GRIGORI MINTS ON PROOF THEORY IN THE USSR, 1925 – 1969:

Grigori Mints, *Proof theory in the USSR 1925 – 1969,* Journal of Symbolic Logic **56** (1991), 385-424.

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Grigori Efroimovich Mints (sometimes spelled Minc) was one of the leading Soviet workers in proof theory of his generation. Formerly a member of the Leningrad Branch of the Steklov Institute of Mathematics of the Academy of Sciences of the USSR (LOMI) and friend and of its director, recursion theorist Yurii V. Matiyasevich, Mints was in the early 1980s a "refusenik" who left his post at LOMI so as not to compromise Matiyasevich and subsisted partly by private tutoring in computer science. In the mid-1980s, Mints moved from Leningrad to Tallinn and joined the Institute of Cybernetics of the Estonian Academy of Sciences as a Leading Research Associate. A member of the American Mathematical Society and the Association for Symbolic Logic, he has since June 1983 been a consulting editor for the *Journal of Symbolic Logic*, and also serves on the AMS/ASL Committee on Translations. In 1991 he obtained a permanent position in the philosophy department of Stanford University in California.

Mints's earliest works, principally on cut-elimination and normal-form theorems, were introduced to the wider (non-Russian-speaking) community of logicians in the early 1960s when they began appearing in English in such translation journals as *Soviet Mathematics Doklady.* In the late 1960s and early 1970s, he contributed to the problem of deducibility in Gentzen's classical and intuitionistic calculi **LK** and **LJ** and to work on Herbrand quantification. However, it was only with the publication in 1970 and 1971 by Jean van Heijenoort of reviews of some of his work on Gentzen and Herbrand that Mints began to attract serious international attention.

A number of van Heijenoort's reviews [*1970; 1970a; 1971*] centered on the work of Mints which examine the relations of Herbrand's work to that of Gentzen and give generalizations of Herbrand's Fundamental Theorem using Gentzen's *Hauptsatz.* Moreover, using the extension of this method from Gentzen's classical sequent calculus **LK** to

Gentzen's intuitionistic calculus **LJ**, Mints was able, as van Heijenoort [*1971*] showed in his review of several of Mints's papers, to obtain an analogue of Herbrand's Fundamental Theorem for intuitionistic predicate calculus. Van Heijenoort's *nachgelassene* notes "Herbrand – Non-classical" deal with the intuitionistic interpretation of Gentzen's sequence calculus through the apparatus provided by Herbrand, and in particular with Mints's extension to Gentzen's **LJ** of Herbrand's Fundamental Theorem. These notes include excerpts from Kreisel's [*1958*] paper "Elementary Completeness Properties of Intuitionistic Calculus" as well as the complete [*1970*] English version of Mints's paper "Disjunctive Interpretation of the **LJ** Calculus" reviewed by van Heijenoort [*1971*]. Mints's work relies upon Kreisel's [*1958*, 326-327; 328-329] proofs of the theorems that *the negation of a prenex formula is provable intuitionistically if and only if it is provable in the classical predicate calculus* and that *there is a Herbrand type theorem for negations of prenex formulae of the predicate calculus*. In fact, as van Heijenoort [*1957*, 351] pointed out in his review of Robert Feys's preface to Ladrière's French translation of Gentzen's "Untersuchungen über das logische Schließen" [Gentzen *1955*], Feys's "preface underlines the fact that Gentzen's methods lead 'naturally' to intuitionistic, and not to classical, logic." Mints's analogue, however, as van Heijenoort noted both in his [*1971*] review of Mints's papers and again in his unpublished paper [*1975*, 9] "Herbrand," is not without difficulty and does not apply to arbitrary formulæ of intuitionistic logic. Van Heijenoort reviews included works relating to the definition of logical connectives for propositional logic in terms of the traditional square of opposition, and, most especially, on the connections, explored by Mints, between Gentzen's **LK** system and Herbrand quantification [van Heijenoort *1970a; 1971*], A.V. Idel'son and Mints's anthology on *Mathematical Theory of Logical Deduction* (van Heijenoort *1970a*), and Mints's [*1966*] paper on "Herbrand's Theorem for the Predicate Calculus with Equality and Functional Symbols" — which was an early and briefer version of the generalization of Herbrand's theorem found in the appendix of the Idel'son and Mints's anthology

Mathematical Theory of Logical Deduction contains the Russian translations, mainly by the editors themselves, of classic papers by Gentzen, Beth, Stig Kanger, Kleene, Schütte, and Gödel, and includes an appendix by Mints which uses Gentzen's *Hauptsatz* for **LK** to prove a generalization of Herbrand's theorem for the classical predicate calculus with identity and function symbols.

Mints's survey paper "Proof theory in the USSR 1925 – 1969" in the *Journal of Symbolic Logic* (*1991*) is intended to be the first of a two-part survey of Soviet work on proof theory, the second part to cover the period 1970 – 1988. It is based upon a talk given at COLOG-88 and published in the *Proceedings* [*1989*]; a short abstract of which appeared in [*1990*]. I am informed by Mints, however, that the plans for completion of this second part are not, however, immediate.

The history which Mints treats in this paper begins with Kolmogorov's famous paper of [*1925*] "On the Principle of Excluded Middle" and ends in work from 1969, although in a few instances it includes work from the early 1970s in order to complete the survey of the contributions of A.A. Markov and N.A. Shanin to provide an effective interpretation of the formulæ of negative arithmetic based on Mints's [*1983*; *1983a*] supplements to the Russian translation of Barwise's *Handbook of mathematical logic* on Markov's ramified semantics and Shanin's majorant semantics.

This history is divided into three periods, and Mints concentrates his attention on details of the work that is less well-known to those outside the USSR. Special emphasis is given in particular to Novikov's work on the cut-elimination method for regular formulæ and to Maslov's inverse method for predicate logic.

The first period covers the years from 1925 to 1950 and discusses the "isolated work by the founders" of Soviet studies in proof theory "and their immediate successors." During this period, the pioneering work of Kolmogorov, and to a lesser extent that of V.I. Glivenko, on intuitionistic systems are very briefly examined (pp. 386–387). The remainder of the section on this period is devoted to a summary of P.S. Novikov's work on cut-elimination and its applications (pp. 387–390), in which Novikov's procedure is shown to be different from the "more Gentzen-like" methods developed by Schütte and his colleagues at the same time.

The second period considered by Mints covers the decade of the 1950s. The work carried out by Markov, Shanin, and their students, such as N.N. Vorob'ev and B.Yu. Pil'chak led in this period to the development of the algorithmic approach to constructive mathematics. This approach began with the desire to develop a method which did not rely upon traditional noneffective methods, and is roughly characterized by Mints (p. 390) as an extension of Heyting's formalization of intuitionistic first-order logic, that is, as "HA + ECT + MP, i.e. intuitionistic first-order arithmetic, extended Church's thesis (equivalent to Kleene's realizability), and Markov's principle" (MP), that is

$$\text{MP:}\ \forall x \in \mathbb{R}\ (\neg x > 0 \rightarrow x > 0)$$

which, as understood and used by Markov himself, is what has been called the *primitive recursive Markov Principle* (MP_{PR}), i.e.

$$\text{MP}_{\text{PR}}\text{:}\ \neg A \neg A(n,m) \rightarrow \exists n\, A(n,m)$$

where A is a property of the constructive objects in the language FIPC (formalized intuitionist predicate calculus).

It is shown that Kolmogorov and Novikov were sympathetic to the constructivist approach but would not accept its restrictions. They and their students, most notably V.A.

Uspenskij, A.V. Kuznetsov, and B.Ya. Falevich, worked on proving the completeness or incompleteness of various systems, including in particular those studied by the constructivists. The foremost of the works along these lines was Novikov's posthumously published lectures of 1953–54 on *Constructive mathematical logic from the classical viewpoint* [*1977*].

The remainder of Mints's survey deals with the work begun in the 1960s. The third section focuses on the theory of proof search in the 1960s and is divided into three main subsections, the first dealing with automatic construction of propositional natural deduction. The work begun by Shanin on Gentzen-style natural deduction with the aim of automating deduction in the early 1960s grew out of the simplification by Shanin and his colleagues in Leningrad [Davydov, et. al. *1965*] of the procedures developed by Gentzen (and based on a suggestion of Gentzen) by transforming Gentzen-type derivations in some suitable sequent calculus into derivations in natural deduction by inserting a series of natural deduction rules and thereby dealing with proof searches having at most one succedent. This was further simplified by Shanin, S.Yu. Maslov and A.O. Slisenko, and is similar to the method presented by Prawitz [*1965*]. Since invertibility is a crucial property of the rules of proof search procedures, Shanin and his group studied various invertible calculi. This led to the development of Maslov's inverse method, one of the main developments of the Leningrad school. In subsection 2 of section 3, Mints sketches the main features of this method; other sketches have appeared in [Anellis *1988*], [Lifschitz *1986*], and by Maslov himself, who compared it with John Alan Robinson's resolution method. Vladimir Lifschitz in particular [*1986*, 78–97] gave a detailed characterization in English of Maslov's method and its applications. Maslov's method became the tool for providing a unified treatment of decidable cases of the predicate calculus, and Mints next turns his attention to this work, after which he gives a comparison of Maslov's method and Robinson's resolution method and discusses computer programs for predicate logic based on Maslov's method.

The relationships between resolution and other automatic theorem-proving methods based upon Gentzen sequences, and in particular Maslov's inverse method, was pointed out by Kuehner [*1971*], who introduced the English version [Maslov *1971*] of Maslov's detailed [*1969*] comparison of resolution with his own inverse method and by Lifschitz [*1986*, 18-19]. It was shown by Maslov that there is a one-to-one correspondence between his inverse method and Robinson's resolution. Davydov [*1971*], in fact, was able to obtain a synthesis of the two methods. In the remainder of this section, Mints describes the work carried out by Mints and his colleagues along the lines of the work by Mints reviewed by van Heijenoort [*1970*; *1970a*; *1971*]. In the final subsection of section 3, Mints turns to work on analysis of derivations in nonclassical, and especially intuitionistic logic, especially the work in this field of the Leningrad school, where he Maslov and V.P. Orevkov were the principle contributors.

Next (in section 4) Mints deals with various other work in proof theory in the 1960s, including work on classical systems, especially first-order arithmetic. This work includes D.A. Bochvar's continued studies of the paradoxes generated by cases of unrestricted comprehension, begun in the 1940s, L.L. Tsinman's work on the paradoxes using Novikov's method of regular formulæ, and the work of A.G. Dragalin and N.V. Belyakin on the ω-rule and of Mints and V.P. Orevkov on properties of embedding operations. Attention is also devoted to some of the less well-known work on constructive systems, and in particular to studies attempting to show the completeness of intuitionistic propositional calculus (IPC) under the recursive realizability interpretation. When it was shown that IPC is not consistent under the recursive realizability interpretation, attention turned on the one hand to deciding which classes of formulæ are those for which IPC is complete and on the other hand at least to attempts to axiomatize the logic of realizability.

The survey ends with a lengthy and detailed discussion first of Markov's ramified semantics for the formulæ of first-order arithmetic (section 5) and then of Shanin's majorant semantics (section 6). The former can be summarized as a "combination of a version of recursive realizability (or rather Shanin's deciphering algorithm...) and a special semantics for the results of this deciphering" (p. 405). The semantics requires a hierarchy of languages in which each lower-level language L_n is embedded into the next level language L_{n+1} ($n > 2$), so that L_{n+1} is L_n with some specified additions. Shanin's majorant semantics is based on the assignment to each arithmetic formula F of a transfinite sequence $\{F^\alpha\}$ consisting of simpler formulæ in a way that allows the use of traditional methods of constructive mathematics and induction up to α to imply $F^\alpha \rightarrow F$ for a majorant F^α of rank α.

Mints's paper belongs to a tradition going back at least to M.M. Troitskij's, *Handbook of logic, with detailed notes on the history and state of this science in Russia and abroad* [*1886*], which surveys both the state of research in logic in the mid-1880s and the history of logic in Europe up to that time. This book serves as a summary of the state of both current knowledge of logic and history of mathematical logic, and thus as a complete general description of logic in the 1880s. The more immediate and direct ancestors of Mints's survey are the famous surveys carried out by S.A. Yanovskaya on the history of work in foundations of mathematics and mathematical logic in the USSR from 1917 to 1947 [*1948*] and on the history of work in mathematical logic and foundations of mathematics in the USSR during the following decade, from 1947 to 1957 [*1959*], Ershov's extremely brief follow-up of Yanovskaya's surveys. [*1983*], and his own expository survey of proof theory (arithmetic and analysis) [*1975*] which explored the main lines of international work in proof theory centered around proving the consistency of classical analysis and the normalization theorem for higher-order logic.

In his introduction, Mints admits (p. 385) that his choice of topics, especially concerning the subject of properties of the set of theorems usually obtained by modeltheoretic methods, is somewhat subjective. In fact, his survey is a very broad survey of only some of the main lines of development, especially of the lesser-known work, by Soviet proof-theorists. The last comprehensive survey of Soviet research in all branches of logic was undertaken by Sof'ya Yanovskaya in the 1950s with the assistance of some of her students. It is easy to understand why Mints chose this course when it is remembered that Yanovskaya's [*1959*] survey of Soviet work in logic for the decade 1947-1957 required more than one hundred pages and in view of the exponential growth of the field since that time.

The following typographical infelicities, all minor, may be noted: on p. 385, line 8, 'into' should be 'to'; on p. 404, we find both 'Tsinman' and 'Ciniman' for 'Циниман'.

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The logic textbooks that were commonly used in the former Soviet Union differed in several respects from those used in the West, for example, in the United States. Some of this disparity is no doubt resultant from the different educational levels of the two societies, but much of the difference results from diverging pedagogical goals and philosophies.