more opinion (that expressed by Tarski, see [Tarski *1944*, 369]) which illustrates this point:

> Being a mathematician (as well as a logician, and perhaps a philosopher of a sort) I had the opportunity to attend many discussions between specialists in mathematics, where the problem of application is especially acute, and I have noticed on several occasions the following phenomenon: if a mathematician wishes to disparage the work of one of his colleagues, say *A*, the most effective method he finds for doing this is to ask where the results can be applied. The hard-pressed man, with is back against the wall, finally unearths the research of another mathematician *B* as the focus of application of his results. If next *B* is plagued with a similar question, he will refer to another mathematician *C*. After a few steps of this kind we find ourselves referred back to the researches of *A*, and in this way the chain closes.
>
> Speaking more seriously, I do not wish to deny that the value of a man's work may be increased by its implication for the research of others and for practice. But I believe, nevertheless, that it is inimical to the progress of science to measure the importance of any research exclusively or chiefly in terms of its usefulness and applicability.

This fragment explains why certain avenues of research carried out in Warsaw were undertaken seriously in spite of their oddity for people looking at logic through the lenses of their applicability.

8. A SURVEY OF RESEARCH AND RESULTS[27]

a. *Propositional calculus.*

Propositional calculus became the specialty of the Warsaw School and a laboratory of its logical research.[28] A special symbolism (bracket-free notation, Polish notation, Łukasiewicz's notation) under which logical operations are always written before their arguments without any need to use typical punctuation signs (dots, brackets) was particularly good for investigations of propositional calculus. More specifically, investigations in this field in Poland concerned:

• formulation of various systems of propositional calculus, including the most popular system based on implication and negation (Łukasiewicz) as well as systems based on negation and implication, negation and disjunction, and the Sheffer function as the sole functor;

• investigation of partial systems of propositional calculus based on equivalence, and implication (Łukasiewicz, Leśniewski, Wajsberg, Tar-ki);

• formulation of propositional calculus with quantifiers binding propositional variables;

• formulation of propositional calculus with variable functors (Łukasiewicz);

• special results: the shortest axiomatic bases, Jaśkowski's natural deduction system.

b. *Predicate logic*

In the interwar period, predicate calculus was not intensively studied in Poland. The Warsaw logicians thought that the functions of quantifiers could be explained in propositional calculus with quantifiers binding propositional variables. Moreover, Leśniewski's ontology was treated as a system which performed most functions of predicate calculus. The situation changed after the war. Although propositional logic was still the Polish specialty, Polish logicians also became more seriously interested in predicate logic. Since first-order logic was almost fully elaborated before 1939, the postwar Polish interests in predicate logic were connected rather with systems going beyond the first-order scheme. Included in this area of research one should mention:

• Wajsberg's works on predicate calculus for finite domains;

• Jaśowski's natural deduction formulation;

• Mostowski's axiomatization of first-order logic valid in every domain, including the empty one;

• algebraization of predicate logic (Rasiowa, Sikorski, Tarski on cylindric algebras)

• infinitary logics (Tarski)

• generalized quantifiers (Mostowski).

This last idea is perhaps the most important from the the perspective of the further development of logic. It opened a new field of logical research, namely model-theoretic logics, e.g., logic with the quantifier "there are countably many".

c. *Non-classical logics*

Certainly, the invention of many-valued logic was one of the most important logical discoveries in Poland. Łukasiewicz discovered three-valued logic in 1918. Then, it was generalized to finitely many-valued logic and infinitely-valued logics. However, not everyone in Poland regarded many-valued systems as interesting from the general point of view. Leśniewski and Chwistek defended the classical logic. As I already mentioned, Łukasiewicz based his modal logic on many-valued system. In the beginning, he interpreted the third value as possibility. Then, after 1945, he decided to construct modal logic as based on four-valued system. Let me add that Łukasiewicz's discovery of many-valued logic was strongly motivated by philosophical reasons, namely he regarded many-valuedness as a weapon against determinism, which he regarded as inconsistent with freedom. For Łukasiewicz, determinism is closely connected with the principle of bivalence (logical determinism). However, Polish logicians, including Łukasiewicz himself, also investigated other non-classical logics, motivated by a general attitude that every formal problem in logic should be considered independently of its philosophical context. Special attention was given to intuitionistic logic. It is interesting that in 1952 Łukasiewicz explicitly said that intuitionistic logic is the best of all non-classical systems [Łukasiewicz *1952*, 333]:

> The intuitionistic theory is richer and consequently more powerful than the classical theory. All the applications of the classical theory to mathematics are also valid in the intuitionistic theory, but in addition many subtle mathematical problems can be dealt with in the intuitionistic theory which cannot be formulated in the classical system. It seems to be that among the hitherto known many-valued systems of logic, the intuitionistic theory is the most intuitive and elegant.[29]

A summary of Polish investigations in the field of non-classical logics can be thusly presented:

• formulation of formal systems for many-valued propositional logic (Łukasiewicz, Wajsberg, Lindenbaum, Sobociński, Słupecki, Tarski);

• formulation of a formal system for many-valued predicate logic (Mostowski);

• modal logic based on many-valued logic (Łukasiewicz);

• formal systems for intuitionistic logic (Łukasiewicz, Wajsberg, Jaśkowski, Mostowski, Grzegorczyk);

• Jaśkowski's natural deduction for intuitionism;

• intuitionistic logic with variable functors (Łukasiewicz);

• Jaśkowski's logic of discussion;

• Łoś's epistemic logic;

• algebraization of non-classical logics (Rasiowa, Sikorski).

d. *Metalogic and metamathematics*

Under Hilbert's influence, metalogic and metamathematics were, in the twenties, reduced to syntax of formal systems. Later, however, model theory was also included in these considerations. Polish logicians studied metalogic (properties of logical systems) and metamathematics (properties of arbitrary formal systems) very extensively. These studies concerned the following topics, among others:

d1. *Metalogic*

• model theory for propositional logic, in particular matrix semantics (Łukasiewicz, Tarski, Kalicki, Łos);

• model theory for predicate logic, in particular the semantic definition of truth (Tarski);

• several new methods of proofs of the completeness theorem for propositional logic (Łukasiewicz, Wajsberg, Tarski, Łos, Rasiowa, Sikorski);

• model theory for several non-classical logics, in particular topological semantics for intuitionism (Tarski) and semantics for modal system *S5* (Wajsberg);

• algebraic semantics for logic (Rasiowa, Sikorski);

• various special concepts and results, for example the separation theorem for intuitionistic logic (Wajsberg), Jaśkowski's matrix for intuitionism, embedding of classical propositional logic in intuitionistic logic with variable functors (Łukasiewicz), an epistemic interpretation of intuitionistic logic (Grzegorczyk), Słupecki's test of functional completeness;

• Pepis' results on decision problems for predicate logic;

• the theory of syntactic categories (Leśniewski, Ajdukiewicz).

d2. *Metamathematics*

• axiomatization of the consequence operation (Tarski)

• calculus of systems (Tarski)

• elimination of quantifiers (Tarski) decidability, undecidability (Tarski, Mostowski, Szmielew, Grzegorczyk)

• limitative theorems (Tarski, Mostowski)

• recursive functions (Mazur, Grzegorczyk)

• the theory of constructive types (Chwistek);

• special results and concepts, for example Lindenbaum's lemma on maximalization, the Lindenbaum-Tarski algebra, degrees of completeness (Tarski), Łoś's theorem on ultraproducts, the Löwenheim-Skolem-Tarski theorem, categoricity in power (Łoś), the Kleene-Mostowski hierarchy, the Grzegorczyk hierarchy.

e. *The Foundation of Concrete Mathematical Theories*

• set theory (Sierpiński, Kuratowski, Tarski, Mostowski);

• arithmetic (Presburger, Tarski)

• Boolean algebra (Tarski, Rasiowa, Sikorski, Jaśkowski)

• general algebra (Tarski, Szmielew, Marczewski)

• special results and concepts, for example the definition of ordered pair (Kuratowski), the definition of finiteness (Tarski), the axiom of choice (Sierpiński, Tarski, Lindenbaum, Mostowski), the continuum hypothesis (Sierpiński), Mostowski-Fraenkel models, the axiom of determinacy (Steinhaus, Mycielski), inaccessible cardinals (Sierpiński, Tarski), the completeness of arithmetic of integers with addition as the sole operation (Presburger), the completeness of elementary algebra and geometry (Tarski), the axiom of induction (Ryll-Nardzewski), decidability of Abelian groups (Szmielew).

f. *Non-standard systems: Leśniewski and Chwistek*

f1. *Leśniewski*

Leśniewski constructed three logical systems intended as the foundations of mathematics: protothetic, ontology, and mereology. Protothetic is a generalized propositional calculus. It contains quantifiers which bind propositional variables and in which variables refer to objects of arbitrary syntactic categories defined from the basic category of propositions. Protothetic is a very rich system in which one can express the principle of bivalence and the principle of extensionality. Ontology is calculus of names or a theory of copula 'is' taken as synonymous with Latin *est*. Mereology is a theory of sets understood as collective wholes. Thus, mereology is a theory of parts and wholes. Philosophically, Leśniewski's systems are based on a very radical nominalism. In particular, Leśniewski's systems are concrete physical units, finite at any stage but freely extendable. Tarski, Wajsberg and Sobociński also contributed to Leśniewski's systems.

f2. *Chwistek*

Chwistek proposed another nominalistic basis for logic (see [Chwistek *1948*]). It was rational semantics, a theory of systems of expression. This system was intended to give a uniform scheme for logic and mathematics. This scheme starts with a list of primitive signs which is as economical as possible. Then, we have the inference rule, and finally, the rules of interpretation. Unfortunately, Chwistek was not able to complete his investigations. Thus, many details are provisional or even unclear. However, Chwistek's semantics is sufficiently strong for expressing arithmetic of natural numbers in it. In particular, Chwistek proved the Gödel incompleteness theorem in his system.

g. *History of Logic*

Łukasiewicz initiated a special program of looking at the history of logic through the glasses of modern logic. In particular, he regarded the old systems as predecessors of modern mathematical logic. According to Łukasiewicz it is not fair to fault or condemn traditional logic; such a blameworthy attitude was quite popular among the originators of modern logic who, including Frege and Russell, who maintained that modern logic completely broke with the past. Łukasiewicz thought that not everything was wrong in the past. In fact, argued Łukasiewicz, it was Descartes who basically caused the degeneration of logic and pushed it to psychologism. Even Leibniz could not stop this process, although he should be considered as a predecessor of modern mathematical logic.

Łukasiewicz's program meant the very revolution in doing the history of logic. Łukasiewicz himself discovered that the Stoics constructed propositional logic. He also rehabilitated the logical inventions of the medieval Schoolmen. Another of Łukasiewicz's results was a reinterpretation of Aristotelian syllogistic in terms of modern logic. He also pointed out that many-valued logic is rather non-Stoic than non-Aristotelian, because the Stoics very strongly defended the principle of bivalence, but Aristotle doubted this principle so far as they concerned statements about the future. Several Polish logicians continued Łukasiewicz's program, among them Ajdukiewicz, Bocheński, Czeżowski, Dąmbska, Korcik and Salamucha. The interests of Polish logicians in the history of logic also resulted in publication of several major works, in particular Bocheński *1956*], [Kotarbiński *1965*], and [Mostowski *1965*].

NOTES

[1] Bibliographical references are to the author(s) and year of publication. Page references are to translations and reprints if they are mentioned in the bibliography at the end of the paper.

Editor's note: For a general treatment in English of the historical background of mathematics in Poland during the period discussed by Professor Woleński, readers may wish to study Sister Mary Grace Kuzawa, *Modern Mathematics: The Genesis of a School in Poland*, New Haven: College & University Press, 1968.

[2] Although I use here some material from [Woleński *1989*], this paper contains several new points.

[3] The manuscripts are in the Archive of Warsaw University.

[4] Fortunately, Łukasiewicz did not cease doing logic.

[5] In 1926-1928, Ajdukiewicz was in Warsaw.

[6] This letter is quoted from [Estreicher *1971*, 212]. Chwistek was also a distinguished painter and writer; hence the title of Estreicher's book.

[7] By the way, two papers by Tajtelbaum-Tarski are quoted in the second edition of the *Principia Mathematica*. There is an anecdote related to this story. Once Russell asked one Polish mathematician what happened to Alfred Tajtelbaum, a young gifted Polish logician. "He is very well, but Tarski is his present name" — answered the person questioned. Tarski changed his name in the early twenties, because he thought that the change (as a way of "Polonization") would make his academic career much easier.

[8] Chwistek was to some extent forced to apply in Lvov. He would certainly have preferred to stay in Cracow. However, his habilitation was accepted in Cracow on condition that he would never apply for a professorship at the Jagiellonian University.

[9] I learned this explanation from personal communications with Jerzy Słupecki.

[10] Although Tarski had many great teachers, including Leśniewski, Łukasiewicz and Sierpiński, he dedicated his *Logic, Semantics, Metamathematics* (see [Tarski *1956*]) to Kotarbiński. According to a report of Jan Tarski (Alfred's son), a picture of Kotarbiński was always on Alfred Tarski's desk. He used to say of Kotarbiński: "He is a true human being".

[11] Let me recall that Ajdukiewicz also spent two years in Warsaw (see Section 2 above).

[12] Tarski was Leśniewski's only doctoral student. I heard that Leśniewski used to say: "I have a hundred percent of genial doctor students."

[13] Mrs. Kotarbiński told me that Tarski's professorship was discussed several times at meetings of the Faculty Council. According to Mrs. Kotarbiński, Leśniewski never prevented by his veto the further steps that were taken on Tarski's behalf. This confirms that he expressed his real attitude in the letter quoted. However, the question of Tarski's

professorship never ceased to be given attention at the university. Why? I often heard, particularly in the United States, that he was not promoted in Poland because of his Jewish origin. According to this opinion, it was a decisive factor in Lvov (see Section 2 above) in 1928-1930 as well as in Warsaw in the thirties. I tried to explain in Section 2 of this paper why Chwistek won in Lvov. As far as concerns Warsaw, the Jewish factor was certainly more important, because anti-semitism in Poland in the thirties was stronger than it had been earlier, and Warsaw was certainly more anti-semitic than Lvov. However, Jewish origin did not preclude academic promotion, although it certainly did not help in this respect. Anyone who discusses the unsuccessful efforts to create a professorship for Tarski in Warsaw must take into account the fact that Warsaw University already had two professors in logic and it would be difficult to apply for creation of a third position. In one discussion in Berkeley, I asked why Tarski had to wait seven years for his professorship in America: because of American antisemitism or perhaps because there was no position for him? Clearly, the second factor was decisive. It is not my intention to minimalize the role of antisemitism in Poland, but neither there is reason to exaggerate this point.

[14] In order to avoid misunderstandings, I would like to add that Tarski's own opinion was probably different as far as concerned the Warsaw case. It is known that he considered himself to be a victim of Polish prewar antisemitism.

[15] According to Hiż, the people in Poznan were afraid that Tarski would apply and win the competition. Poznan was perhaps the most anti-semitic region in Poland. This would explain the situation.

[16] Note also that the students of colleges underwent a quite intensive teaching in logic. [Tarski *1935*] was written as a textbook for colleges. The fourth English edition of this book appeared in the series *Oxford Logic Guides*, which certainly is not directed to students of lower university levels.

[17] If I am right, Russell's remark on the fate of the Polish readers of the *Principia* is not correct because neither Chwistek nor Leśniewski nor Tarski was liquidated by the Nazis.

[18] My report on the postwar period is much more sketchy than in Sections 2-4. Firstly, we have not yet attained a sufficient historical

distance to this period. Secondly, geography and personalities concerning logic after 1945 are not as important as they were before. Thus, there is no reason to enter very deeply into details.

19 Warsaw University offered a post for Tarski in 1946, but only as a docent. He rejected this proposal, which was one of the biggest mistakes in the history of Polish academic life.

20 An English translation of this book was announced in the plan of the series "Mathematical Monographs". Unfortunately, this project was never realized.

21 Tarski at that time acted as the President of the Association for Symbolic Logic and Church as the editor of *The Journal of Symbolic Logic*. The letter concerned the editorial policy of the journal; its original is in the Bancroft Library in Berkeley.

22 Tarski said this at the Colloque Internationale de Logique at Brussels in 1953.

23 There is a well-known anti-Platonic slogan: *Platon amicus sed veritas amicus major*. Tarski changed it in this way: *Platon inimicus, sed veritas amicus major.*

24 Łukasiewicz believed at that time that the real world corresponds to infinitely many-valued logic.

25 Since Leśniewski uses "ogólne zasady bytu", but not "ogólne zasady istnienia", the phrase "general principles of being" should replace "general principles of existence".

26 There is also a satirical picture of the perfect logician given by Stanisław Ignacy Witkiewicz (Witkacy), a philosopher, writer and painter, who did not like logic or logicians. In one of his dramas, Witkacy pictures a logician who constructed the simplest system. This system was based on the sole primitive term 'point', the sole axiom 'point is point' and the sole rule of inference 'nothing more to do'.

27 This survey is very selective. In particular, I skip bibliographical references. One can find details in [Jordan *1945*], [Prior *1956*] and [Woleński *1989*]. Also compare [McCall *1967*] and English collections

of particular Polish logicians: [Tarski *1956*], [Tarski *1986*], [Łukasiewicz *1970*], Wajsberg *1977*], [Mostowski *1979*] and [Leśniewski *1992*]. [Rasiowa & Sikorski *1970*] is a good example of logic "from the Polish point of view".

28 There is a nice story which shows this points. When Tarski met Emil Post for the first time (in 1939 or 1940) he told him: "You are the only logician who achieved something important in propositional calculus without having anything to do with Poland". Post answered: "Oh, no, I was born in Białystok." Białystok is a town in northeastern Poland.

29 Note that Łukasiewicz's remarks concern intuitionistic logic with variable functors.

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