CRITICAL REMARKS ON
MICHAEL DUMMETT'S *FREGE AND OTHER PHILOSOPHERS*

(Review of Michael Dummett, *Frege and other philosophers*, Oxford, Clarendon Press, 1991. xiv + 330 pp.)

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After describing the general structure of Dummett's volume (1), I comment on some aspects of Dummett's discussions of, and with, "other philosophers" (2). Section (3) is devoted to Dummett's study of Frege's analysis of the concept of number, and section (4) to Dummett's views on Frege's context principle.

Dummett claims that Frege was better equipped than Husserl in the search for "objective correlates" of expressions other than singular terms, and that Husserl was "indifferent" to the nature of those correlates: I disagree (2.1). Dummett's (and Sluga's) view of Frege's historical connections is not broad enough (2.2). Dummett points to a text that, he claims, shows Frege "running headlong" into the "paradox of analysis". I think that Frege's general philosophy rules out the alleged paradox, and that the text in question is not so supportive of such a reading (3.1).

My principal disagreement concerns the interpretation of Frege's analysis of the concept of number. Dummett does not appear to recognize clearly enough the peculiar two-stage structure of Frege's method, and when he does he misrepresents it by referring to it as "definition by logical abstraction" (3.2). Finally, Dummett sees the "context principle" in the post-*Grundlagen* Frege whereas I do not (4).

A unifying theme through most of this essay has to do with the contrast between two methods not distinguished by Dummett: one pioneered by Frege in his analysis of number which I call, because of Carnap's presentation of it, the "looking-around method", and another preluded by Peano's *definizioni per astrazione* (to which I refer as the "abstraction method"). The full disentanglement and clarification of the curious, interrelated sequence of events originating in Peano's and Frege's ideas — a tale not of two cities but of two methods — still form a desideratum in the historiography of modern logic.

1. INTRODUCTION. Dummett's volume includes the following fifteen essays: 1. "Frege on the consistency of mathematical theories," 2. "Frege and the paradox of analysis," 3.

"On a question of Frege's about right-ordered groups," 4. "Frege's 'Kernsätze zur Logik'," 5. "Frege as a realist," 6. "Objectivity and reality in Lotze and Frege," 7. "Frege and Kant on Geometry," 8. "An unsuccessful dig," 9. "Second thoughts," 10. "Which end of the telescope?", 11. "Frege and Wittgenstein," 12. "Frege's myth of the third realm," 13. "Thought and perception: the views of two philosophical innovators," 14. "More about thoughts," 15. "The relative priority of thought and language." Essays 8, 9 10, 13, 14, and 15 have to do with Frege as much as any of the others.

The fifteen essays have been put together in one volume for the following reason:

> This volume contains all the essays I have written that were too late for inclusion in *Truth and Other Enigmas* and are wholly or largely about Frege. All of them concern other philosophers as well... (Preface).

The reader should be advised that the essays collected in this volume are only a fragment of Dummett's important contribution to the Frege literature. In this connection it is fortunate that an "Appendix: Writings on Frege by Michael Dummett," has been inserted towards the end of the volume (pp. 325–326). The appendix, alas, overlooks *Frege: The Philosophy of Language* [*1973*]; also, it is a bit confusing that the papers published in *Frege and Other Philosophers* are omitted from the list in the appendix.

2. COMMENTS ON DUMMETT AND SOME OF THE "OTHER PHILOSOPHERS"

2.1. HUSSERL AND FREGE. The non-contemporary philosophers most frequently mentioned by Dummett in connection with Frege are Kant, Brentano, Husserl, Wittgenstein, and Russell. I restrict myself to comments on one point concerning Husserl.

Husserl "was at one with Frege in regarding all meaningful expressions as having, or purporting to have, objective correlates" (p. 269). It appears that in this context (pp. 269–270) Dummett regards as equivalent the expressions "objective correlate", "objective reference", "reference", "objective reference". The problem was how to determine these correlates for expressions other than the paradigmatic singular terms (sentences, predicates...).

According to Dummett, first, Husserl was "indifferent" to the nature of the objective correlates, whereas Frege was not indifferent; secondly, even if Husserl had not been indifferent, he would still have lacked any method of determining what the objective correlates were, contrary to Frege who had, in this respect, a "systematic theory" or "general principle" (pp. 269–270).

My examination of the second of these two comparative judgments will become a rather complex business; the first one will require only a short paragraph at the end of this section.

The principle that Frege had and Husserl lacked, according to Dummett, was the following: "the reference of an expression is what is common to every expression which could be substituted for it in any sentence without affecting the truth-value of that sentence" (p. 269).

I wonder, *where* in Frege is this principle to be found? Rather than re-scanning Frege's corpus in order to establish that the principle does not occur in Frege, I will evaluate the help that such a principle may offer in the search for objective correlates or references. Besides, Dummett's principle can be modified into a formulation that, if not in Frege, is very Fregean, as we shall see.

Furthermore, what does Dummett mean by "what is common"? He does not say. One reasonable interpretation of that phrase is in terms of abstraction, but here we are even worse off: not only has Frege nothing to say about abstraction (how to do it, etc.) but he banished abstraction from logic (*cf.* my "Frege and Abstraction" [*1984*]). Thus, an abstraction reading of the principle that Dummett ascribes to Frege would be totally unhistorical. In particular, it would be inappropriate to say that Frege was better equipped than Husserl because of a principle based on a notion rejected by Frege.

Even though an abstraction reading of Dummett's principle is ahistorical, it is nevertheless theoretically interesting to find out how much help the principle would then offer to anyone searching for objective correlates of (say) sentences and predicates.

Dummett does not explain what abstraction is, or how to do it. But this is an almost universal defect among philosophers. This is particularly true in the analytic tradition, as a consequence of Frege's and Russell's distaste for abstraction, but not even in the refined scholastic or phenomenological analyses do we find exact instructions for performing abstraction, except for "closing one's eyes" to certain aspects, while "retaining" others.

Only one precise theory of abstraction seems to be available in the philosophy market today, namely, that method, first hinted at in Peano's "*definizioni per astrazione*", then recognized briefly by H. Weyl, and finally fully introduced as an essential part of the logical propaedeutic by P. Lorenzen.

According to this abstraction method (or "modern abstraction" as I have called it in contrast with traditional abstraction theories, *cf.* my *Abstracción moderna y tradicional* [*1981*]) one starts with a domain of objects a, b, ... and an equivalence relation $x \sim y$. Then one may decide to do abstraction, not by the ill-defined, traditional ceremony of "closing one's eyes" to certain aspects while "retaining" others, but by the logico-linguistic exact procedure of disregarding any predicate that is not invariant with respect to the chosen equivalence relation.

Under this abstraction, Dummett's principle tells the philosopher who is searching for the reference of sentences and predicates that that reference is an *abstractum*, which (in view of the nature of the equivalence relation relative to which the abstraction is performed) may be described as "what is important for truth" in a given sentence or predicate. Interesting as this may be, it immediately raises one anomaly, namely that the notion of reference for sentences and predicates turns out to be quite different from that for singular terms (is the reference of "Paris" the importance for truth of the singular term "Paris"?).

There is one way (only one, as far as I know) to preserve the content, if not the appearance, of Dummett's principle as something ascribable to Frege. This requires a double surgery: to remove the non-Fregean (to the extent that it requires abstraction) phrase "what is common", and to reshape the principle into a statement that, while not occurring in Frege's corpus, is structurally very Fregean. The result is: "the reference of an expression E = the reference of an expression E^* iff the expressions E and E^* are

interchangeable *salva veritate*". (I assume Dummett means a biconditional, "iff", rather than a conditional from right to left, "if".)

The modified principle is structurally identical to two crucial, fundamental Fregean biconditionals: 1) the number of a concept F = the number of a concept G iff F and G stand in the relation of there being a one-one correspondence between their individuals (*Grundlagen* [*1884*, §§64–68]); 2) the course of values of a function F = the course of values of a function G iff F and G stand in the relation of yielding the same value for each argument (*Grundgesetze* I, especially §10).

Now, these biconditionals constitute, in Frege, only the first stage of a two-stage procedure which I have called "Frege's method" or, with pejorative connotations more applicable to its use by later authors such as Carnap, Russell, and Quine, the "looking around method" (*cf.* my "Abstraction, Looking-Around and Semantics" [*1979*]).

The purpose of the method is to produce an analysis of expressions already in use, such as (in Frege's case) "the number of the concept F", "the course of values of the function F...". An equivalence relation $x \sim y$ must be previously defined over a domain a, b, ... (concepts, functions....). In the *first* stage one stipulates that $F(a) = F(b)$ iff $a \sim b$, with "$F(a)$" representing the singular terms of the form "the number of the concept a", "the course of values of the concept a...". These singular terms are, at first, deliberately thought of as denotationless. It is the goal of the *second* stage to assign a denotation to them. To this end, as Carnap explains, one "looks around" for entities that are "suitable" for playing the role of the desired denotations; "suitable" just means the same as "compatible with the biconditional stipulated in the first stage." In other words, *anything* will do, provided it does not conflict with the biconditional. The equivalence classes are the most popular choice.

Obviously, the second stage of the method suffers essentially from conventionalism, lends itself easily to playfulness, and cannot be admitted as a philosophically satisfactory tool of conceptual analysis.

Unfortunately, this is in fact how Frege proceeded in connection with number and course of values, thereby pioneering such an unsatisfactory method in philosophy. As a denotation for singular terms of the form "the number of the concept F" Frege chose the equivalence class (*Grundlagen* §68), and for some of the singular terms of the form "the course of values of the function F" he chose the truth-values (*Grundgesetze* I, §10).

Dummett's principle, in my proposed modified version, is the type of biconditional required for the first stage of Frege's method. The singular terms of the form "the reference of the expression E" that occur in the biconditional — temporarily without a denotation — should receive a denotation in the second stage of the method.

Now, the help provided by the principle, understood this way, in the search for objective correlates is merely negative: the principle will never help to determine *what* the objective correlates are. Besides, what comes from the second stage is not acceptable help at all since the assignments made therein are unjustifiably arbitrary.

We may try a variant of Dummett's modified principle, which is to replace the word "reference" by the word "importance". The result is: "the importance of an expression E = the importance of an expression E^* iff the expressions E and E^* are interchangeable *salva veritate*". Obviously the importance is *for truth*.

The switch from "reference" to "importance" is justified because the German word "*Bedeutung*" means not only semantic *Bedeutung*: "Julius Caesar" *bedeutet* (denotes, designates...) the Roman leader; but also *Bedeutung*-importance: the *Bedeutung* of Julius Caesar in the history of Rome is enormous. (The English word "reference", and all the other alternative translations found in the literature, are defective in that they do not convey the double meaning of the German term; they only translate the semantic *Bedeutung*.[1])

Replacing "reference" with "importance" does not seem to deviate from Dummett, since he also thinks of *Bedeutung* as having two "ingredients": the "semantic role" or "value", and the name-bearer relation (p. 84), which seem to coincide with my distinction of *Bedeutung*-importance and semantic *Bedeutung* respectively. Dummett says that the "the semantic value of an expression is its contribution to determining any sentence in which it occurs as true or false" (p. 84), which obviously focuses more on importance (for truth) than on denotation. (Quite rightly Dummett blames Tugendhat for omitting "the appeal to the name-bearer relation", p. 85).

This new variant of Dummett's principle is identical to the "principle of *Bedeutung*" I proposed in "Frege's notion of *Bedeutung*" [*1979a*] as the starting point for the study of Frege's semantical theory. The principle of *Bedeutung* does not occur in Frege, but is structurally very Fregean for the same reason as given in my first modification of Dummett's principle.

However, while the principle of *Bedeutung*, taken as the biconditional of the first stage of Frege's method, leads to a clarification of some oddities in Frege's semantics (as shown in "Frege's Notion of *Bedeutung*"), it does not help a philosopher interested in determining the nature of the objective correlates. As before, the help provided by the biconditional is merely negative, and the arbitrariness of the choices made in the second stage of Frege's method cannot be accepted as an aid in that determination.

Thus, I fail to see the point of Dummett's second Husserl-Frege comparison in any of the ways in which I can understand the principle that Dummett attributes to Frege.

Finally, Dummett's first comparative judgement appears as false, and quite inappropriate, as soon as one observes the supreme "indifference" built into the "looking around method" pioneered by Frege.

2.2. DEBATES WITH CONTEMPORARIES. The volume is largely written in a dialectical style: Dummett meticulously reports the sequences of criticisms, responses to criticisms, and new criticisms between himself and other recent writers on Frege, such as Sluga (especially essays 5, 6, 7), Hacker and Baker (especially essays 8, 9), and Shanker (essay 10).

With regard to the debates with Hacker and Baker, and with Shanker, one sometimes wonders why Dummett has spent so much time on them. For instance, it does not seem to be worthwhile to discuss arguments such as the following:

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[1]I defended the usefulness of considering the ambiguity of the German word for a better understanding of Frege in my [*1967*] *Studies* (2.26, 2.62) and earlier in my 1965 Fribourg dissertation. For the sake of chronology, I would like to point out that Tugendhat's remarks on *Bedeutung* as importance, which Dummett has discussed at length, date from 1970.

> Shanker reasons that, since assertion is for Frege the outer expression of the inner mental act of judgement, and since inference essentially involves judgement, his account of logical validity must itself be psychologistic. (p. 223)

Sluga's criticism of Dummett's work on Frege is essentially that Dummett's work is unhistorical. To this Dummett replies that he "suspects" that being historical, as Sluga demands, is "largely sterile" (p. viii). Ironically, it is Dummett himself who has offered a good proof of its non-total sterility by discovering (essay 4 of this volume) that one of Frege's posthumous pieces is a series of remarks "prompted by reflection" (p. 66) on Lotze's introduction to his *Logik*. But all in all Dummett is right to be suspicious.

Perhaps Dummett has tended to take history only in the sense of concrete influences of one author on another, Frege in this case, and indeed, even more particularly, influences of an author's specific idea on another's idea. A much discussed example in this volume is the question of whether Frege's use of "objective" was inspired by Lotze's notion of *Geltung* (validity, value).

In my view, a broader sense of history would help Dummett to be more optimistic about it. Even in the case of Kant, the interesting connection with Frege lies far beyond the point-to-point concrete influence: Frege's philosophical project as a whole is a spectacular challenge to Kant's philosophy of arithmetic (stated in the 1879 *Begriffsschrift*) — with a Kantian finale (avowed in the very last notes left by Frege, posthumously published).

A connection of a still broader nature exists between Frege and the Aristotelian-scholastic tradition, quite overlooked by both Dummett and Sluga, who limit their histories to the nineteenth-century German context. Frege had not read the Aristotelian-scholastic sources, and I cannot detect any direct influence from such authors on Frege. Nevertheless, it is relative to the Aristotelian-scholastic tradition that some of Frege's finest conceptual distinctions appear as amazing rediscoveries (second intentions, objective *vs.* subjective concepts, etc), and that Frege emerges as a most revolutionary philosopher. This revolutionary aspect of Frege's thought is to be seen first of all in his predication theory (consider the opening lines of *Grundlagen*, §53: marks are not predicated of the concept of which they are marks, cf. my *Studies* [*1967*], and "Freges Ort" [*1975*]). Frege's sharp criticism of the bimillenary "Euclidean" definition of number as set of units — a definition with a signifi-cant role in the history of Aristotelian-scholastic metaphysics rather than of mathematics — is another instance of Frege's profound historical roots.

3. THE ANALYSIS OF THE CONCEPT OF NUMBER. In Dummett's second essay, "Frege and the Paradox of Analysis", the following four theses may be pinpointed: (1) Frege "runs headlong" into the so-called paradox of analysis in the 1914 "Logik in der Mathematik" (p. 18); (2) this causes Frege to "put a very low value" on the analysis of concepts (p. 18); (3) none of "Frege's writings of any period" offer "an adequate account of the status of the definitions [here: analyses of concepts] that he used in his logical construction of number theory" (p.30); (4) neither do they offer a "resolution" of the paradox of analysis (*ibid*).

I will comment on points (1), (2) and (4), in section 3.1, and on (3) in section 3.2.

3.1. THE PARADOX OF ANALYSIS: A PARADOX? IN FREGE?

The so-called paradox of analysis, in Dummett's exposition (p. 17), is as follows:

(1) There is an expression A, already meaningful: *the analyzed expression*; there is another expression A*: *the analyzing expression*, or in short, the *analysis*;

(2) The purpose of constructing the new expression A* is to capture the meaning of "A";

(3) But then a paradox begins to emerge: if the analysis is successful it cannot yield any new meaning or information; the statement that the expressions A and A* coincide in their meaning will be an "analytic truth *par excellence*", and every analysis, if correct, should be immediately recognizable as an obvious triviality.

My comment is that this is a paradox only if one presupposes that analysis is the merely passive discovery of, so to speak, already carved precious stones. There is no paradox for those who think that the analysis may involve, as it usually does, a patient, normative clarification and reconstruction of traditional notions that are unfinished, unclear and entangled with misconceptions of all sorts. For such people the principal goal is *not* necessarily to produce an analysis which fully coincides in meaning with those received notions. Perhaps the philosopher in some rare occasion ends up just "repeating" what was "always there"; then the analysis turns out to be superfluous, although even in such cases there may be the advantage of a greater precision.

Especially in the case of Frege, for whom the task of philosophy includes liberation from the dominion of ordinary language, the paradox of analysis as described by Dummett appears as quite foreign.

The best example of analysis in Frege is precisely one not mentioned by Dummett: the ancestral. Here the analyzing expression A* ("for every property F, if all children of Julius Caesar had F, then Frege has F too") is quite different from the traditional, initial expression A ("Julius Caesar is an ancestor of Frege"). A* expresses what, according to Frege, earlier human beings *should* have meant by "ancestor". Frege, in proposing A*, not only intends to make things more precise but also to free the old notion A from unnecessary associations with order in space or time.

Although there seems to be no room for any paradox of analysis in Frege's general philosophy, there is a justification for Dummett's writing on "Frege and the Paradox of Analysis". In fact, it is Dummett's merit to have found a text ("Logik in der Mathematik," *Nachgelassene Schriften*, pp. 224–229) which contains all the ingredients required to create the alleged paradox.

In that text Frege says or implies that a correct analysis (*Zerlegung*) must express the same sense (*Sinn*, or the same thought, *Gedanke*, when the expression is a sentence), as the traditional, given analyzed expression. Moreover, Frege says that the recognition of the correctness of the analysis must be by "immediate intuition", not by a proof.

This immediately strikes the reader as doubly inconsistent with other aspects of Frege's thought. (1) There is an inconsistency with the above described general feature of Frege's philosophy *vis à vis* the analysis of ordinary, traditional notions. (2) The view presented by Frege in the text in question is inconsistent with other general features of his notion of *Sinn*. Wherever there is some amount of work to be done in order to discover the identity that

underlies an apparent diversity (with the single exception of *definiens – definiendum*), the *Sinn* of both sides is likely to be regarded by Frege as different. For example, *within* the same essay Frege says that the sense of "17 – 3" is not the same as the sense of "16 – 2" (p. 251). It is hard to believe that if the expressions "17 – 3" and "16 – 2" have for Frege a different sense, any analyzed expression *A* will have for him the same sense as its analysis *A**. Just consider the case of the ancestral!

However, both inconsistencies remain purely potential because Frege declares that the identity of the senses of the analyzed and analyzing expressions is unattainable in practice. Although in one place (p. 227, line 6) he uses a conditional: "If it is uncertain..." ("*wenn es fraglich ist...*"), which suggests that one cannot rule out the possibility of establishing that identity, in another place he writes in a way that seems to deny that possibility *simpliciter*: "But that also the thought is the same, is uncertain" ("*Aber dass auch der Gedanke derselbe bleibt, ist fraglich,*" p. 226).

Thus, with regard to Dummett's first point ("Frege runs headlong into the paradox of analysis") it must be observed that, at any rate, Frege does not get hurt, insofar as he says that the identity needed to generate the paradox is out of reach.

Frege however insists on saying that the sense of the analyzed expression and the sense of the analyzing expression are identical, and naturally he has to wonder how it is that in practice we fail to recognize that identity. He explains (p. 228) that the source of the uncertainty lies in the fact that, whereas the sense of the analyzing expression is clear, the sense of the analyzed expression *is not clearly grasped*: "its outlines are confused as if we saw it through a mist." Finally, Frege says that "the result of the logical analysis" will be "to have articulated the sense clearly ("*der Sinn deutlich herausgearbeitet worden ist,*" p. 228).

A curious novelty in Frege's theory of *Sinn* arises from these remarks. Just as we are used to regarding the senses as different "ways of being given" of the same object, now we have to consider the possibility that the same sense be given in different ways: one clear, the other misty; one articulated, the other not.

This novelty in the theory of *Sinn*, dreadful as it may be, provides a quick rebuttal of Dummett's *fourth* point. Frege did anticipate the correct solution to the alleged paradox of analysis. (At any rate, for those who take such a paradox seriously, among whom we may perhaps count Frege qua author of the text studied by Dummett.) Even if an identity of sense between the expressions *A* and *A** is established, that same, unique sense is "given" by the singular terms "*A*" and "*A**" in different ways — as different as the ways in which Venus is given by "the Morning Star" and by "the Evening Star".

Let us turn to the *second point* of Dummett's paper, which might be called "Frege's low self-esteem in matters of conceptual analysis". Dummett devotes the first six and a half pages of his essay to his astonishment (pp. 19, 23) at Frege's remarks on "analytic" definitions in the 1914 essay "Logik in der Mathematik". Such remarks are, according to Dummett, negative. He writes, for example, that Frege "put a very low value on such [analytic] definitions" (p. 18), and that they contradict the fact that the analytic definitions "had lain at the very heart of the project to which Frege had devoted his life" (p. 19).

I am myself astonished at Dummett's astonishment. As I read Frege's text (*Nachgelassene Schriften* [*1969*, 224–229) I see nothing more than a mere terminological issue: Frege wants to restrict the use of the word "definition" to stipulations whereby a thus-far

meaningless symbol is said to mean the same as an already meaningful expression; he does not want to call "definition" an analysis of an already meaningful expression. This, of course, does not imply at all any negative view of the analysis but only a stubborn, but licit attitude *vis à vis* the use of the term "definition", with which of course it is not obligatory to agree.

Perhaps Dummett has been misled by the fact that, as a remedy for the above-noted uncertainty in the identity of the meanings of the analysis and of the analyzed expression, recommends *forgetting* about the traditional expression (p. 228). But this does not entail that Frege "attached little importance to conceptual analysis," as Dummett claims (p. 17). On the contrary, it is perfectly compatible with holding that analysis is most important, except that it is useless to continue to discuss for ever the adequacy of the analysis versus the traditional notion, given that the latter is "misty" and that Frege's plan is, after all, to reconstruct traditional concepts, not to describe them (*cf.* again the ancestral).

3.2 THE TWO METHODS AGAIN: LOOKING-AROUND AND ABSTRACTION.

Let us turn now to Dummett's *point* 3 in his essay 5: Frege gives no explicit account of how he goes about "definitions" not in the strict Fregean sense of stipulations but in the sense of analysis of concepts. In view of this, all we can do is "scrutinize [Frege's] practice to elicit from it an attitude" to conceptual analysis (p. 30). The results of this scrutiny occupy most of Dummett's essay 5 (p. 30 to the end of p. 52).

Dummett restricts his consideration to the conceptual analysis of number (p. 30), and regrettably fails to indicate the great difference that exists between, for example, the case of number and Frege's analysis of ancestor. The latter is plain analysis, where no systematic procedure can be discerned; in the case of number we observe a *structured, two-step procedure*: Frege's looking around method (§2.1 above).

Dummett refers to Frege's method in terms of a "device introduced [by Frege] which has subsequently become standard," involving "the step from the equivalence relation to the new objects" and "the definition of the latter as equivalence classes" (p. 43).

In spite of this recognition, it is not always clear whether Dummett perceives *Grundlagen* [*1884*, §§62-69] as exhibiting *one* procedure in *two coordinated stages*. For example, Dummett says that in *Grundlagen* [*1884*, §68] Frege "substitutes" the previous definition "the number of *F*s = the number of *G*s iff there are just as many *F*s as *G*s" with the new definition "the number of *F*s = the class of concepts *G* such that there are just as many *F*s as *G*" (p. 32). Contrary to Dummett, it must be emphasized that Frege is *not* substituting in that text one definition for another definition: the second definition *builds on* the first one.

Dummett notes the "conventionalist" (p. 34) or "arbitrary" (p. 37) side of Frege's procedure, but these appear as merely occasional remarks in the many pages devoted to Frege's analysis of number. One misses a frontal evaluation. In my view (§2.1) Frege's method (let alone the definitely frivolous, later, Carnapian "looking-around" presentation of it) is unacceptable as a tool of conceptual analysis because of the arbitrariness of the choice of entities in the second stage.

It is also my view that, ironically, once the negative evaluation has been stated, it is in the very arbitrariness of the choice of entities of the second stage that one may discover the good, albeit obscure, motivation of the method, as well as the key to rescue it for philosophy.

The chosen entities do not matter *per se*: all that matters is their compatibility with the biconditional set forth in the first stage. Thus, it is easy to realize that Frege's method *really* aims at abstraction, in terms of which it should be overhauled and reconstructed. Here the crucial question arises: how exactly to do that? Only one answer seems to be available: to reshape Frege's method into modern abstraction (described above, §2.1).

Now, although it is correct to interpret the looking-around method as obscurely aiming at abstraction (and to rescue it for philosophy accordingly), it is false to say that that method is abstraction: none of its official moves can be described as abstractive. Amazingly, however, this error has persistently recurred since the late nineteenth century. The peak of the misnaming (and concomitant conceptual confusion) was reached by Russell when he applied the label of "principle of abstraction" to what is in fact a pure looking-around procedure (for instance, in *Principia Mathematica*, *72). Later on ("Logical Atomism"), Russell's philosophical instinct led him to remove the misnomer and rather speak of "the principle that dispenses with abstraction" (the truth being, of course, that Russell neither does abstraction nor dispenses with abstraction), but the error persisted and now we see it repeated by Dummett at a time approximately coinciding with the centenary of Peano's *definizioni per astrazione*.

In fact, Dummett is so convinced that Frege's looking-around method is genuine, good abstraction that he even proposes to call it not merely "definition by abstraction", but more precisely, "definition by logical abstraction" (p. 43) in order to avoid the wrong associations with "psychological abstraction" (which is "a myth" or "internalized magic", p. 50).

A particular, subtle aspect of the error consists in proclaiming the equivalence classes as the modern, scientific explication of the old notion of abstraction. Thus, we read in Reichenbach: "the notion of class finds an important application in the interpretation of a logical operation which traditionally has been called *abstraction*" (*Elements* [*1947*, §37]). In suggesting (p. 43) that the equivalence classes may be thought of as the *abstracta* resulting from a "logical abstraction", Dummett appears to continue this specific misconception as well.

The first who dared to turn things upside down (i.e. to put them back in the right position) was Lorenzen in his *Einführung* [*1955*, 101]. The following passage is the key text for anyone interested in fully disentangling the confused vicissitudes of Peano's "definitions by abstraction" in the history of modern logic:

> Dadurch [that is according to what is customary since Frege and Russell] soll die Abstraktion auf die Einführung von "Klassen" zurückgeführt werden. Wir werden jedoch weiter unten sehen, dass die Klassen nichts anderes als ein spezieller Fall von abstrakten Objekten sind.
>
> (Thereby [that is according to what is customary since Frege and Russell] abstraction is to be reduced to the introduction of "classes". However, we will see below that classes are nothing else but a special kind of abstract objects).

In *Grundgesetze der Arithmetik, II* [*1903*, §146], Frege describes his courses of values, classes, as what is common to functions yielding the same value for each object. If the "common" is read as the result of some abstraction, we have in the cited text an anticipation of Lorenzen's quoted view. But Frege's remark, if it was a hint at abstraction, was only a belated gesture, not further developed. The looking-around misconception prevailed, and pervaded most philosophical-foundational work in the twentieth century.

4. THE CONTEXT PRINCIPLE. In *Grundlagen der Arithmetik* [*1884*] Frege states several times the so-called "context principle", namely, "never to ask for the meaning [*Bedeutung*] of a word in isolation, but only in the context of a sentence". The context principle is not stated again after *Grundlagen* in Frege's writings (*Grundgesetze, II*, §97, contains only a pseudo-occurrence).

Essay 10 includes a section titled "The Context Principle Again" (pp. 229–233), which provides, according to the author, "a complete answer" to the question of "how Frege viewed the context principle in the mature period" (p. 233). Dummett proposes two distinctions. The first distinction is between a weak reading of the context principle, according to which "the meaning of an expression consists in its contribution to the content of any sentence containing it", and a strong reading, where "an expression has been given a meaning once the truth-conditions of all sentences containing it are determined" (p. 230). The second distinction is based on whether the old term "*Bedeutung*" that occurs in the formulation of the principle in *Grundlagen* should be updated, in view of Frege's post-*Grundlagen Sinn-Bedeutung* theory, to coincide with the new *Sinn* or with the new *Bedeutung* (or both).

Dummett claims (pp. 229–233) that in the post-*Grundlagen* Frege we find: (1) the weak-*Sinn* interpretation of the context principle in *Grundgesetze, I* [*1893*, §32], and (2) the strong-*Bedeutung* interpretation of the context principle in *Grundgesetze* I, §10.

First, an inconsistency must be noted between (2) and another part of the volume, where Dummett writes: "It still appears to me [...] that Frege abandoned the context principle, construed as a thesis about reference" (p. 82, n.)

But my central objection to Dummett's discussion is that he assumes that Frege *had* to continue to hold the principle after *Grundlagen*. In essay 5 Dummett says that "the context principle was explicitly stated only in *Grundlagen*, before Frege had arrived at the distinction between sense and reference, hence [...] we have to decide whether to take the principle as one about reference or only about sense" (p. 81, emphasis mine). But it was not obligatory for Frege to keep the principle after *Grundlagen*, let alone for us to decide whether it was on sense or on reference.

What in fact happened was that Frege rejected the principle:

Es ist für die Strenge der mathematischen Untersuchungen durchaus wesentlich, dass der Unterschied zwischen Definitionen und allen anderen Sätzen in aller Schärfe durchgeführt werde. Die andern Sätze (Axiome, Grundgesetze, Lehrsätze) dürfen kein Wort enthalten und kein Zeichen, *dessen Sinn und Bedeutung oder dessen Beitrag zum Gedankenausdruck nicht bereits völlig feststände*, sodass über den Sinn des Satzes, den darin ausgedrückten Gedanken kein Zweifel ist.

("It is absolutely essential for the rigor of mathematical investigations that the distinction between definitions and all other statements be respected most thoroughly. The other statements (axioms, basic laws, theorems) cannot contain any word and any symbol *whose sense and reference or whose contribution to the expression of the thought did not fully exist beforehand,* so that there is no doubt with regard to the sense of the sentence, the thought expressed by the sentence.) (Letter to Hilbert, dated December 1899, Frege, *Wissenschaftlicher Briefwechsel,* [*1975,* 62]; emphasis mine.)

The text used by Dummett to support his thesis that in the post-*Grundlagen* Frege we find a weak-*Sinn* version of the context principle is not incompatible with this unequivocal rejection of the principle. *Grundgesetze* §32 (quoted by Dummett on p. 230) simply says that the sense of an expression is fulfilled in a context (as the voice of a singer is fulfilled in a choir), not that it does not exist outside of a context.

Dummett's reading of *Grundgesetze* §10 as stating the context principle in its strong-*Bedeutung* interpretation takes us again to the issue of "Frege's method." In the cited text Frege proceeds according to the same pattern first instantiated in *Grundlagen* §§64–68 for the definition of number. Now we have functions, and the purpose is to define the notion of course of values of a function. Frege begins with his typical biconditional (now his axiom V) and then moves to choose suitable denotations for the singular terms of the form "the course of values of the function *F*". Through complex considerations he claims the truth-values are suitable denotations for certain types of singular terms of that form; once these denotations are given, all singular terms of that form will have a denotation. New functions introduced in the future may require an updating of the assignments.

Now, neither the contextual air of the first stage of Frege's method, nor the reference to future updatings of the assignments made according to the second part of the method, amount to anything like the context principle of *Grundlagen*. Moreover, the assignments made in the second part of the method ignore the context principle: certain names of courses of values receive a denotation and keep it regardless of their occurrence in a sentence.

In my view the source of confusion concerning the context principle is the failure to distinguish between the extreme thesis expressed by the principle and various degrees of "contextuality" (the reference of a word changes when the word occurs in indirect speech, the first part of Frege's looking-around method, etc.), none of which however imply the principle. The rejection, by the later Frege, of the *Grundlagen* "context principle" does not entail the rejection of all contextuality.

The ultimate question is not "What does the principle mean?" but "What did Frege actually *intend to do* with the context principle in *Grundlagen*?" Frege's use of the principle in his 1884 book is twofold: *negative* when it deters wrong views of number; and *positive* when it paves the way for the première of "Frege's method".

The negative use of the context principle is ridiculously excessive, and even offendingly trivial for any educated reader of *Grundlagen*. In my *Studies [1967]* I ventured the suspicion that Frege realized his thesis that numbers are non-material, non-psychological "subsistent objects" was too "metaphysical". Also, that somehow the context principle (together with the trivial remarks about Frege's readers being unable to distinguish between

a picture and an abstract notion) provided an escape from the embarrassing metaphysical path he should have pursued. Dummett objected ("The Interpretation," p. 365) that I was accusing Frege of unethical behaviour. I may drop the charges, but then Frege's strange behaviour in his negative use of the principle in *Grundlagen* still has to be explained.

Frege in *Grundlagen* uses the principle positively to justify the contextuality of the first part of his method. But here too, as in the case of the negative use, the context principle is excessive: surely Frege does not intend to introduce *all* singular terms by means of his method. Worse than that, Frege's method in its second stage has the curious effect of killing the context principle: singular terms like "the number of the concept *F*" receive a denotation regardless of the sentences in which they occur.

Excessive and self-defeating as it is, Frege's *positive* use of the context principle in *Grundlagen* is quite understandable. With his method he was breaking new ground and he knew it (*cf.* §63). Probably the structure of his method and its division into two stages, described above, were not yet clear to him, as shown by his painful consideration of one doubt (*Bedenken*) after the other (§§62–67). It must have seemed very strange to begin by using singular terms in identity sentences while pretending that one does not know their meaning. At a certain point in this pioneering work the notion of "words have meaning only in a sentence" must have appeared as a great relief.

Still, understandable as Frege's use of the context principle in *Grundlagen* is, it is no less understandable that, after *Grundlagen,* he ignored it and even made statements that amount to a rejection of it.

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