DE MORGAN, VICTORIAN SYLLOGISTIC AND RELATIONAL LOGIC

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"A *form* is an empty machine."
– A. De Morgan

The world of Augustus De Morgan (1806–1871) is fundamentally not unfamiliar; this present age being an heir to the consequences of that world. It is a world, as Charles S. Peirce (1839–1914) imagining [64, p. 4] a scientific personage of the seventeenth or eighteenth century saying in 1863, ' "that all this or something very like it ... is nothing more than the certain consequence of the principles laid down by me and my contemporaries for your guidance." '

Merrill ([54, 55, 56]; see [78]) has assumed factotum, [55, 56] tracing the descent of De Morgan's work to the logic of relations; [56] rendering primarily an historical study of De Morgan's work that would traverse contexts of the nineteenth and twentieth centuries.

The Victorian age spans the nineteenth and twentieth centuries, from 1837 to 1901; *the* sunset or sunrise (see [3, pp. 13–16]; [15, pp. 9, 15–87, 298–332]; [51, pp. xvii–xxix]), an epoch of the mind, if not of the ubiquity of a body politic that are essentially British, and not for the age being merely in measure and appellation that of the British Queen Victoria's reign.

It is the epoch of the machine technology; [36], [63, VII pp. 172–173] pre-electronic 'computers,' the Englishman Charles Babbage (1792–1871) designing 'Difference' and 'Analytic' Engines. It is a sense ([4, p. 478]; see [35, p. 382]) of "insular originality" within British mathematics, no more than in the British sciences, and logic.

Bronowski [11, p. 291] writing of Charles R. Darwin (1809–1882) and Alfred Russell Wallace (1823–1913),

> that biology as we understand it begins with naturalists in the eighteenth and nineteenth centuries; observers of the countryside, bird-watchers, clergymen, doctors, gentlemen of lessure in the country houses. I am tempted to call them, simply, 'gentlemen in Victorian England', because it cannot be an accident that the theory of evolution [by natural selection] is conceived twice by two men living at the same time in the same culture—the culture of Queen Victoria in England.

The memoir of De Morgan [25, pp. 208–246] on the logic of relations, is dated the twelfth of November 1859 (read the following year, on the twenty-third of April); with Peirce ([62, V, p. 44]; see [22, p. 279]; [60, p. 18]) "surveying in the wilds of Louisiana," for the United States

Coast and Geodetic Survey, the same year (on the twenty-fourth of November 1859), as Darwin publishing *The Origin of Species.*

The Victorian age ([64, pp. 336, 350, 337]; see [63, VII, p. 45]) "is the age of Methods"; methodology, which "is itself a scientific result," emerging, "adapting the methods of one science to the investigation of another." For [2, p. 399]

> Victorian science was, in most areas, self-consciously concerned with its own methods. In few other eras before or since were men of science so given to elaborate analyses of their own practices.

Victorian mathematics increasingly shows a similar atttention to methods, or, as it were, (see [33, p. 138n.13]; [35, p. 387]; [37, pp. 63–64]) to the 'logic' of mathematics.

De Morgan ([25, pp. 78n.1, 345, 337, see pp. 91, 184n.1, 247–249, 336]; [42, p. 58]; [56, pp. 170–174]) writing in 1858, and in 1862, envisions,

> As joint attention to logic and mathematics increases, a logic will grow up among the mathematicians, distinguished from the logic of the logicians by having the mathematical element properly subordinated to the rest. This *mathematical* logic—so called *quasi lucus a non nimis lucendo*—will commend itself to the educated world by showing an actual representation of their form of thought—a representation the truth of which they recognize—instead of a mutilated and onesided fragment, founded upon canons of which they neither feel the force nor see the utility.
>
> I believe, and I am joined by many reflecting person, among students both of logic and mathematics, that as the increasing number of those who attend to both becomes larger and larger still, a serious discussion will arise upon the connexion of the two great branches of exact science, [logic,] the study of the necessary laws of thought, [and mathematics,] the study of the necessary matter of thought. The severance which has been widening ever since physical philosophy discovered how to make mathematics her own special instrument will be examined, and the history of it will be written.
>
> To the mathematician I assert that from the time when logical study was neglected by his class, the accuracy of mathematical reasoning declined. An inverse process seems likely to restore logic to its old place. The present school of mathematicians is far more rigorous in demonstration than that of the early part of the century: and it may be expected that this revival will be followed by a renewal of logical study, as the only sure preservative against a relapse.

The logician and mathematician, on this 'mathematical logic,' neither the one nor the other converging, De Morgan ([25, p. 78n.1]; see [1]) apparently suggesting, in the words of Peirce [67, II, p. 892], "are in a backward state of development."

"With the exception of De Morgan," William Stanley Jevons (1835-1882) observes [41, p. 43] that George Boole (1815–1864) "was probably the first English mathematician since the time of [John] Wallis (1616–1703) who had also written upon logic."

The major 'logical' work of Boole [9, pp. 1, 37–38, see pp. 6–7,10–11, 26,31]; [35, pp. 382–383, 386] is, in

> design . . . to investigate the fundamental laws of those operations of the mind by which reasoning is proformed; to give expression to them in symbolical language of a Calculus, and upon this foundation of establish the science of Logic and construct its methods . . . and, finally, to collect from the various elements of truth brought to view in the course of these inquiries some probable intimations concerning the nature and construction of the human mind.
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> Let us conceive, then, of an Algebra in which the symbols x, y, z, &c. admit indifferently of the values 0 and 1, and of these values alone. The laws, the axioms, and the processes, of such an Algebra will be identical in their whole extent with the laws, the axioms, and the processes of an Algebra of Logic. Difference of interpretation will alone divide them. Upon this principle the method of the following work is established.

Jevons ([41, p. 43]; compare [25, pp. 22–23, 82, 255-256]) seminally notes that Boole "did not regard logic as a branch of mathematics, as the title of his earlier pamphlet [*Mathematical Analysis of Logic*, 1847] might be taken to imply, but he pointed out such a deep analogy between the symbols of algebra and those which can be made, in his opinion, to represent logical forms and syllogisms, that we can hardly help saying that logic is mathematics restricted to the two quantities, 0 and 1."

De Morgan represents the subject, logic and mathematics, from a different perspective. De Morgan ([25, pp. 78n.1,184n.1]; see [4, p. 485]) asserting that "Logic considers the *laws of action* of thought," that "Mathematics [used plurally] are concerned with ***necessary matter of thought***," and hence "having the mathematical element properly subordinated," that "mathematics [used singularly, as customary today] applies these *laws* of thought to necessary *matter* of thought."

From the standpoint of De Morgan [25, pp. 117, 118, see pp. 26, 74–83, 89–107, 116–119, 131, 153–154, 178–179, 185, 188–189, 208–210, 247–252, 332–333, 345, Heath, p. xxiv]; [24, pp. 29–52]; [9, pp. 1–3]; [18, p. 547]; [76, pp. 60n., 61n., 131–133]; [77, pp. 116–132],

> Logic is the science and art, the theory and practice, of the form of thought, the law of its action, the working of its machinery; independently of the matter thought on.
>
> Logic considers both [objective names representing objects and qualities, and subjective names representing classes

> and attributes, which are respectively] first and second intentions, because both are forms of thought; but the first chiefly as leading to the second: and in both it considers *quae non debentur rebus secundum se, sed secundum esse quod habent in anima.* That is, logic belongs to psychology, not to metaphysics.

The words of De Morgan, on the subject of logic, here seem traditionally hidebound and irredeemably quaint.

The 'quaintness' of De Morgan's express "devotion to the traditional lore of the subject," as Heath [25, p. xxiv] remarks, yet belies something of De Morgan's "admirable insight into its workings."

De Morgan ([25, p. 82, see pp. 247–249, Heath, p. xxiv]; [66, IV, pp. 247–248]) regards the province of psychology, in somewise, as thought, and logic, as "the *form* of thought, the law of action of its machinery," and mathematics, as "a branch of thought," in the sense of thought being "genus" and mathematics "species." He might well concur with [77, p. 125] that in a sense, "*All logic is limited by the limitations of the human mind when it is engaged in that activity known as logical thinking.*"

Wiener ([77, p. 125, see pp. 124–127], from the vantage point of a, then, bourgeoning 'computer science,' his considering that "the study of logic must reduce to the study of the logical machine, whether nervous or mechanical," concedes,

> It may be said by some readers that this reduces logic to psychology, and that the two sciences are observably and demonstrably different. This is true in the sense that many psychological states and sequences of thought do not conform to the canons of logic.

"Psychology," Wiener ([77, p. 125]; see [76, p. 60n.]) would contend, "contains much that is foreign to logic, but—and this is the important fact—any logic which means anything to us can contain nothing which the human mind—and hence the human nervous system—is unable to encompass."

Suffice it to say that De Morgan ([25, pp. 26, 50, 74–82, 89–100, 117–118, 153, 208n.1, 211n.1, 215, 218, 228–229, 238–241, 247–249]; see [34, p. 40]) and Boole ([9, pp. 1–51, 399–424]), although both being not without some bent to psychologism, would variously mind the logical and mathematical p's and q's.

De Morgan and Boole,—the one reforming the old logic, and the other creating a new one,—historically confront (see [25, pp. 247–248, Heath, p. xxiv]; [35, p. 382]) a largely, then sterile rule of the syllogism, with Kant's dictum that logical perfection and Aristotelian logic are one.

Richard Whately (1781–1863) though being no precursor of a "*mathematical* logic," which De Morgan [25, p. 78n.1] envisages; De Morgan

[25, p. 237] in 1859, yet credits Whately with his having been "restorer of logical study in England." Whately [76] is, indeed "a work," as Prior ([68, p. 103]; see [56, pp. 1–10]) states, "which did much to revive the study of the subject [of logic] in England in the earlier part of the nineteenth century." Seth ([70, p. 559]) retrospecting, similarly stating that

> By this work, which gave a great impetus to the study of logic not only in Oxford but throughout Great Britain, Whately has been known to generation after generation of students; and, though it is no longer so much in use, the qualities of the book make much of it as admirable now as when it was written. Whately swept the webs of scholasticism from the subject, and raised the study to a new level.

The 'elements of logic,' in Whately's title, not surprisingly (see [62, II, p. 36]), are language and the syllogism.

De Morgan ([25, pp. 247–248]; see [4, pp. 484–487]; [34, p. 40]; [35, p. 382]; [67, II, pp. 892–894]) considers, in 1860, since Whately [76] that

> The study of logic in this country has [lately] undergone three important changes. First, much more attention is paid to the subject; secondly, innovations have been listened to in a spirit which seems to admit that Kant's dictum about the perfection of the Aristotelian logic may possibly be false; thirdly, a disposition has arisen to distinguish logic from metaphysics and psychology, without losing sight of the psychological and metaphysical discussion which is necessary to a sound view of the meaning, province, and first principles of the science ... which may be styled an exact science.
>
> The changes to which we have alluded arise from that revival of the taste for philosophy which has commenced and is continuing, both in England [meaning the three kingoms] and France. So far as England is concerned this revival was preceded by the publication of Dr. Whately's [1826] work .... The author had for years taught the subject at Oxford, and had trained some, at least, to see the low state into which logic had fallen .... The Archbishop of Dublin [Whately] possesses the talent which distinguished [William] Paley [1743–1805] from his predecessors; the talent of rendering a dry subject attractive in a sound form by style, illustration, and clearness combined. And to him is due the title of the restorer of logical study in England .... In 1833 appeared the celebrated criticism [of Sir William Hamilton (1788–1856), the Scots philosopher, the *other* Hamilton, and not the Irish mathematician, Sir William Rowan Hamilton (1805–1865)] on Dr. Whately's logic, of which the first thing to be said is that it is due to Dr. Whately himself that it had an audience to listen to it: its history, philosophy, and philology, would have fallen

> dead upon the previous generation. Accordingly this criticism ... is nothing as a criticism; for it neither is nor proceeds upon a true view of the place and office of the work criticized.

So much, then, for Whately effectively restoring, in England, singlehandedly, the study of logic.

Whately [76] is significant, however, in the United States, in 1852, as the work (see [62, II, p. 408]; compare [66, IV, pp. vi, vii]; Eisele, [67, I, pp. 20–21]; [60, pp. 16–17]), introducing Peirce to logic. Whately ([76, pp. 315–327]; see [63, VIII, p. 11]; [34, pp. 58–59]), opposing realism, with a British penchant for nominalism, may be the work also introducing Peirce to the Medieval controversy of nominalism–realism (see [58, pp. 47, 48, 53n.18]), which (see [35, p. 387]; [37, pp. 65–69]; [38, §III, §§5–6]) later so pervades Peirce's work, in his forcefully proffering realism.

What Whately ([76, pp. 166, 14–15, see pp. xii–xiii, 31–33, 35, 38–41, 59–61, 65, 70, 80–82, 107–112, 132–133, 257, 259, 431]; [9, pp. 238–242]; [24, pp. 25–28]; [25, p. 237]; [56, pp. 1–11, 20, 93, 106–107] expressly considers, are such "rules" that

> enable us to develop the principles on which all reasoning is conducted, whatever be the Subject-matter of it, and to ascertain the validity or fallaciousness of any apparent argument, as far as the *form of expression* is concerned; that being alone the proper province of Logic.
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> For Logic, which is, as it were, the Grammar of Reasoning, does not bring forward the regular Syllogism as a *distinct mode of argumentation*, designed to be *substituted* for any other mode; but as the form to which *all* correct reasoning may be ultimately reduced.

Boole [9, p. 239] quoting the latter, remarks that "The language of Archbishop Whately, always clear and definite, and on the subject of Logic entitled to peculiar attention, [and not merely of the supremacy, then, but of the universal sufficiency of syllogistic inference in deductive reasoning, he] is very express on this point."

Whately [76, p. 61, see pp. 14–15, 58–62, 66, 80, 82] takes the view that on "the *proper use of language*," overlapping are the 'grammar' of language, and the 'grammar' of reasoning,—terms, propositions, and arguments. Thus, for Whately [76, pp. 66, 63, 62, 80, 81, 81n.*, 81], a proposition consists of such "*words*," as compose ' "*a sentence indicative*," *i.e.* affirming or denying,' with terms, which "may consist either of one Word or of several," being "the *subject*" and "the *predicate* of a proposition "and, in the middle, the Copula," which "must be either *is* or *is not*"; while "in its *regular form*, is called a *syllogism*," an argument "is the strict *logical form*" of "the *premises*" and "the Conclusion," the two "*antecedent*" propositions and a third proposition 'introduced by some *illative* conjunction, as "therefore." ' Whately ([76, p. 61, see p. 61n.]) defining Logic,

> "the Art of employing language properly for the purpose of Reasoning; and of distinguishing what is properly and truly an argument, from spurious imitations of it."

Whately [76, p. 60n.] introduces "the mention of *language* previously to the definition of Logic," noting he has "departed from the established practice, in order that it may be clearly understood, that Logic is *entirely conversant about language*." Logic, for Whately ([76, pp. 82, 61]; see [46, pp. 1–2 or pp. xiv–xv (1956)]), indeed "is wholly concerned in the use of language," for "our thoughts are influenced by expressions, and ... error, perplexity, and labour are occasioned by faulty use of language." Whately ([76, p. 60n., see p. 61n.]) contending,

> If any process of reasoning *can* take place, in the mind, without any employment of language, orally or mentally, (a metaphysical question which I shall not here discuss) such a process does not come within the province of the science [of Logic] here treated of. This truth, most writers on the subject, if indeed they were fully aware of it themselves, have certainly not taken due care to impress on their readers.

Whately [76, see pp. xxiii–xxiv, 14–15, 59–61, 82, 166], in his usage of 'grammar,' then, is not merely posing an analogy, simply drawn between 'language' and 'logic'; although it is not suggested by the comment of Boole [9, p. 239], Whately's ([56, p. 93]) is a "very specific language-oriented conception of form."

The deference to Whately [76] that De Morgan ([25, pp. 247–249, see pp. 75–83, 208–212, 218–222, 227–229, Heath, p. xxiv]; [34, pp. 40–42, 57]; [56, p. 124]) expresses, in 1860, would seem not to extend to Whately's logic of language, however, as a sense of 'logic' that De Morgan could readily embrace; though De Morgan's own outlook remaining essentially syllogistic, which on a range of some but not all issues of the traditional logic, would liable to impede him, unfortunately, from definitively taking a radically novel point of view.

De Morgan [25, pp. 218–220] does not so much eschew "the idiom," and such remarks of his, as on prepositions even have ([52, p. 223]) "an extraordinarily modern ring," but *à l'anglaise*, he generally is not, [62, II, p. 36; see III, p. 218] "with appeals to the ordinary usages of speech as determinative of logical doctrines."

De Morgan ([25, pp. 219, 220, 312, see pp. 75–83, 208–210, 221, 227, 250, 269–270]) cautions, indeed, against the "difficulty which usage [of language] puts in the way of logic," remarking that "habits of thought of a nation silently accomplish many changes which we call caprices of language," while "original tendencies of language, partial, one-sided, stopping at just enough, have tied some of our mental muscles until they only act by special volition and a good deal of it." There are, De Morgan [25, p. 219, see p. 221] asserts, "distinctions of which grammar takes no cognizance," with the logician exercising "his privilege of making language for every distinction which exists in thought."

"The general character of De Morgan's development of logical forms," Jevons [42, p. 58] regards, "was wholly peculiar and original on his part." Jevons here alludes (see [24, pp. 345–376]; [25, pp. 271–345, Heath, pp. x–xxiv, xxvii]; [31, pp. 18–20]; [34, pp. 40, 54]; [62, II, pp. 322-326]; [68, pp. 146–156]) to the circumstances that onwards of 1847, would so affect De Morgan's work: his famous (and, at times comic) controversy, over Sir William Hamilton's scheme of the quantified predicate, in conception and execution, a predicative quantification, exemplary of how,—yea, even by logical standards of the day,— it should not be done; provoking the guileless De Morgan, initially, Hamilton, *non heroico more*, imputing the 'principle' of a quantified predicate, in common, alleging his priority to De Morgan, and impugning De Morgan's independence.

The 'controversy,' difficult as the dispute is in the profusion of details, with the disputants including a partisanship of Hamilton's 'pupils'; De Morgan ([24, pp. 345, 346]) considers the "two questions involves, one concerning my character, the other purely literary," with Hamilton, in the *guerre de plume*, enlisting "half quotations, on which hang sundry jokes and sneers, some of them at mathematicians in general, and myself as one of the body." De Morgan ([24, pp. 350–352, see pp. 53–87, 177, 345–350]; [25, pp. 17–50, 271–345]; [68, pp. 148–151]) discerning, however, in Hamilton's accounts,

> Three schemes of quantity are here mentioned.
>
> First, the ordinary one.
>
> Secondly, that in which the *ordinary quantities*, *all* and *some*, are applied in every way to both subject and predicate.
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> Thirdly [De Morgan's], that in which numerically definite quantity is applied to subject or predicate or both: the essential distinction of this case is *numerical* definiteness: it really contains the second system, when numerical quantity is algebraically expressed. Of these, it appears, Sir W. Hamilton claims the second, or rather, the application of such a scheme to the syllogism. What then is it? I suppose it to be the following. My order of reference is XY [in such propositions, as $A_,$ and A', having names contrary of those in the other].
>
> *All X is all Y* means that X and Y are identical: it is my D. *All X is some Y* is $A_,$. *Some X is all Y* is A'. *Some X is some Y* is $I_,$. As to negative propositions, *All X is not all Y* is $E_,$. *Some X is not all Y* is $O_,$. *All X is not some Y* is O'. *Some X is not some Y* is true of all pairs of terms one of which is plural. In its indefinite form, it is what I have in Chapter VIII called *spurious*.

The propositions of this system are then the complex D, or $A_{\prime} + A'$, the six Aristotelian forms $A_{\prime}$, A', $E_{\prime}$, $O_{\prime}$, O', $I_{\prime}$, and the spurious form which may be called U. In looking over ... Sir W. Hamilton's pamphlet, I happened to light on the assertion (incidentally made) that his system gives *thirty-six* valid moods in each figure. On examining the preceding system, I find this to be the case. I should not have published the results, had not Sir W. Hamilton made it necessary for me to comment on them. I shall denote the proposition U, or "Some Xs are not some Ys" by $X :: Y$; and I shall, supposing each case to be formed in the first figure, then transpose it into my own notation.

1. There are *fifteen* forms in which D enters. Whenever D is either of the premises, the other premise and conclusion agree. Thus we have $A_{\prime}DA_{\prime}$, DUU, &c. &c.

2. *Fifteen* Aristotelian forms $A_{\prime}A_{\prime}A_{\prime}$, $A'A'A'$; $A_{\prime}E_{\prime}E_{\prime}$, $E_{\prime}A'E_{\prime}$; $A_{\prime}O'O'$, $O_{\prime}A'O_{\prime}$; $A'O_{\prime}O_{\prime}$, $O'A_{\prime}O'$; $A'I_{\prime}I_{\prime}$, $I_{\prime}A_{\prime}I_{\prime}$; $E_{\prime}I_{\prime}O'$, $I_{\prime}E_{\prime}O_{\prime}$; $A'A_{\prime}I_{\prime}$; $A'E_{\prime}O_{\prime}$, $E_{\prime}A_{\prime}O'$.

3. *Six* more U syllogisms $A'O'U$, $O_{\prime}A_{\prime}U$; $A'UU$, $UA_{\prime}U$; $I_{\prime}O'U$, $O_{\prime}I_{\prime}U$.

The two things to be considered are:—the introduction of the identical proposition; and that of the spurious one, as I call it.

It is, I suppose, a fundamental rule of all formal logic, that every proposition must have its denial, its contradiction. Now D has no simple contradiction in this system: that O' and $O_{\prime}$ both contradict it (and also $E_{\prime}$) is true: but the mere contradiction is the disjunction "O' or $O_{\prime}$." A person who can show that one or the other of these is true, has demonstratively contradicted D, even though it could be proved impossible to dertermine which of the two it is.

The proposition U is usually spurious. But if we introduce it, we must introduce its contradictory also. Now if either X or Y be plural names, it must be true: consequently, the contradiction of U is "X and Y are singular names, and X and Y." When a syllogism having the premise U is introduced, either that premise may be contradicted, or it may not. If it may, there is no form to do it in: if it may not, then it is a spurious proposition, and cannot, by combination with others, prove anything but a like spurious conclusion.

Let $X :: Y$ denote "Some Xs are not some Ys," and X_1Y_1 denote "there is but one X and one Y, and X is Y." The either $X :: Y$ or X_1Y_1 must be true, and one only. A logical system which admits one and not the other, which contains an assertion incapable of contradiction without going out of the system ... is not self-complete. The proposition X_1Y_1 includes in itself the condition of D, and is a kind of *singular form* of D.

I presume, from the number of Sir W. Hamilton's moods, *thirty-six*, as above obtained, that the contradiction neither of D nor of U finds a place. Admit them, and the contradiction of U alone (call it V) demands sixteen new moods in each figure.

This is a remarkable text, with comments that transcend the immediate bounds of issue, in terms suggesting almost the modern sense of a 'deductive system,'* which seem otherwise disregarded in the precis; full of words that bear upon the present general discussion of De Morgan's work. For his system of *numerical* definite 'quantity,' De Morgan ([24, pp. 351, 352]; see [68, pp. 148–151]; [74, pp. 69–70,93–94]) argues, "really contains" the scheme of 'quantity' that Hamilton advocates, "when numerical quantity is algebraically expressed"; directing his criticisms primarily against the two forms D and U, in Hamilton's system, neither 'All X is all Y' nor 'Some X is not some Y' having a contradiction "without going out of the system," the system "is not self-complete," which almost tempts saying, 'incomplete.' Peirce [62, p. 324] referring to the "controversy," in 1893, judges,

> The reckless Hamilton flew like a dor-bug into the brilliant light of De Morgan's mind in a way which compelled the greatest formal logician that ever lived to examine and report upon the system. There was a considerable controversy; for Hamilton and several of his pupils were as able in controversy as they were impotent in inquiry.

De Morgan [25, p. 345] himself remarking, in 1862, that "perhaps many do [think], that the whole question is about Hamilton and myself; I, from the beginning, in 1847, have never considered it in this light," but "it is clear that the great change to which Hamilton's name must be attached, the *expressed* quantification of the predicate, must have its history." De Morgan [25, p. 345] yet adds ironically that "I have over them this undeniable advantage: if right, I shall be known to have been right; if wrong, I shall not be known to have been wrong."

The remark of Jevons [42, p. 58] having been earlier quoted, is yet perhaps apt in a sense, albeit 'logical form,' which Peirce [62, II, p. 324] characterises De Morgan, as "the greatest formal logician that ever lived," that is 'the general character of De Morgan's development of logical form is wholly peculiar and original on his part.'

De Morgan ([25, pp. 68, 75–83, 86–87, 96, 117–118, 140n.1, 153, 183, 183n.1, 184n.1, 188, 210–213, 217–220, 228–230, 240–241, 248–249, 252–253, 269, 303–304, 345]; see [34, pp. 35–37]; [56, pp. 89–112]) certainly develops, in his work, whether being philosophy or praxis,

*The import of his own words, De Morgan would appear never to have fully appreciated, and, as will be later seen, *in actu*, a modern sense of the 'deductive system' would not be among his logical conceptions, for "the whole notion of a deductive *system*," as Merrill ([56, p. 196, see pp. 120–124]) remarks, "is foreign to De Morgan." The notion of a logical 'system' present in De Morgan's text (see [68, p. 119]) has its antecedents, especially, in the work of Aristotle; in hindsight, however, a reader today recognises De Morgan here expanding Aristotle's notion, teasing something of its essentials, in surprising details, which instance that ripe was the time for the deductive system, awaiting its development in the work of Frege and Peirce.

a distinction of 'matter,' and marked discrimination of 'form.' For it is a stance of De Morgan [25, p. 82, see pp. 75, 117] that "Logic considers, not *thought*, but the *form* of thought, the law of action of its machinery."

De Morgan's is effectively the position that in logic, laws are principles or properties, and hence 'formal,' proposes Merrill ([56, p. 249n.5, see p. 104]; [25, p. 183n.1]), "which can be defined solely in terms of variables and logical constants." De Morgan (quoted, [56, p. 249n.4]) himself remarking, "A *form* is an empty machine." Thus, for De Morgan [25, pp. 75, 248, 79n.2, see pp. 79–80n.3, 241],

> The *form* or *law* of thought—asserted differences between these words being of no importance here— is detected when we watch the machine in operation without attending to the matter operated on. The form may again be separable into form of form and matter of form: and even the matter into form of matter and matter of matter; and so on. The *modus operandi* first detected may be one case of a limited or unlimited number, from all of which can be extracted one common and higher principle, by separation from details which are still differences of form.
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> When many things are thought of in one way, which way is neither governed nor modified by the difference between one thing and another, there is a common *form* of thought about distinct *matters*. Thus arithmetic is a formal science with reference to concrete magnitude: that 8 and 4 make the same as 4 and 8 is a form, a law of thought, which is not affected by the objects counted being yards rather than sheep, or pints rather than men. Pass into algebra, and the differences which are formal in arithmetic become only material: thus $8+4=4+8$ is but one material instance of the form $a+b=b+a$. Are we therefore to say that arithmetic and algebra are parts of logic? Certainly not: logic deals with the pure form of thought, divested of every possible distinction of matter.
>
> But ... I am compelled to have recourse to the difference between the ideas of form belonging to the mathematician and to the logician. Is there any *consequence* without *form*? Is not *consequence* an action of the machinery? Is not logic the science of the action of this machinery? Consequence is always an *act* of the mind: on every consequence logic ought to ask, What kind of act? What is the *act*, as distinguished from the *acted on*, and from any inessential concomitants of the action? For these are of the form, as distinguished from the matter.

The logician's 'form,' De Morgan ([25, pp. 75–83, 183n.1, 217–221, 228–229, 241]; see [34, pp. 40–42, 57]; [56, p. 124]; [62, III, p. 218]) argues, diverges from inquires of 'matter'; distinctions in language however modifying usage to language, as the logician's exposition of formulae may require, the 'matter,' not the 'form,' being the idiom of the subject.

De Morgan's formal-material distinction, and the promise of his 'mathematical logic,' or the remarks of De Morgan ([24, p. 352]; [25, p.

234, see pp. 78n.1, 91, 184n.1, 337, 345]; [34, pp. 35–37]; [56, pp. 1–10, 89–112, 170–195]; [74, pp. 69–70, 93–94, 167, 210, 251]), almost casually that Hamilton's system "is not self-complete," and that logical "rules would give mechanical canons of inference, if such things were wanted," variously, all address a 'modern' logic of his time; in yet various ways, whether the point of view be always De Morgan's, each looks to a 'modern' logic of the present century. The Whatelian language-oriented sense of 'form,' in contrast, foreshadowing, it seems, as (quoted, [56, p. 2, see p. 93]; [73, pp. 176–181, 199–200, 250–258]) "roughly Gilbert Ryle, vintage 1826."

De Morgan ([25, pp. 247–248, see Heath, p. xxix]; [42, p. 59]) observes, in 1860, the "changes," in England, since Whately [76] that have "undergone" the "study of logic." De Morgan could assuredly claim, albeit outmoded now seem his aims, some contribution to those "changes."

De Morgan's logical writings, however, do show him subject to something of a procedural and notational lag. "Although the work of De Morgan is strictly contemporary with that of Boole." as Lewis [48, p. 38] observes, "his methods and symbolism ally him rather more with his predecessors than with Boole and those who follow." The logical work of De Morgan represents, simply, Victorian 'innovative accommodation,' adapting a system rather than abandoning it (see [52, pp. xxv, xxvii]), which, as in the case of Boole rather than De Morgan, not infrequently would have the effect of creating a wholly new system. Thus, De Morgan (see [34, pp. 37–39, 42–43, 48–49, 55–56]; [35, p. 382]; Heath, [25, p. xxv]; [48, pp. 37–51]; [56, pp. vii–ix]; [62, II, pp. 326–327]) introduces 'x,' as a contrary term or negative of 'X,' proposes (so limiting their scope) the convention of a 'universe of discourse,' and develops a logic of relations, but De Morgan's primary concern is always logical reform (improving and enlarging) of the traditional syllogism.

De Morgan [25, quoted, Heath, p. xxi], writing to Boole in 1861, admits to "always grubbing at logic in some shape or other." The one such "shape or other," (Heath, [25, pp. xxv, xxix]) "at which he tinkered a good deal without ever quite achieving finality," is his "rather makeshift" notation. It is a feature of this notation, De Morgan's (quoted, [42, p. 58]; see [25, pp. 31, 33–36, 88, 106, 129–130, 157–158, 166, 191, 195, 203, 234, 261, 318–319, 324]; [31, pp. 3–21]; [34, pp. 33–39, 43–54]; [48, pp. 38–51]; [68, pp. 133–134, 141–142, 148–156]; compare [56, pp. 63, 76, 140–142, 152, 155, 164–165]) "horrent with mysterious spiculae," that each term on the inner side of a spicule (or parenthesis) is distributed, on the outer side is undistributed; indicating quality, a dot between spicules marks a proposition 'negative,' and two dots, or none, 'affirmative.'

Prior [68, p. 126] observes that De Morgan "certainly did more than anyone else had done with the kind of logic that takes the Aristotelian syllogistic as its starting-point."

De Morgan ([24, pp. ix, 53–62, 312]; [25, pp. 50–66, 68, 74–83, 98, 117–119, 154, 173–176, 183, 183n.1, 184n.1, 208–220, 225, 228, 234, 240–241, 248, 250–254, 271, 312, 345]; see [34, pp. 35–43]; [56, pp. 80, 98–101, 103–105, 107–112, 117, 123, 192–194]; [76, p. 70]) regarding, as he does,—improving and enlarging,—the syllogism, as central to logic,—the formal laws of the machinery of thought,—it is initially puzzling, then, he considers the copula, as essentially being material. De Morgan ([25, pp. 79n.2, 51, 75, see pp. 50–51, 53, 58, 74–83, 117–118, 173, 187, 214–218, 253–254]; [24, pp. ix, 53–62, 312]), however, proffers the copular sign, in "abstraction," disregarding the material difference between "*equality*" and "*identity*," considering only the formal principles, which determine syllogistic inference; and thus, as formal properties of the copula, "*transitiveness*" and "*convertibility*" being, as it were, "detected when we watch the machine in operation without attending to the matter operated on."

De Morgan's is an interesting usage of 'machine,' living in this Victorian age, as he did, when (see [36]) machines were at the cutting edge of technology. De Morgan ([25, pp. 75–78, 79n.2, 79–80n.3, 82, 234]; see [39, pp. 673–674]; [56, p. 249n.4]) apparently suggesting the analogy,—comparing 'form' of machine versus 'form' of inference, and hence 'principle' being the commonable tenor,—drawn between 'machine' and 'inference.'

De Morgan [25, p. 79n.3] refers to "Nasmyth's steam-hammer," which is to that of James Nasmyth (1808–1890), whose 1842 patent having proved so crucial to development of heavy industry in Victorian Britain. It is the view, pertinently expressed by a Victorian writer ([71, p. 380]) that

> By the simple device of attaching the hammer head to the lower end of the rod of a piston working in an inverted steam cylinder, he [Nasmyth] produced a machine capable of being made to deliver its blows with a force to which no limit has yet been found, and yet so perfectly under control as to be able to crack a hazel nut without injuring the kernel. To the introduction of this invaluable tool is due more than to any other single cause the power which we now possess of producing the forgings in iron and steel which are demanded by the arts of modern times; and ... it is now met with in every workshop in which heavy work is carried on.

The words of the Victorian writer that Nasmyth's steam-hammer is "able to crack a hazel nut without injuring the kernel," occasion those of De Morgan [25, p. 79n.3, see pp. 75, 79n.2] that "the pure form [of nut-cracking], which ... applies to ... Nasmyth's steam-hammer," is the "form of ... being pressure enough applied to opposite sides of the nut."

"The syllogism is the nut to be cracked," De Morgan [25, p. 79n.3] asserts, and "cracked," as it were, with transitiveness and convertibility,—the formal principles, which determine syllogistic inference.

De Morgan seems without some indisposition to technology, then driving the industrialisation of Britain; interestingly, in the way of his (above) instancing Nasmyth's steam-hammer, even though his (quoted, Heath, [25, p. x]) having "spent only $1\frac{3}{4}$ hours in the Great Exhibition" of 1851, at London. "The Exhibition illustrated Britain's superiority in technology," which as Beales [3, pp. 210, 209] remarks, "the promoters ... saw in it a triumph of peace and internationalism," if "the triumph of [Britain's] industrialization, bringing mass production and a flood of new materials, together with pride and confidence in material progress, were fatal to taste." It would have likely been to De Morgan's 'taste,' with "$1\frac{3}{4}$ hours" being, indeed quite enough for him!

The artistic, intellectual, scientific, and technological strides,—continuous and incremental,—which mark the initial thirty-six years of the nineteenth century, and then the subsequent three decades, which are the first flush of Victorian Britain, must have seemed to many Victorians, however, the panoply of achievements. The impression would yet be mixed with a sense of 'anxiety.' Thus, "Retrospect of Literature, Art, and Science in 1867" of the *Annual Register*, sounding such concern (quoted, [15, pp. 299–300]) that

> Superior to every other nation in the field of battle, she [England] nevertheless owes her great influence, not to military successes, but to her commanding position in the arena of industry and commerce. If she forgets this, she is lost; not perhaps to the extent of being conquered and reduced to a province, but undoubtedly to the extent of having to give up the lead, and ceasing to be a first-rate power. The signs, for those who can read, are present, and can be plainly seen ... which should induce Englishmen to reflect seriously.

The signs also plainly present, as "England passed through the most painful phase of its industrialization," Marshall [52, pp. xvii, xix] noting, the "poverty and degradation," the "displaced agrarian workers," and "the plight of those men and women suddenly cast out of work by unexpected overproduction or by the invention of a machine that replaced them." For Henry Adams (1838–1918), winnowing the English society of 1858 to 1865 (quoted, [15, p. 86]), "History muttered down Fleet Street, like Dr. Johnson, in Adam's ear," and "The eighteenth century held its own." Henry Adams also observes (quoted, [3, p. 169]; see [15, pp. 300–304]) that

> Never had the British mind shown itself so *décousu*—so unravelled, at sea, floundering in every sort of historical shipwreck. Eccentricities had free field. Contradictions swarmed in State and Church.

It is (see [3, pp. 16, 213]; [52, p. xxvii]) Darwin's 'evolution,' by 'natural selection,'— exerting, as it did, amongst, then, developing sciences moving into the public domain, a signal influence on intelligence in the 1860s,—that seemed so "desultory" (*décousu*), disjointing, as it were, the received conventions of thought. And it is this 'disjointing' that [30, pp. 23–24] contributed to "the so-called nineteenth-century nightmare, a curious complex of theoretical beliefs and emotional reactions."

The Victorian response to the new dynamics of the nineteenth century and the paradigms of the eighteenth century, typically, would be,—as De Morgan, advancing a logic of relations, enlarging the syllogism,—one of 'innovative accommodation.' Marshall [52, pp. xxv, xxvii] pointing out,

> The Victorian thinker, whether he were a philosopher or a scientist or a literary man, committed himself—perhaps more intensely than any intellectual in history—to the search for truth in a universe of rapidly changing concepts and to the establishment, upon whatever degree of truth he might find, of propositions explaining the universe and the nature of man.
>
> But the most significant aspect of the Victorian crisis of belief was to be found among the intellectuals themselves. Their painful appraisals of the ontological and moral implications of the new science bear witness to the fact that they were intent upon saving rather than abandoning [a system or] value, although frequently to do so they had to build a new system to replace the old.

Thus, the ([15, p. 276n.]; see [22, pp. 160, 178–179]) Victorian "grand thesis of metaphysical evolutionism," some proponents erroneously assuming sanction of Darwinism; for many metaphysical evolutionists, so far as they could arrogate Darwin "one of themselves they were cushioned against being seiously shocked or alarmed by him," simply not understanding the import of Darwin's conclusions. Peirce (see [21]; [30, pp. 23–24, 180–242]; [60, pp. 13–15, 100–101, 173, 218, 294–295, 322–326, 328–329, 348–350, 403–404]), with his grasp of Darwinism, being here, of course, *the* obvious exception. "Unbroken Evolution under uniform conditions," ironically remarks Henry Adams (quoted, [3, p. 214]), "pleased every one—except curates and bishops; it was the very best substitute for religion; a safe, conservative, practical, thoroughly Common Law diety."

De Morgan seems to have [42, p. 59] "left no published indications of his opinions on religious questions, in regard to which he was extremely reticent." De Morgan in his publications, whether he does or does not likewise indicate 'anxiety' to Darwinism; it is the syllogism, to paraphrase Henry Adams (quoted, [15, p. 86]), that a sense of at least the eighteenth century holds its own; history muttering in De Morgan's ear, being [48, p. 38] "bent upon improving the traditional Aristotelian logic."

De Morgan, a professional mathematician, erudite, writing on logic with an admixture of the old and the new, and widely on mathematics, science, and history (see [42]); who in the Victorian milieu, his calling himself (quoted, Heath, [25, p. vii]; see [3, pp. 158, 276–278]; [15, pp. 46–48, 270–278, 302–304]; [22, pp. 266–276]) "Christian unattached," may not have been a euphemism, and unlike Darwin, not agnostic.

Jevons [42, pp. 59, 57] relates that De Morgan "seldom or never entered a place of worship, and declared ... he could not listen to a sermon, a circumstance perhaps due to the extremely strict religious discipline under which he was brought up," and indeed "strong repugnance to any sectarian restraints upon the freedom of opinion was one of De Morgan's most marked characteristics throughout life," which "prevented [him] from taking his M.A. degree, or from obtaining a fellowship, ... by his conscientious objection to signing the theological tests then required from masters of arts and fellows at Cambridge."

Jevons having (quoted, Heath, [25, p. viii]) "adored" his late teacher and perhaps recognising a sense of propriety, Jevons [42, p. 59] attributes "a deeply religious disposition" to De Morgan, "founded not on any tenuous method of inference, but on personal feeling." The ascription is certainly at some variance with De Morgan, for whom ([25, p. 26, see pp. 8, 26–29, 119, 214–215, 237, 249, 251–252, 301–304]; [24, pp. 2,8–9]) "the nobel act of the mind [is] called by us *inference.*"

De Morgan ([25, p. 88, see pp. 22-23, 74, 82, 88–89, 120n.2, 139n.1, 208–246, 254–256, 330, and Heath, pp. x, xv, xxiv]; [9, pp. 24–51, 226–242]; [34, pp. 33–34]; [35, pp. 381–382, 386]) despairs, not theologically, but the notational development in logic:

> Every science that has *thriven* has thriven upon its own symbols: logic, the only science which is admitted to have made on improvements in century after century, is the only one which has *grown no symbols.*

De Morgan's claim, itself in a logical memoir (read on the eighth of February 1858), begging the question or not, is historically in context, *inter alia*, De Morgan [24], Boole [8], [9], and the memoir (dated the twelfth of November 1859, but read on the twenty-third of April 1860) of De Morgan [25, pp. 208–246], on the logic of relations.

De Morgan [24], his logical papers, articles, and memoirs over the years 1839 to 1863, altogether embody (see [34, pp. 33–35]) the endeavours of De Morgan to produce syllogistic improvements, and interesting, if not always viable, attempts to also engender notational exactitude. His principal logical treatise, [24], is a virtual grab-bag, chapters elaborating his syllogistic tenets, including the numerically definite syllogism, a reprint of his *First Notions* of 1839 as chapter one, others on probability, induction, old logical terms, and fallacies. Jevons [42, p. 58] describing De Morgan's chief publications in logic, states that

> The severity of the [1847] treatise is relieved by characteristic touches of humor, and by quaint anecdotes and allusions furnished from wide reading and perfect memory.

> There followed at intervals, in the years 1850, 1858, [1859 or] 1860, and 1863, a series of four elaborate memoirs on the "syllogism," printed in volumes ix. and x. of the *Cambridge Philosophical Transactions.* These papers [25, pp. 22–68, 74–146, 208–246, 271–345] take together constitute a great treatise on logic, in which he substituted improved systems of notation, and developed a new logic of relations, and a new [relationship of names, considered purely as names, or] onymatic system of logical expression. Apart, however, from their principal purpose, these memoirs are replete with acute remarks, happy illustrations, and abundant proofs of De Morgan's varied learning. Unfortunately these memoirs are accessible to few readers, otherwise they would form invaluable reading for the logical student. In 1860 De Morgan endeavored to render their contents better known by publishing a *Syllabus of a Proposed System of Logic* [[25, pp. 147–207]], from which may be obtained a good idea of his symbolic system, but the more readable and interesting discussions contained in the memoirs are of necessity omitted. The article "Logic" in the *English Cyclopaedia* (1860) [[25, pp. 247–270]] completes the list of his logical publications.

De Morgan's is actually a series of five memoirs, his publishing "On the Syllogism" (see [34, p. 33]; [56, pp. 49–50]); *his initial installment*, De Morgan [25, pp. 1–21] publishing in 1846. Jevons' "list," however, only comprises entries of De Morgan's four subsequent memoirs; and in his entries, Jevons perhaps not including the 1846 memoir of De Morgan, since De Morgan [24, see pp. 378–379] does incorporate the syllogistic system of De Morgan's 1846 memoir.

The fruition of De Morgan putting hand to pen and paper, in fields of 'exact science' and beyond, is yet extraordinary,—speed of work counting for nothing (see [22, pp. 275–276]; [42, p. 59]), and labours literacy and scientific being prodigious,—in the age and society of Queen Victoria. If quirky though his prose may be, De Morgan's publications seem finished with an eye to accuracy of expression (see [42, p. 59]; [65, p. 41]); and in quantity or polemics, quality and recitude seem sacrificed nary a jot. "It is possible," Jevons [42, p. 59] opines, appraising De Morgan's prose, "that his continual efforts to attain completeness and absolute correctness injured his literary style, which is wanting in grace; but the estimation in which his books are held is shown by the fact that they are steadily rising in market price." Jevon's criterion of a 'market price' would seem that of the economist,—Jevons, of course, having been an economist, as well as being a logician.

Jevons ([42, p. 58]; see [25, pp. 22–23, 75–83, 184n.1, 337, 345, Heath, p. xxiv]; [56, pp. 170–195]) does present insight, when correctly predicting of De Morgan, that despite "the excellence and extent of his mathematical writings, it is probably as a logical reformer that De Morgan will be known to future times," although "it may be doubted whether De Morgan's own system . . . is fitted to exhibit the real analogy between quantitative and qualitative reasoning, which is rather to be sought in the logical works of Boole."

The logical writings of De Morgan, though "extensive," as Merrill [56, p. 171] points out, reflect but one side of "the sheer breadth of his intellectual interests, covering almost all aspects of Victorian culture,"—a son (see [4, p. 486]; Heath, [25, p. viii]), William Frend De Morgan (1839–1917), interestingly, being artist and novelist. The Victorian era is the great age of scientific development in the Western world (see [2, pp. 3–21]); the professional *cognoscenti* emerging more and more, while remaining an era of the *amateur*, but in actual terms of scientific work, by no means, as presently, being an exclusive province of the professional. Thus, "ironic," it may be to Merrill [56, p. 171], but less so would it perhaps be (or have been) to the Victorian that De Morgan's work in logic, his "most original work, and that which probably had the most lasting impact, was in a field that was something of an avocation for him."

De Morgan, the 'logician,' seems little recognised nowadays (see [56, pp. 170–195]), for his even having been a professional mathematician; and although eyeing all ways his Victorian culture, mathematics and logic (see Heath, [25, pp. viii–xi]; [65, pp. 41–42]) appear, as yet the two most conspicuous, if not convergent aspects of De Morgan's interests. "Unlike most mathematicians [of his day],", Jevons [42, p. 58] asserting that "De Morgan always laid much stress upon the importance of logical training."

It is perhaps worthy of note (see Heath, [25, pp. vii–viii]; [42, p. 57]) that De Morgan's maternal great grandfather, James Dodson, F.R.S., in his day, having been well-known, a mathematician and the author of such works, as *Anti-logarithmic Canon*; William Frend, in occupation a mathematician and actuary, and in faith a Unitarian, having been De Morgan's friend, neighbour, and later (1837) his father-in-law, and hence, De Morgan twice being related, by descent and marriage, to mathematicians.

Jevons ([42, p. 57]; see Heath, [25, pp. vii–viii]) states that "De Morgan's attention [as an undergraduate at Trinity College, Cambridge,] was by no means confined to mathematics, and his love of wide reading somewhat interfered with his success in the mathematical tripos, in which he took the [A.B.] fourth place in 1827." The Fourth Wranglership does not seem to have proven an obstacle to De Morgan's election, professor of mathematics, at the new and nondenominational University of London (now, the University College) in 1828; the sole academic post, De Morgan holding, albeit intermittently, the rest of his working life. De Morgan has perhaps the rarest of distinctions, as a professor, for his having twice resigned the same chair, and each time on a point of principle, protesting against the powers that be of the university;

first retiring, with his later being unanimously recalled, a hiatus lasting the years 1831 to 1836, and, only reluctantly, finally quitting in 1866. "As a teacher of mathematics," Jevons ([42, p. 57]; see [65, pp. 41–42]) remarking on his late mentor, "De Morgan was unrivalled." De Morgan, it is also said (quoted, [42, p. 59]), "was the kindliest, as well as the most learned of men—benignant to every one who approached him, never forgetting the claims which weakness has on strength."

De Morgan is easily a caricatural kind of Victorian Englishman (see [22, pp. 275–276]; Heath, [25, p. vii]; [42, p. 57]), though, as it were, an 'Anglo-Indian,' in the sense of circumstance and connexion: Augustus having been born at Madura (in Madras presidency), India, but residing in England, from his infancy, with his intellectual heritage being purely British; De Morgans (Huguenot, in origin) of the previous three generations having been military officers in the East India Company's service,—a congenital defect of Augustus' right eye having precluded any prospect of his following the family career, fortunately, it being perhaps for the history of logic.

De Morgan can be imagined, "the good-humoured" butt of the likes of *Punch*, ensuing (1847–1848) its change in tone and outlook, emphasising (quoted, [15, p. 73]; see [2, p. 437]) "the kindly and urbane notation of oddity," but a "manly exposure of affectation and meanness" would be of De Morgan, neither manly nor just. De Morgan's 'character' is a happy combination of qualities, Heath [25, p. x] stating that

> De Morgan's generosity and lack of self-interest, his high principles and honest indignation at all worldly shams and expediencies, were somewhat reminiscent of Mr. Pickwick, whom he came to resemble in appearance, and whose creator he greatly admired. Such qualities, allied to a certain natural pugnacity, were apt to involve him in controversy, made him something of a foe to the Victorian 'Establishment', and led, indeed, to his refusal even of such conventional honours as would ordinarily have come his way.

Jevons [42, p. 59n.1] relates "that De Morgan was [both] a great reader and admirer of Dickens; he was fond of music, and a fair performer on the flute." De Morgan [25, pp. 231–233n.2] humourously admits to having an apparent, native inability, at recognising certain musical and logical notation. Peirce ([65, p. 41]; see [25, p. 52]) reporting of De Morgan, "his genial and hearty manners," and "a man of full habit, much given to snuff-taking; and those who have seen him at the blackboard, mingling snuff and chalk in equal proportions, will not soon forget the singular appearance he often presented."

De Morgan's care of the word, written and printed, for perhaps [42, p. 59] "no writer can be more safely trusted in everything which he wrote"; his wide, and love of reading, partly bespeak the bibliophile. It would have been abhorrent to De Morgan, the habits of Darwin [20,

p. 111] to "cut a heavy book in half, to make it more convenient to hold," and to "tear out [of pamphlets], for the sake of saving room, all the pages except the one that interested him." Thus, Jevons [42, p. 59] bethinks,

> One marked character of De Morgan was his intense and yet reasonable love of books. He was a true bibliophil, and loved to surround himself, as far as his means allowed, with curious and rare books. he revelled in all the mysteries of watermarks, title-pages, colophons, catch-words, and the like; yet he treated bibliography as an important science. As he himself wrote, "the most worthless book of a bygone day is a record worthy of preservation; like a telescopic star, its obscurity may render it unavailable for most purposes; but it serves, in the hands which know how to use it, to determine the places of more important bodies."

"A sample of De Morgan's bibliographical learning," Jevons [42, p. 59] regards, "is to be found in his account of *Arithmetical Books, from the Invention of Printing* (1847), "which remains, remarks Heath [25, p. xin.1], "a classic of mathematical bibliography, and (as any antiquarian bookseller's catalogue will testify) still in daily use."

De Morgan's library, not otherwise being dispersed over the years, and (see Heath, [25, pp. x–xi]; [42, p. 59]; [56, p. x]) most of his papers (manuscripts and letters), now are such properties of the University of London.

"The writer of a book," Peirce [62, I, p. 98] observes, "can do nothing but set down the items of his thought." Peirce ([66, IV, p. 239]; see [19, pp. 302–347]) considers that

> Giving to the word *sign* the full scope that reasonably belongs to it for logical purposes, a whole book is a sign; and a translation of it is a replica of the same sign. A whole literature is a sign.

Peirce ([62, II, pp. 130, 165–169; VI, pp. 359–362, 389–390; V, pp. 169–189; VI, pp. 175–177, 235–236]; [63, VII, pp. 212–218, 347–358; VIII, pp. 148–150]; [66, IV, pp. xxi, 238–240, 262–263]) would virtually deem that a book be a sign of its author, *ipso facto*, the book *is* the man. For the thought being a sign, in the sense of a semiosis, and the man *is* a sign, in the sense of the man, as the sign user *is* the sign processor, with Peirce ([62, V, p. 189]; see [14]; [59, pp. 3–13]; [80, (German) pp. 114, 116, 118, (English) pp. 115, 117, 119]) concluding, "my language is the sum total of myself; for the man is the thought."

The qualities of De Morgan's writings,—take a book, memoir, or an article of his, any that comes to hand, being learned, discursive, perspicacious, prolix, awkward, elegant, polemic, varacious, genial, serious, hearty, quirky, or Pickwickian,—are certainly no less than those of the man himself,—a work of De Morgan's, *is* De Morgan. His thought being in its entirely, living thought that is of service, as De Morgan writes (quoted, [42, p. 59]; see [62, I, p. 98], "in the hands which know how to use it."

'De Morgan' is not infrequently to be seen in text or footnote of logic's annals; but with the thought that *is* De Morgan, seldom the reader venturing forth to meet and to acquaint himself. It is, in words [53, p. 543] of Herman Melville (1819–1891), as it were,

> Signs and wonders, eh? Pity if there is nothing wonderful in signs, and significant in wonders!

Peirce ([62, I, p. 301; see IV, p. 510]; [25, pp. 234–235] considers

> that no decent semblance of justice has ever been done to De Morgan, owing to his not having brought anything to its final shape. Even his personal students, reverent as they perforce were, never sufficiently understood that his was the work of an exploring expedition, which every day comes upon new forms for the study of which leisure is, at the moment, lacking, because additional novelties are coming in requiring note. He stood indeed like Aladdin (or whoever it was) gazing upon the overwhelming riches of Ali Baba's cave, scarce capable of making a rough inventory of them. But what De Morgan, with his strictly mathematical and indisputable method, actually accomplished in the way of examination of all the strange forms with which he had enriched the science of logic was not slight and was performed in a truly scientific spirit not unanimated by true genius.

"The only upheaval of the century," regards Peirce [62, I, p. 263], reflecting on the course of thought, in the nineteenth century "that stands amid the tempest of philosophical opinion unshaken and citadel-crowned is the exact logic of Boole and De Morgan; and this was the product of pure scientific study," which "would leave no practical maxim of reasoning unaffected, but would extend some, curtail others."

The well-known pair of sentential 'laws' currently bearing De Morgan's name,—and, for most students of logic, generally sufficing to commemorate his name,—are actually, the analogues of such principles, as De Morgan ([25, pp. 119, 182, see Heath, pp. xxv, xxvii]; [24, p. 133]; [17, p. 104n.188]; [18, p. 547]; [34, pp. 37–40]; [48, p. 37]; [68, pp. 14, 141]) variously formulates (notably 1847, 1858, and 1860), which he renders in the logic of terms:

> The contrary of an aggregate is the compound of the contraries of the aggregants [where negative (or 'contrary') terms are represented by small letters, the conjunction of terms by simple juxtaposition, and their disjunction by the interposition of a comma]: the contrary of a compound is the aggregate of contraries of the components. Thus (A, B) and AB have ab and (a, b) for contraries.
>
> The contrary of an aggregate is the compound of the contraries of the aggregants: either one of the two X, Y, or

> both not-X and not-Y; either (X, Y) or (xy). The contrary of a compound is the aggregate of the contraries of the components; either both X and Y, or one of the two, not-X and not-Y; either (XY), or (x, y).

These principles or 'laws,' however, are in point of fact, De Morgan's 'rediscovery'; implicitly or explicitly, as they are, in work (see [7, pp. 207, 236]; [17, p. 104n.188]; [45, pp. 295, 314–315]; [49, p. 197]; [50, pp. 211–212]; Boehner, [61, p. xxxvii]; [68, pp. 14, 78]) of Peter of Spain (†1277), William of Ockham (1280–1349), Walter Burleigh (1275–1345), and Arnold Geulincx (1624–1669).

De Morgan [25, pp. 208–246] is commonly credited, with his having initiated the moderm logic of relations. "Here," extending the subject-predicate concept, De Morgan ([25, p. 220, see pp. 119, 221, 225]; [7, p. 375]; [34, pp. 43–44]; [67, II, p. 738]) stating that "X and Y are *subject* and *predicate*: these names having reference to the mode of entrance in the relation [L], not to order of mention."

De Morgan's statement of relational logic does, indeed, constitute something of a precedence, though, it should be said that ready were the conditions (see Aristotle, *Topics*, ii 8 (114a13); [13, pp. 109–143]; [18, p. 547]; [32]; [33]; [34, pp. 32–33, 53–54, 56–59]; [35, pp. 386–389]; [45, pp. 41–42, 313, 324, 427–434]; [47, pp. 88–89, Parkinson, pp. xix–xx, lxii]; [48, pp. 50, 79–106]; [55]; [56, pp. vii, 1–25, 79]; [72, pp. 96, 101, 114n.43, 125, 131, 155, 158, 159]; [75, p. 88n.1]): there being, on the one hand, sporadic examples of relational theory, in work of Aristotle (384–322 BC), William of Sherwood (†1249), John Buridan (†1358), Joachim Jungius (1587–1657), and G.W. Leibniz (1646–1716): on the other hand, Peirce extending in conception, the theory of relations (or 'relatives'), discovering the connexion between relational theory and Boole's algebra of logic, independently of De Morgan, and developing the calculus of relations.

Peirce ([67, I, p. 493]; see [66, III, p. 740]) observes that "Boole created a method of miraculous fruitfulness, which aided [me] in the development of the logic of relatives," and while "De Morgan ... brought the logic of relatives into existence," Peirce ([67, I, p. 143]; see [66, IV, p. 152]) remarks that "I still say without affectation, that I at once left his work far behind."

Peirce ([62, IV, pp. 8–9, see p. 510; I, pp. 3, 301–305; II, pp. 284–312, 508–517; III, pp. 3–98, 404–409]; [65, p. 42]; [66, III, p. 740; IV, pp. 152, 334–335, 882–883]; [67, I, p. 143]; [10]; [57, p. 75n.32]; [55]; [56, pp. 10–25, 28, 32, 170–195]) reminiscing in 1898, recalling his

> logical studies in 1867, various facts proved to me beyond a doubt that my scheme of formal logic was still incomplete. For one thing, I found it quite impossible to represent in syllogisms any course of reasoning in geometry, or even any reasoning in algebra, except in Boole's logical algebra. Moreover, I had found that Boole's algebra required

> enlargement to enable it to represent the ordinary syllogisms of the third figure; and though I had invented such an enlargement, it was evidently of a makeshift character, and there must be some other method springing out of the idea of the algebra itself. Besides, Boole's algebra suggested strongly its own imperfection. Putting these ideas together I discovered the logic of relatives. I was not the first discoverer; but I thought I was, and had complemented Boole's algebra so far as to render it adequate to all reasoning about dyadic relations, before Professor De Morgan sent me [and not before May or June of 1868, received] his epoch-making memoir ["On the Syllogism, No. IV, and on the Logic of Relations," [25, pp. 208–246]] in which he attacked the logic of relatives by another method in harmony with his own logical system. But the immense superiority of the Boolian method was apparent enough, and I shall never forget all there was of manliness and pathos in De Morgan's face when I pointed it out to him in 1870. I wondered whether when I was in my last days some young man would come and point out to me how much of my work must be superseded, and whether I should be able to take it with the same genuine candor.

Lest a credibility of Peirce's 'originality' here be discounted, Gallie ([30, p. 12]; see [60, pp. 1–19]) points out that "Harvard, the nursing ground of ... [Peirce], was in the second half of the last century a cultural centre at least the equal of Oxford and Cambridge; it had long and deep, if somewhat narrow, intellectual traditions of its own, and in the opinion of Charles Darwin it contained enough brilliant minds in the 1860s to staff all the universities in England."

Peirce ([66, III, pp. 882–883]; see [55]; [57]; [58]), in a letter of December 5, 1908, responding to queries of P.E.B. Jourdain (1879–1919), diverges from his (above quoted) account of 1898, in the particulars, regarding his receipt of De Morgan's memoir, although he perhaps does somewhat clarify, in a sense, the 'originality' of his logic of relatives, in the 1860s, being independently of De Morgan:

> Undoubtedly De Morgan's paper on the Logic of Relations influenced me much when I came to know it; but a paper of mine [[62, II, pp. 284–312]] in the Proceedings of the American Academy of Arts and Science (of Boston, Mass.) Vol. VII has a passage [[62, II, p. 306]] on p. 281 which shows I had been thinking of the matter, though it also shows that I had not advanced but very little in it. This was read 1867 April 9, when it is clear that I had not seen De Morgan's paper. Moreover, there is internal evidence that a paper by me [[62, III, pp.27–98]] of 1870 published in Vol. IX of the Memoirs of the same Academy (I will send you a copy) was nearly complete before I had much acquaintance with De Morgan's paper. For having used [[62, III, pp. 46, 69–70]] the notation ι^W to mean lover of every women, it was only

> as an afterthought that I introduced 'W to signify lover of nothing but women, which a reading of De Morgan's paper [see [25, pp. 220–224]] would have shown me to be necessary. As far as my recollection goes, I was in London in 1870 for some months and called on De Morgan and carried him my paper [[62, III, pp. 27–98]] and he then presented me with his [[25, pp. 208–246]]; and I should say from memory unchecked, that almost all my acquaintance with De Morgan's system was derived from that and his [[25, pp. 147–207]] Syllabus which he gave me the same day. But I suppose I must have been more influenced by him at first than this would imply. It was Boole whom I was chiefly thinking of in those days. My point of view remained quite opposed to some chief features of De Morgan's such as that a proposition implies the existence of its subject [see [24, pp. 123–130]; [25, pp. 6–7, 62, 72–73, 83, 97, 119, 128, 154n.1, 156, 160, 206, 220, 221n.1, 222–223, 225–226, 229–246, 250–251, 269, 300–345]], which is bed-rock truth for him. All I admit is that the interpreter of the proposition must have a previous acquaintance with its subject.

The 1870 paper of Peirce [62, III, pp. 27–98], on the logic of relatives, is the product of an 'originality,' in the 1860s, which is at the very least, in terms of 'method,' quite independent of De Morgan (see [18, p. 547]; [48, p. 50]; [52, pp. 224, 228–229]; [55, pp. 280–281]; [57, pp. 71–73]; [58]): De Morgan's relational logic, which is not "Boolian," in his conception of method, notes Peirce ([62, IV, p. 9]; see [25, pp. 22–23]), but "another method in harmony with his own logical system," and "Boole" is "whom" Peirce [66, III, p. 883, see p. 740] "was chiefly thinking of in those days."

"Just as Peirce has his Procrustean 'Boolian' equations," asserts Martin [52, p. 224, see pp. 228–229], "so De Morgan has his Procrustean syllogisms."

Boole's is a 'logic,' of course, cast as an algebra ([35, pp. 382–383, 386]; [41, p. 43]; [48, pp. 51–57]), operators and operands, being restricted to 1 or 0, or extended to classes. And De Morgan ([25, pp. 22–23]; see [42, p. 58]) specifically states, in 1850, that his "methods ... have nothing in common with that of Professor Boole, whose mode of treating the forms of logic is most worthy the attention of all who can study that science mathematically, and is sure to occupy a prominent place in the ultimate system."

De Morgan construing the copular sign, in 'abstraction,' if not in a sense, his bicopular syllogism, does tempt, however, saying that De Morgan ([24, pp. ix, 53–62, 90–91, 312, 334–338]; [25, pp. 8, 50–51, 55-59, 77–80, 79n.2, 173–174, 190–194]) almost suggests, as it were, something of Boole ([9, p. 38, see pp. 6–7, 31, 36–38]; [35, pp. 382–383, 386]; [56, pp. 165–166, 189]), for whom, an abstract calculus, subject to different interpretations, operators and operands, "interpretation will alone divide them."

De Morgan's is, primarily, a 'logic' of names, 'common' or 'general' (see [24, pp. 48–49, 99–100]; [25, pp. 52–53, 80, 82, 89–100, 104–107, 110, 116–127, 139n.1, 154, 178–182, 185, 188–205, 218, 241, 249–256, 267–269, 304–305, 309, 312, 330, Heath, pp. xxiv, xxviii]; [18, p. 547]; [34, pp. 35–39, 43]); extension and intension he posits of both classes and attributes, with union and intersection of classes receiving his notational expression, as disjunction and conjunction of names, and aggregation and composition of terms.

The interest of De Morgan [25, p. 220], in his even developing a logic of relation, is "to consider the formal laws of relation, so far as is necessary for the treatment of the syllogism." There is, in the calculus of relations, of course (see [24, pp. 53–62, 312]; [25, pp. 160, 212–218, 227–235, 238–239]; [33, p. 132]; [34, pp. 38–39]; [45, pp. 427–428]; [48, p. 271]; [68, pp. 150–151,159–161]), an exactly corresponding theorem, for each theorem of the calculus of classes. "The whole of the system of relations of quantity remains undisturbed," De Morgan [25, p. 235]), not too surprisingly, writing in 1859, "if for the common copula '*is*' be substituted any other relations: so the usual laws of quantity may be applied to the ... *unit-syllogisms* ... precisely as if [the relations] L and M only meant 'is.'" De Morgan's "Procrustean bed," as Martin [52, p. 228] reminds, "is that of the syllogism."

"In the form of the proposition," as having twice alluded to here, De Morgan ([24, p. ix, see pp. 53–62, 312]; [25, pp. 50–51, 56, 77–80, 79n.2, 173–174, 217–218, 225, 228–229, 252–253]; [34, p. 56]; [45, pp. 427–428]; [68, pp. 150–151, 159–161]) maintains that "the copula is made as abstract as the terms: or is considered as obeying only those conditions [of transitivity, and convertibility or symmetry,] which are necessary to inference."

De Morgan's logic of relations develops,—his word is "emerges,"—in three progressions (see [25, p. 241]; [55, pp. 247–257]), involving the abstract copula, composition of relations, and quantified relations. Thus, De Morgan ([25, pp. 228–229]; see [34, p. 56]) states that

> The universal and all containing form of syllogism is seen in ... $[(X)(Y)(Z)((XLY \cdot YMZ) \supset X(L/M)Z]$. Whether the compound relation LM be capable of presentation to thought under a form in which the components are lost in the compound—in the same manner, to use Hartley's simile, as the odours of the separate ingredients are not separately perceptible in the smell of the mixture—is entirely a question of matter.

"In the doctrine of syllogism," De Morgan (quoted, [34, p. 34]; [25, p. 221]) considers that "it is necessary to take account of combinations involving a sign of *inherent quantity*," with "attention to forms in which *universal* quantity is an inherent part of the compound relation, as belonging to the notion of the relation itself, intelligible in compound, unintelligible in the separated component." De Morgan ([24, p. 70]; [25, p. 222, see pp. 221–222, 221n.2, 226–227, 238–246]; [34, pp. 35, 46–47, 52]) suggesting almost something of a reassignment of the "*sub*" and "*super*" ($_{\prime}$ and $'$), already veterans of his 1847 fount, proffers "LM, and L of an M; LM', an L of every M; $L_{\prime}M$, an L of none but Ms," as being the "three symbols of compound relation," which "will be needed in syllogism." De Morgan ([25, p. 221n.2]; see [34, pp. 46–47]) remarking,

> Simple as the connexion [of LM' and $L_{\prime}M$] ... may appear, it was long before the *quantified relation* suggested itself, and until this suggestion arrived, all my efforts to make a scheme of syllogism were wholly unsuccessful. The quantity was in my mind, but not *carried to the account of relation.*

De Morgan ([25, pp. 234, 241, 235, see pp. 221–223]; [34, pp. 33–34, 55]) observes, "quantification itself only expresses a relation," and "here [within his memoir of 1859, a justifiable sense of pride that] the general idea of relation emerges, and for the first time in the history of knowledge, the notions of relation and *relation of relation* are symbolized," so that "in logic, as in mathematics, the horizon opens with the height gained: generalization suggests detail, which again suggests generalization, and so on *ad infinitum.*"

The logic of relations, De Morgan's major effort at reforming the traditional syllogism, in 1859, even in the context of his own work, yet seems isolated more than does it ongoing (see [35, p. 382]; [56, pp. 115, 142–143, 194–195, 250n.1]); his work of 1859, he effectively leaving otherwise fallow. The work of Peirce, however, in terms of lineage and accumulative detail, is continuous with the development of modern relational theory (see [26]; [32]; [33]; [34, pp. 32–33, 46, 54, 56–58]; [35, pp. 382, 386–387]; [38, §II, §1]; [48, pp. 3–5, 118–119, 279]; [75, p. 88n.1]); whether it be the quantificational-theoretic or algebraic-theoretic tradition, as in the work of Ernst Schröder (1841–1902), in all, largely being based upon Peirce's work,—extending Boole's algebra of logic, in 1870, developing the logic of relatives, and over the years, pursuing various subsequent elaborations with the whole of his logic. "In 1870 I made a contribution to this subject," Peirce [62, III, quoted, p. 27n.*, see pp. 27–98] remarks, in 1903, referring to his classic memoir, on the logic of relatives, "which nobody who masters the subject can deny was the most important excepting Boole's original work that ever has been made." Merrill ([56, pp. 142–143, see pp. 115, 194–195, 250nn.1, 10]; [25, pp. 77, 221–223, 234–235, 241]; [34, pp. 42–43, 49–52, 55]; [62, I, pp. 301–305]) pointing out,

> There are no indications in De Morgan's published and unpublished papers and letters that he ever attempted to pur-

> sue the logic of relations any further. He did not touch the general logic of relations or the logic of quantified relational syllogisms again. De Morgan never explained why he did not develop the logic of relations more fully. Perhaps the sheer complexity of the task led him to abandon it: De Morgan apparently did not have the tenacity we associate with the very finest mathematicians. Undoubtedly, he also was limited by his concentration on that part of the logic of relations which is needed for the theory of the syllogism. The formulation of an extended logic of relations would have to wait some ten years until Peirce's classical memoir [[62, III, pp. 27–98]] of 1870 on the logic of relatives.

De Morgan, in words [24, p. 789] of George Eliot (1819–1880), have, as it were, "An endless vista of fair things before, repeating things behind."

De Morgan's logical notation, if splendidly chronicling certain shifts, in his thought, is yet "rather makeshift," and does not always lend perspicuous expression to his work ([25, Heath, p. xxix]; see [34, pp. 34–35, 42–44]); an archaism, for the reader being only accustomed to the modern notations that are currently in use. De Morgan's notation, [25, Heath, pp. xxv, xxvi] "at which he tinkered a good deal without ever quite achieving finality," *ab initio*, "harbours ambiguities in practice, and perhaps blinded its author to weaknesses in his notion of distribution which he might otherwise have been acute enough to detect."

De Morgan ([25, pp. 5–6, see pp. 260–270, Heath, pp. xxv–xxvii]; [48, pp. 38, 38n.62, 43, 43nn.71–72]) employing 'universally' or 'particularly spoken of,' his so expressing 'distribution,' considering,

> It is usual in modern works to say that a term which is universally spoken of is *distributed*. But in truth every proposition distributes, wholly or partially, among the individuals of the predicate, or its contrary. It will be sufficient to call a term universal or particular, according to the manner in which it is spoken of. It will then be found that every proposition speaks in different ways of each term and its contrary; making one particular or universal, according as the other is universal or particular. The manner in which the subject is spoken of is expressed; as to the predicate, it is universal in negatives but particular in affirmatives. And of the two terms and their contraries, each proposition speaks universally of two, and particularly of two.

The notation of De Morgan, at once the very medium of formulating his syllogistic reform, yet at bed-rock is logically conservative, assuming unfortunately, as he does, *distribution*, which, as Geach ([31, pp. 21, 21n.8, see pp. 3–21]; [23, p. 33]) notes, "supplies easy mechanical rules for judging the validity of inferences," though the "rules are in fact not fool-proof."

The logical notation of De Morgan is like a clock,—a 'machine,' as in De Morgan's analogy,—that should in principle, keep time, and does keep time, but badly!

Merrill ([56, pp. 56–58, 63–66, 79–88, 117–124, 140–142, 152–155, 164–169, 199]; see [48, pp. 38–43]) seems not to appreciate, in De Morgan's notation, the full scope of distribution. His supplementing or amending De Morgan's notation, a jury that would seem only to emphasise the problems, amgibuity and distribution, inherent.

Merrill ([56, p. 125, see pp. 124–129, 221–229]; [25, pp. 225–227]) notes that although De Morgan does use the word "reflexive," he "propounds a striking thesis," which Merrill shows, as not universally true "that every convertable relation is reflexive." The discussion (pp. 124–129) of De Morgan's "thesis" suffers the desiderata of Merrill including in quotations, something more of the texts that he would explicate,—distractive, as here, are the errors, paginal (references) or typographic.

Merrill [56, pp. 208, 113, see viii, 120–124, 150, 166–169, 208–212, 238–239] rightly draws attention to "De Morgan's demonstration of Theorem K," as being "one of the most interesting parts of his logic of relations, for it ([25, pp. 224–227, see pp. 186–187]; [34, pp. 49–53]) comes the closest to being a formal proof," and "the closest to being a formal proof of all the results in this memoir."

Merrill [56, pp. 120–124, 208–212], construing Theorem K, states two versions of a proof, with the initial version perhaps most approximating De Morgan's own 'proof.' The 'approximating' is here used advisedly, Merrill appearing to exceed De Morgan, even supersede, in his explications, resorting to the more modern notation and deductive procedures. The points and counterpoints, in any case, only seem properly understood, with recourse to De Morgan's original text. The reader, then, best consult De Morgan ([25, pp. 224–227, see pp. 186–187]; [34, pp. 49–53]), comparing Merrill's modern 'improvements.'

There is a question that some formulae Merrill expresses, in modern notation, are even proper translations of the formulae, in De Morgan's notation (see [16, pp. 42, 120, 176, 184, 226, 241]), or he otherwise expresses, conflating notations, modern and De Morgan's, and (see [34, pp. 42–44, 49–52]; [48, p. 45n.81]; [54]; [56, pp. 120–124, 199, 208–212, 231–235]; [78]) problematically is, indeed, the desirability at all of a conflation.

Merrill [56, p. 124, see pp. 54, 78, 111, 123–129, 139, 156, 210–212, 222, 225–227, 229–235] rightly cautions a morass, declaring "sceptical," De Morgan's "blanket assumption" of "existential import." Merrill seems yet not to notice, himself treading none too gingerly, or else he makes no mention that (see [27, pp. 387–388]; [33, pp. 134, 138nn.17–18]; [34, pp. 38, 42–44, 53–54]) existential import is effectively tacit, in the very logic, he employs, in his translations of De Morgan's formulae.

There is also the errancy of Merrill [56, p. 232] that Boole "had to express the particular proposition, 'some x are y,' by the form '$x = vy$,' where v is the 'elective' symbol whose function is to generate an unspecified subclass of y." It is actually the form '$v = xy$' that Boole ([9, pp. 52–65]; see [48, pp. 52, 56–57]) expresses 'some x is y,' whereas '$x = vy$' is the form that Boole expresses 'all x is y.'

Merrill [56, Chapter VIII, pp. 196–244] presents De Morgan's contributions to the logic of relations, as "A Rigorous Formulation," which proves only to be (p. 196), and hence a "mirror," in no sense of "De Morgan's reasoning," a modern "deductive *system*." Merrill's 'formulation,' as a "reconstruction" of "De Morgan's reasoning" (p. 196), apparently serving Merrill's *pièce de résistance*, seems more unpalatable than interesting; as a modern, axiomatic, systemic 'reduction' of De Morgan's 'logic,' it seems sometimes amiss, not uninteresting (notably, pp. 235–244), and to be something of Merrill's *tour de force*.

De Morgan's 'reasoning,' in all of Merrill's discussions, informal and formal alike, would seem historically more sustainable, even more interesting, however, were Merrill's 'rigour' *entirely limited* to De Morgan's own 'rigour' of notation (see p. 199), formulation, and deductive technique. For, as it were, in words of George Eliot [28, p. 890], "Every limit is a beginning as well as an ending."

The "formal discussion of De Morgan's logic of relations," Merrill [56, pp. 209–210, see p. 233] remarks, has the revisory "task ... of bringing order to a somewhat disordered series of laws." Merrill [56, pp. 196–197, see pp. 120–124, 170–195] admits

> the whole notion of a deductive *system* is foreign to De Morgan. This would require the explicit statement of a class of immediate inferences, together with rules for linking them to form derivations in the system. De Morgan provides none of this. His interest is in results rather than process. He has discovered a great many relational arguments and he wants to convince the reader of their validity. The principles which he uses in doing so may or may not be stated explicitly.
>
> As the same time, it is possible to determine which principles De Morgan used, if only implicitly, and to systematize them .... In this reconstruction, we will attempt to mirror De Morgan's reasoning as closely as possible. We will select our axioms and rules from principles which he used, and our proof techniques will be very similar to his.

For De Morgan, as it were, in the oftentimes quoted (for example, [12, p. 25]) words of William Blake (1757–1827), "What is now proved was once only imagin'd."

The modern deductive system has to await until 1879 and 1885, in the work of Gottlob Frege (1848–1925) and Peirce (see [32]; [33, p. 133]; [35]), a modern sense of the "deductive *system*" would simply not be (hints, aside) among De Morgan's logical conceptions; in "proof techniques," comparing Merrill's with De Morgan's, a modern axiomatic system would not "mirror," could not be, indeed, a "reconstruction" of "De Morgan's reasoning."

Frege [29, p. VIII] remarks that "*Die Länge eines Beweises soll man nicht mit der Elle messen.*" The measure of a 'formulation' and 'proof' is as much a matter of history, as it is a point of logic. It seems but a truism that (see [4, pp. 478–479]; [6]; [25, pp. 78n.1, 91, 128, 183–187, 214–215, 224–226, 234, 312, 336–337, 345]; [34, pp. 36, 42–43, 49, 57]) neither De Morgan's canons of 'rigour' nor those of most of his contemporaries, nor *ipso facto*, their canons of 'proof' meet 'modern' standards. The historian of logic should yet consider the work of forebears, with some measure of tolerance. For 'modern' is not invariable. "The present school of mathematicians," observes De Morgan [25, p. 337] in 1862, "is far more rigorous in demonstration than that of the early part of this century." Frege [29, p. VIII, see pp. V–VIII] considering standards of 'rigour' (*Strenge*) and 'proof' (*Beweis*), complains that "*Den Mathematikern kommt es ja gewöhnlich nur auf den Inhalt des Satzes an, und dass er bewiesen werde.*"

Merrill [56, pp. 196, 222] would "attempt to capture De Morgan's own logic," in system D, and augment, obtaining a "system of De Morgan's logic with identity, DI." Merrill [56, pp. 233, 199, see pp. 36–38, 43–46, 54–55, 73–76, 93–94, 98, 116–129, 196–244, especially pp. 121, 201–202, 205–207, 209, 211–210, 222–223, 225–235, 237–239], "in reconstructing and ... augmenting De Morgan's logic," via a deductive system, is to "set up" the deductive system, "in a standard and completely formal way." It appears that (see [5, pp. 504–505]; [17, pp. 47–68, 76–77nn.167–168, 171n.305]; [23, pp. 89–94]; [29, pp. 69–80, 141–149]; [37, p. 67]; [44, pp. 59–65]; [74, pp. 167, 173, 280–282, 402–403]) some blurring yet are the language and metalanguage of Merrill's system, particularly ambiguous are '=' and definitions.

Merrill [56, pp. 227–228] encounters a 'principle,' which "is obviously valid, and, since it contains only the symbols of D, it ought to be a theorem of D, not just of DI," but with "many failed attempts at deductions," in D, and no "firm metalogical theorems about the scope of D it is impossible to tell," the 'principle,' then,

> is bothersome because it raises the question of whether there are other valid laws of relational logic that are not derivable in D. Such questions cannot be settled in the absence of metalogical results.

Merrill [56, p. 228, see pp. 124, 228–229] concedes, "there are other valid laws which I have not been able to deduce in D," which bodes none too well that D or DI, in the bygone words of De Morgan [24, p. 352], "is not self-complete," were the general question of completeness, any particular interest to Merrill, not merely "bothersome." The general questions (see [45, pp. 689–742]) of independence and consistency, seem likewise not to hold Merrill's interest; in the index, there not even being entries of 'independence,' 'consistency,' and 'completeness.'

Merrill [56, for example, pp. 30, 32, 45, 101–102, 108, 110–112, 132–133, 134, 152–153] would bring an acuity to De Morgan [25, see pp. 80–81, 119, 156–157, 231, 252–253], were always quotations accurately presented; were (sometimes) quotation or citation, and discussion better coordinated or synchronised; were always text cited, with the pagination; were not something of De Morgan's work better met, now and then, with the reader perusing the textuary of De Morgan; were subjects and entries of the index extensively expanded; and were misprints not rife,—too numerous, as to list. There too is Merrill [56, p. 185] employing "pragmatic," which could be mistaken for an adjective form of the technical noun 'pragmatics'; an ethics of terminology (see [62, II, pp. 129–133]) would avoid using words and phrases of technical origin, as vernacular terms. These are but vexatious flaws, however, which, for the general reader, should not otherwise derogate this book.

The informed specialist or historian of logic is apt to find that Heath [25] has largely anticipated Merrill [56], at first hand; in his commentary on De Morgan's work, Merrill has yet lent, albeit sometimes amiss, a contextual sense of the nineteenth and twentieth centuries to the work of De Morgan, which in a single volume, could be rather a boon, for the general reader.

Perhaps one should not quibble, criticism passing to cavil, for the flaws or faults of [56] are problems born of a welcome excess of ambition: to create at once, a panorama of the work of De Morgan, an extraordinary Enghlishman and Victorian logician, and a profile of De Morgan's logic of relations, where hitherto there has been no extended study (book), on De Morgan's logic.

Whatever be the faults, then, [56] is an interesting, if not an important book; an historical account (pp. 1–195), well worth reading, and the deductive system (pp. 196–244), studying, but as a 'reduction' of De Morgan's logic of relations, not a 'reconstruction' of his 'reasoning.'

The world of De Morgan is indeed not unfamiliar,—Merrill [54], [55], [56] having perhaps made a part of that world somewhat more familiar.

References

[1] R. Adamson. *Logic*. Encyclopedia Britannica XIV (9$^{\text{th}}$ ed.), 1878, 787–812.

[2] G. Basalla, W. Coleman, R.H. Kargon (editors). *Victorian Science*. Doubleday, Garden City, 1970.

[3] D. Beales. *From Castlereagh to Gladstone, 1815–1885.* W.W. Norton, New York, 1969.

[4] E.T. Bell. *Men of Mathematics.* Vol. 2. Penguin Books, Middlesex, 1965.

[5] E.W. Beth. *The Foundations of Mathematics.* 1959. Reprint, Harper and Row, New York, 1966.

[6] O. Bird. *The History of Logic.* Review of metaphysics **16**, 1963, 491–502.

[7] I.M. Bocheński. *A History of Formal Logic.* I. Thomas (editor and translator). University of Notre Dame Press, Notre Dame, 1961.

[8] G. Boole. *The Mathematical Analysis of Logic.* 1847. Reprint, Basil Blackwell, Oxford, 1948.

[9] G. Boole. *An Investigation of the Laws of Thought.* 1854. Reprint, Dover, New York, 1961.

[10] C. Brink. *On Peirce's Notation for the Logic of Relatives.* Transactions of the Charles S. Peirce Society **14**, 1978, 285–305.

[11] J. Bronowski. *The Ascent of Man.* Little, Brown and Company, Boston, 1973.

[12] J. Bronowski. *A Sense of the Future, Essays in Natural Philosophy.* P.E. Ariotti (editor). MIT Press, Cambridge, London, 1977.

[13] J. Buridan. *Sophisms on Meaning and Truth.* T.K. Scott (translator). Appleton-Century-Crofts, New York, 1966.

[14] A.W. Burks. *Man: Sign or Algorithm? A Rhetorical Analysis of Peirce's Semiotics.* Transactions of the Charles S. Peirce Society **16**, 1980, 279–292.

[15] W.L. Burn. *The Age of Equipose, a Study of the Mid-Victorian Generation.* W.W. Norton, New York, 1965.

[16] R. Carnap. *The Logical Syntax of Language.* A. Smeaton (translator). Paterson, Littlefield, Adams and Company, 1959.

[17] A. Church. *Introduction to Mathematical Logic.* Princeton University Press, Princeton, 1964.

[18] A. Church. "Review of [25]." The Journal of Symbolic Logic **41**, 1976, 546–547.

[19] E.R. Curtius. *European Literature and the Latin Middle Ages.* W.R. Trask (translator). Pantheon Books, New York, 1953. (*Europäische Literatur und lateinisches Mittelalter.* A. Franke AG Verlag, Bern, 1948.)

[20] C.R. Darwin. *Autobiography, Notes, and Letters.* 1887. Sir F. Darwin (editor). Reprint, Colliers Books, 1961.

[21] H.W. Davenport. *Peirce's Evolutionism and his Logic: Two Connections.* In: [43], 307–312.

[22] G. De Beer. *Charles Darwin.* Thomas Nelson, Melbourne, 1968.

[23] H. De Long. *A Profile of Mathematical Logic.* Addison-Wesley, Reading, 1970.

[24] A. De Morgan. *Formal Logic.* 1847. A.E. Taylor (editor). Reprint, Open Court, London, 1926.

[25] A. De Morgan. *On the Syllogism and other Logical Writings.* P. Heath (editor). Yale University Press, New Haven, 1966.

[26] R.R. Dipert. *The Life and Work of Ernst Schröder.* Modern Logic **1**, 1990/1991, 119–139.

[27] J.T. Dunlap, B.S. Hawkins. *Cogito, Ergo Sum: Neither Inference Nor Performance.* The Personalist **19**, 1976, 386–390.

[28] G. Eliot. *Middlemarch.* 1874. W.J. Harvey (editor). Reprint, Penguin Books, Baltimore, 1973.

[29] G. Frege. *Grundgesetze der Arithmetik.* Vol. I. Pohle, Jena, 1893. Reprint (with Vol. II, 1903), Georg Olms Verlagsbuchhandlung, Hildesheim, 1962.

[30] W.B. Gallie. *Peirce and Pragmatism.* Dover, New York, 1966.

[31] P.T. Geach. *Reference and Generality.* Cornell University Press, Ithaca, 1962.

[32] B.S. Hawkins, Jr. *A Compendium of C.S. Peirce's 1866–1885 Work.* Notre Dame Journal of Formal Logic **16**, 1975, 109–115.

[33] B.S. Hawkins, Jr. "Review of [69]." Transactions of the Charles S. Peirce Society **11**, 1975, 128–139.

[34] B.S. Hawkins, Jr. *A Reassessment of Augustus De Morgan's Logic of Relations: a Documentary Reconstruction.* International Logic Review **10**, 1979, 32–61. (The author disclaims the many misprints, but not otherwise the paper.)

[35] B.S. Hawkins, Jr. *Peirce's and Frege's systems of notation.* In: [43], 381–389.

[36] B.S. Hawkins, Jr. "Review of [79]." Mathematical Reviews, 1983, 83d:01040.

[37] B.S. Hawkins, Jr. *J.W. Dauben on C.S. Peirce's Place in Mathematics: Some Reflections.* International Logic Review **17**, 1986, 62–69.

[38] B.S. Hawkins, Jr. *Peirce and Russell: The History of a Neglected Controversy.* In: [40].

[39] D.R. Hofstadter. *Gödel, Escher, Bach: An Eternal Golden Braid.* Penguin Books, New York, 1980.

[40] N. Houser, D.D. Roberts, J. Van Evra (editors). *Studies in the Logic of Charles S. Peirce. Proceedings of the C.S. Peirce Sesquicentennial Congress, 1989.* Indiana University Press, Bloomington, forthcoming, 1996.

[41] W.S. Jevons. *Boole.* Encyclopedia Britannica IV (9^{th} ed.), 1878, 42–43.

[42] W.S. Jevons. *De Morgan.* Encyclopedia Britannica VII (9^{th} ed.), 1878, 57–59.

[43] K.L. Ketner, J.M. Ransdell, C. Eisele, M.H. Fisch, C.S. Hardwick (editors). *Proceedings of the C.S. Peirce Bicentennial International Congress, 1976.* Texas Tech Press, Lubbock, 1981.

[44] S.C. Kleene. *Introduction to Metamathematics.* 1952. Reprint, Wolters-Noordhoff, Groningen, and North-Holland, Amsterdam, London, 1971.

[45] W. Kneale, M. Kneale. *The Development of Logic.* Clarendon, Oxford, 1962.

[46] A.L. Lavoisier. *Traité élémentaire de Chimie.* 1784. Reprint, Johnson Reprint Company, London, 1965. (*Elements of Chemistry.* R. Kerr (translator). Dover, New York, 1956.)

[47] G.W. Leibniz. *Logical Papers.* G.H.R. Parkinson (translator and editor). Clarendon, Oxford, 1966.

[48] C.I. Lewis. *A Survey of Symbolic Logic.* Dover, New York, 1960.

[49] J. Łukasiewicz. *Aristotle's Syllogistic.* Clarendon, Oxford, 1958.

[50] J. Łukasiewicz. *Selected Works.* L. Borkowski (editor). North-Holland, Amsterdam, 1970.

[51] W.H. Marshall (editor). *The Major Victorian Poets*. Washington Square Press, New York, 1967.

[52] R.M. Martin. *Some Comments on De Morgan, Peirce, and the Logic of Relations*. Transactions of the Charles S. Peirce Society **12**, 1976, 223–230.

[53] H. Melville. *Moby-Dick*. 1851. H. Beaver (editor). Reprint, Penguin Books, Baltimore, 1972.

[54] D.D. Merrill. *On De Morgan's Argument*. Notre Dame Journal of Formal Logic **17**, 1977, 133–139.

[55] D.D. Merrill. *De Morgan, Peirce and the Logic of Relations*. Transactions of the Charles S. Peirce Society **14**, 1978, 247–284.

[56] D.D. Merrill. *Augustus De Morgan and the Logic of Relations*. Kluwer Academic Publishers, Dordrecht, Boston, London, 1990.

[57] E. Michael. *Peirce's Early Study of the Logic of Relations, 1865-1867*. Transactions of the Charles S. Peirce Society **10**, 1974, 63–75.

[58] E. Michael. *Peirce's Earliest Contact with Scholastic Logic*. Transactions of the Charles S. Peirce Society **12**, 1976, 46–55.

[59] C. Morris. *Foundations of the Theory of Signs*. University of Chicago Press, Chicago, 1970.

[60] M.G. Murphey. *The Development of Peirce's Philosophy*. Harvard University Press, Cambridge, 1961.

[61] W. of Ockham. *Philosophical Writings*. P. Boehner (translator). Bobbs-Merrill, Indianapolis, 1964.

[62] C.S. Peirce. *Collected Papers of Charles Sanders Peirce*. Vols. I–VI. 1931–1935. C. Hartshorne, P. Weiss (editors). Reprint, Harvard University Press, Cambridge, 1960, 1965.

[63] C.S. Peirce. *Collected Papers of Charles Sanders Peirce*. Vols. VII–VIII. 1958. A. Burks (editor). Reprint, Harvard University Press, Cambridge, 1966.

[64] C.S. Peirce. *Selected Writings*. P.P. Wiener (editor). Dover, New York, 1966.

[65] C.S. Peirce. *Charles Sanders Peirce: Contributions to the Nation Part One: 1869–1893*. K.L. Ketner, J.E. Cook (compilers and annotators). Texas Tech Press, Lubbock, 1975.

[66] C.S. Peirce. *The New Elements of Mathematics*. Vols. I–IV. C. Eisele (editor). Mouton, The Hague, Paris, 1976.

[67] C.S. Peirce. *Historical Perspectives on Peirce's Logic of Science, A History of Science*. Parts. I–II. C. Eisele (editor). Mouton, Berlin, New York, Amsterdam, 1985.

[68] A.N. Prior. *Formal Logic*. Clarendon, Oxford, 1962.

[69] D.D. Roberts. *The Existential Graphs of Charles S. Peirce*. Mouton, The Hague, 1973.

[70] A. Seth. *Whately*. Encyclopedia Britannica XXIV (9^{th} ed.), 1878, 558–560.

[71] C.P.B. Shelley. *Hammer*. Encyclopedia Britannica XI (9^{th} ed.), 1878, 379–381.

[72] W. of Sherwood. *Introduction to Logic*. N. Kretzmann (translator). University of Minnesota Press, Minneapolis, 1966.

[73] H. Skolimowski. *Polish Analytical Philosophy*. Routledge & Kegan Paul, London, 1967.

[74] A. Tarski. *Logic, Semantics, Metamathematics*. J.H. Woodger (translator). Clarendon, Oxford, 1956.

[75] A. Tarski. *Introduction to Logic and the Methodology of Deductive Sciences*. O. Helmer (translator). Oxford University Press, New York, 1959.

[76] R. Whately. *Elements of Logic*. 1826. 5^{th} ed., revised, B. Fellowes, London, 1834.

[77] N. Wiener. *Cybernetics*. 1948. Reprint, M.I.T. Press, Cambridge, 1975.

[78] R.G. Wengert. *Schematizing De Morgan's Argument*. Notre Dame Journal of Formal Logic **15**, 1974, 165–166.

[79] M.V. Wilkes. *The Design of a Control Unit—Reflections on Reading Babbage's Notebook*. Annals of the History of Computing **3**, 1981, 116–120.

[80] L. Wittgenstein. *Logisch-philosophische Abhandlung*. With an English translation (*Tractatus Logico-Philosophicus*) by D.F. Pears and B.F. McGuinness. Routledge & Kegan Paul, London, 1961.