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# REMARKS ON SIMPLE EXTENDED LIE ALGEBRAS

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**We continue the discussion of finite dimensional simple extended Lie algebras over an algebraically closed field $F$ of characteristic zero with nondegenerate form $(x, y) =$ trace $R_xR_y$ where $R_x$ (or $R(x)$) denotes the mapping $A \to A: a \to ax$; for brevity we call such an algebra a *simple el-algebra*. The main result of this paper is that those simple el-algebras which are not Lie or Malcev algebras probably cannot be analyzed by the usual desirable Lie-type methods.**

First if we assume the simple el-algebra [3] $A$ has a diagonalizable Cartan subalgebra [3] such that for any weight space $A(N, \alpha)$ of $N$ in $A$ we have $A(N, \alpha)^2 = 0$ or $A(N, \alpha)^2 \subset A(N, \beta)$ for some weight $\beta$ (which is a function of $\alpha$), then $A$ is a Lie or Malcev algebra. Thus if one attempts to remedy the situation that $A(N, \alpha)^2$ is difficult to locate by the rather desirable above assumptions and tries to construct a multiplication table for a new simple el-algebra, then actually nothing new is obtained. Next we show that if the derivation algebra $D(A)$ is used to analyze a simple el-algebra, using [1, page 54] or possibly Lie module theory, then again a difficult situation is encountered: If $A$ is simple el-algebra, then $A$ is not a simple Lie or Malcev algebra if and only if there exists a nonzero element $a \in A$ such that for every derivation $D \in D(A)$ we have $aD = 0$. The element $a \in A$ reflects the structure of $A$ and so it appears that the structure of $A$ is not accurately reflected in its derivation albgebra.

The proofs of the above results use the following lemma.

LEMMA 1.1. *If $A$ is a simple el-algebra, then $A$ is a Lie or 7-dimensional Malcev algebra if and only if $u(x) =$ trace $R_x$ is the zero linear functional.*

*Proof.* A linearization of the defining identities of an extended Lie algebra

$$xy = -yx \qquad \text{and} \qquad J(xy, x, y) = 0$$

where $J(x, y, z) = xy \cdot z + yz \cdot x + zx \cdot y$ yields

$$J(wx, y, z) + J(yz, w, x) = J(wy, z, x) + J(zx, w, y) \tag{1.2}$$

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$$wJ(x, y, z) - xJ(y, z, w) + yJ(z, w, x) - zJ(w, x, y) = 3[J(wx, y, z) + J(yz, w, x)] \tag{1.3}$$

for all $w, x, y, z \in A$. From (1.2) we obtain by operating on $w$ that

$$\begin{aligned}(xz, y) - (x, zy) &= \text{trace } R(xz)R(y) - \text{trace } R(x)R(zy) \\ &= \text{trace } R(xz\cdot y + x\cdot zy) \\ &= u(xz\cdot y + x\cdot zy)\ .\end{aligned} \tag{1.4}$$

Now if $u(x) = 0$ for all $x \in A$, then from (1.4) we see $(x, y)$ is a nondegenerate invariant form and from [3], $A$ is a simple Lie or 7-dimensional Malcev algebra. Conversely, from the identities for these algebras [2] we see that $u(x) = 0$ for all $x \in A$.

We continue the use of the notation in [3] for sets and algebraic operations.

**2. On the construction.** We shall first investigate the assumption that a simple el-algebra $A$ has a diagonalizable Cartan subalgebra $N$ [3]. That is, $N$ is a nilpotent Lie subalgebra of $A$ such that for all $m, n \in N$,

$$R_{mn} = [R_m, R_n] \equiv R_m R_n - R_n R_m\ ;$$

furthermore, decomposing $A$ into its weight spaces relative to $R(N) = \{R_n : n \in N\}$ we have [1; 3]

$$A = A(N, 0) \oplus \sum_{\alpha\neq 0} A(N, \alpha)$$

where, since $R(N)$ is diagonalizable,

$$A(N, \lambda) = \{x \in A : xR_n = \lambda(n)x\}$$

is the weight space of $N$ corresponding to the weight $\lambda$ and, since $N$ is Cartan [3],

$$N = A(N, 0)\ .$$

Since we are using a fixed Cartan subalgebra we use the notation $A_\sigma$ or $A(\sigma)$ for $A(N, \sigma)$ and the convention $A(\sigma) = 0$ if $\sigma$ is not a weight of $N$ in $A$. From [3] we have the identities

$$A_\alpha A_\beta \subset A_{\alpha+\beta} \qquad \text{if } \alpha \neq \beta \tag{2.1}$$

$$\begin{aligned} J(A_\alpha, A_\beta, A_\gamma) &= 0 \qquad \text{if } \alpha \neq \beta \neq \gamma \neq \alpha \\ \text{and } J(A_\alpha, A_\beta, N) &= 0 \qquad \text{if } \alpha \neq \beta\ .\end{aligned} \tag{2.2}$$

Let $K$ denote the kernel of the linear functional $u : x \rightarrow \text{trace } R_x$, then we have

$$(\alpha+\beta)(n)(x, y) = (\alpha-\beta)(n)u(xy) \quad \text{if } n \in N, x \in A_\alpha, y \in A_\beta \tag{2.3}$$

$$(A_\alpha, A_\beta) = 0 \quad \text{if } \alpha \neq 0 \text{ and } \beta \neq 0 \text{ and } \alpha \neq -\beta \tag{2.4}$$

$$A_\alpha A_\beta \subset K \quad \text{if } \alpha \neq 0 \text{ and } \beta \neq 0 \text{ and } \alpha \neq \beta\ . \tag{2.5}$$

For (2.3), let $n \in N$, then $xn = \alpha(n)x$, $yn = \beta(n)y$ and using (1.4) we have

$$\begin{aligned}(\alpha(n)+\beta(n))(x, y) &= (xn, y) - (x, ny) \\ &= u(xn\cdot y + x\cdot ny) = (\alpha(n)-\beta(n))u(xy)\ .\end{aligned}$$

For (2.4) and (2.5), let $x \in A_\alpha$, $y \in A_\beta$ and first assume $\alpha \neq 0$ and $\beta \neq 0$, $\pm\alpha$. If $xy = 0$ for all $x, y$ as above, then the results follow from (2.3). So assume $0 \neq xy \in A(\alpha)A(\beta) \subset A(\alpha+\beta)$, then $\alpha+\beta$ is a weight of $N$ in $A$. Let $z \in A(\alpha+\beta)$, then since $\alpha \neq \alpha+\beta \neq \beta \neq \alpha$ we use (2.2) to obtain $J(x, y, z) \in J(A(\alpha), A(\beta), A(\alpha+\beta)) = 0$. Therefore

$$\begin{aligned}zR(xy) = zx\cdot y + yz\cdot x \in A(2\alpha+\beta)A(\beta) \\ + A(\alpha+2\beta)A(\alpha) \subset A(2(\alpha+\beta))\ .\end{aligned}$$

Using this result and (2.1) we see that for any weight $\gamma$,

$$A(\gamma)R(xy) \subset A(\gamma+(\alpha+\beta)) \neq A(\gamma)$$

and therefore the matrix for $R(xy)$ has zeros on its diagonal so that $u(xy) = \operatorname{trace} R(xy) = 0$. Next we relax the assumptions on $\beta$, use the above result and (2.3) to see that (2.4) and (2.5) now follow.

Now we shall start using the hypothesis that if $\alpha$ is any weight of $N$ in $A$, then $A_\alpha^2 = 0$ or there exists a weight $\pi(\alpha)$ such that $A_\alpha^2 \subset A_{\pi(\alpha)}$. Thus we are assuming that if $A_\alpha^2 \neq 0$, then there exists a weight $\pi(\alpha)$ such that for each $x, y \in A_\alpha$, $xy \in A_{\pi(\alpha)}$; that is, $\pi$ is a function of the weight and not a function of the particular elements used in forming the products. Using this assumption we shall show that for any weight $\alpha$, $A_\alpha \subset K(=\text{kernel of } u)$ and therefore by Lemma 1.1 conclude that $A$ is Lie or Malcev.

First for $\alpha = 0$ we have $A_0^2 = A_0N = 0$. So assume $\alpha \neq 0$. If $xy = 0$ for all $x, y \in A_\alpha$, then using (2.1) we see that for any $x \in A_\alpha$, $u(x) = \operatorname{trace} R_x = 0$ and therefore $A_\alpha \subset K$. So next we consider $0 \neq A_\alpha^2 \subset A_{\pi(\alpha)}$ where $\alpha \neq 0$.

LEMMA 2.6. *If $\alpha \neq 0$ and $0 \neq A_\alpha^2 \subset A_{\pi(\alpha)}$, then $\pi(\alpha) \neq 0$ .*

COROLLARY 2.7. $N = \sum_{\alpha\neq 0} A(\alpha)A(-\alpha) \subset K$.

Suppose Lemma 2.6 has been proven, then to prove the corollary we first note $\sum_{\alpha\neq 0} A(\alpha)A(-\alpha) \subset A(0) = N$. Next set $B = \sum_{\alpha\neq 0} A(\alpha)(-\alpha) \oplus \sum_{\alpha\neq 0} A(\alpha)$; we shall show $B$ is an ideal of $A$. For any weight $\beta \neq 0$,

$$BA(\beta) \subset (\textstyle\sum_{\alpha\neq 0} A(\alpha)A(-\alpha))A(\beta) + A(\beta)^2 + A(\beta)A(-\beta) + \sum_{\alpha\neq 0, \pm\beta} A(\alpha+\beta)\,.$$

Then using $A(\beta)^2 = 0$ or $A(\beta)^2 \subset A(\pi(\beta))$, where from Lemma 2.6 $\pi(\beta) \neq 0$, we see that $BA(\beta) \subset B$. For $\beta = 0$ we note that

$$(\textstyle\sum_{\alpha\neq 0} A(\alpha)A(-\alpha))A(0) \subset A(0)N = 0$$

and use (2.1) to obtain $BA(0) \subset B$. Thus $BA \subset B$ so that $B$ is an ideal of $A$ and since $A$ is simple, $B = 0$ or $B = A$. If $B = 0$, then $A_\alpha = 0$ for each $\alpha \neq 0$ and $A = A_0 = N$ so that $A^2 = A_0 N = 0$, a contradiction. Thus $B = A$ and from this $N = \sum_{\alpha\neq 0} A(\alpha)A(-\alpha) \subset K$, using (2.5).

For Lemma 2.6 assume $\pi(\alpha) = 0$ and let $x, y \in A_\alpha$, then $xy \in A_\alpha^2 \subset A_0 = N$. We shall show for any weight $\beta$ that $\beta(xy) = 0$, then for any $z \in A_\beta$ we have $z(xy) = zR(xy) = \beta(xy)z = 0$. Therefore $(xy)F$ is an ideal of $A$ which must be zero and so $A_\alpha^2 = 0$, a contradiction. For $x, y \in A_\alpha$ we have from the defining identity

$$0 = J(xy, x, y) = (xy\cdot x)y + (y\cdot xy)x$$

which implies, since $xy \in N$, $2\alpha(xy)xy = 0$. From this and the fact that $\alpha$ is a linear functional on $N$ we have $2\alpha(xy)^2 = 0$ and so $\alpha(xy) = 0$. Thus for $\beta = 0, \alpha$ we have $\beta(xy) = 0$ so we now assume $\beta \neq 0, \alpha$ and let $z \in A_\beta$, $n \in N$, then using (2.1) and (2.2) we obtain

$$\begin{aligned} J(zx, y, n) + J(yn, z, x) &= \alpha(n)J(y, z, x) \\ &= -\alpha(n)\beta(xy)z + \alpha(n)(yz\cdot x + zx\cdot y) \end{aligned}$$

and

$$\begin{aligned} J(zn, x, y) + J(xy, z, n) &= \beta(n)J(z, x, y) \\ &= -\beta(n)\beta(xy)z + \beta(n)(yz\cdot x + zx\cdot y)\,. \end{aligned}$$

We combine these equations by using (1.2) to obtain

$$\alpha(n)(-\beta(xy)z + zx\cdot y + yz\cdot x) = \beta(n)(-\beta(xy)z + zx\cdot y + yz\cdot x)\,.$$

From this equality we obtain, since $\beta(n) \neq \alpha(n)$ for some $n$, that

$$\beta(xy)z = zx\cdot y + yz\cdot x \in A(2\alpha + \beta)\,.$$

But since $\beta(xy)z \in A(\beta)$ we have

$$\beta(xy)z \in A(\beta) \cap A(2\alpha + \beta) = 0\,.$$

Thus if $z \neq 0$, $\beta(xy) = 0$ and this proves the lemma.

Thus far we have considered for $\alpha \neq 0$: (1) $A_\alpha^2 = 0$ which implies $A_\alpha \subset K$; (2) $A_\alpha^2 \neq 0$ which implies $\pi(\alpha) \neq 0$ and consequently $N = A_0 = \sum_{\alpha \neq 0} A(\alpha)A(-\alpha) \subset K$. So we next investigate (2) more closely and note that it suffices to consider $0 \neq A_\alpha^2 \subset A_{\pi(\alpha)}$ where $\pi(\alpha) = \alpha$. For if $\pi(\alpha) \neq \alpha$, then using (2.1) we see that the matrix of $R_x$ for any $x \in A_\alpha$ has zeros on its diagonal and therefore $u(x) = 0$ so that $A_\alpha \subset K$ which is what we eventually want to show for any weight $\alpha$.

Thus we are considering $0 \neq A_\alpha^2 \subset A_\alpha$. Since $(x, y)$ is nondegenerate and $A_\alpha^2 \neq 0$, there exists a weight $\beta$ so that

$$(A_\alpha^2, A_\beta) \neq 0 .$$

But since $A_\alpha^2 \subset A_\alpha$ this means $(A_\alpha, A_\beta) \neq 0$ and from (2.4) and the assumption that $\alpha \neq 0$ we conclude $\beta = 0$ or $\beta = -\alpha$. We shall consider these two cases and show that the situation $0 \neq A_\alpha^2 \subset A_\alpha$ actually does not exist so that we may conclude that for any weight $\alpha$, $A_\alpha \subset K$.

Case $\beta = 0$. Let $x, y \in A_\alpha$, $n \in A_0$ and $xy \in A_\alpha$, then using $(A_\alpha, A_\alpha) = 0$ (from (2.3)) we have

$$\begin{aligned}(xy, n) &= (xy, n) - (x, yn)\\ &= u(xy \cdot n + x \cdot yn)\\ &= u(\alpha(n)xy + \alpha(n)xy)\\ &= 2\alpha(n)u(xy) . \end{aligned} \tag{2.8}$$

However from (2.3) and $xy \in A_\alpha$ we have

$$\begin{aligned}\alpha(n)(xy, n) &= (\alpha + 0)(n)(xy, n)\\ &= (\alpha - 0)(n)u(xy \cdot n)\\ &= \alpha(n)^2 u(xy) . \end{aligned} \tag{2.9}$$

From (2.8) we also have $\alpha(n)(xy, n) = 2\alpha(n)^2 u(xy)$ and therefore from

$$\alpha(n)^2 u(xy) = 0 \qquad \text{for all } n \in N, x, y \in A_\alpha . \tag{2.9}$$

Now there exists $x, y \in A_\alpha$ so that $u(xy) \neq 0$, otherwise from (2.8) we would have $(A_\alpha^2, A_0) = 0$, contrary to our assumption for case $\beta = 0$. But from the previous equation this implies $\alpha(n) = 0$ for all $n \in N$, contradicting the assumption $\alpha \neq 0$. Thus case $\beta = 0$ does not exist.

Case $\beta = -\alpha$. That is, $\alpha \neq 0$, $A_\alpha^2 \subset A_\alpha$ and $(A_\alpha^2, A_\beta) \neq 0$ with $\beta = -\alpha$; in particular we are assuming $-\alpha$ is a weight. We shall show in this case that the dimension of $A_\alpha$ is one and therefore $A_\alpha^2 = 0$, a contradiction; thus case $\beta = -\alpha$ does not exist. So assume the dimension of $A_\alpha$ is greater than one and let $x, y \in A_\alpha$, $z \in A_{-\alpha}$ and $n \in N$, then using $xy \in A_\alpha$ and (2.2) we have

$$J(ny, z, x) + J(zx, n, y) = -\alpha(n)J(y, z, x)$$

and $$J(nz, x, y) + J(xy, n, z) = \alpha(n)J(z, x, y) .$$

Applying (1.2) to these equations we have, since $\alpha \neq 0$,

$$\begin{aligned} 0 = J(y, z, x) &= yz\cdot x + zx\cdot y + xy\cdot z \\ &= xy\cdot z - \alpha(yz)x - \alpha(zx)y . \end{aligned}$$

Therefore since $xy\cdot z \in A_0$ and $x, y \in A_\alpha$ we have $xy\cdot z = 0$ and $\alpha(yz)x + \alpha(zx)y = 0$. But since we have assumed the dimension of $A_\alpha > 1$ and $x, y$ are arbitrary in $A_\alpha$ we have $\alpha(zx) = 0$ for any $z \in A_{-\alpha}$; for just choose $0 \neq x$ arbitrary in $A_\alpha$ and $y$ to be linearly independent of $x$, then for any $z \in A$, $\alpha(yz)x + \alpha(zx)y = 0$ which yields the result.

Next we shall show $\beta(zx) = 0$ for any weight $\beta$ of $N$ and any $z \in A(-\alpha)$, $x \in A(\alpha)$. If $\beta = q\alpha$ where $q$ is a rational number, the results follow. Next suppose $\beta \neq q\alpha$ and let $M = \sum_k A(\beta + k\alpha)$, $k = 0, \pm 1, \pm 2, \cdots$. Using (2.1) and $\beta \neq q\alpha$ we see that $M$ is $R_x -$, $R_z -$, and $R(xz)$-invariant and for any $y = \sum_k y_k \in M$ where $y_k \in A(\beta + k\alpha)$ we have

$$J(y, x, z) = \sum_k J(y_k, x, z) = 0 ,$$

using (2.2). Thus $y([R_x, R_z] - R(xz)) = 0$; that is, on $M$ we have $R(xz) = [R_x, R_z]$ so that

$$\text{trace}_M R(xz) = 0 , \tag{2.10}$$

where $\text{trace}_M$ denotes the trace function restricted to $M$. However calculating the $\text{trace}_M R(xz)$ from the matrix of $R(xz)$ on $M$ we see that

$$\begin{aligned} \text{trace}_M R(xz) &= \textstyle\sum_k N_k(\beta + k\alpha)(xz), \qquad N_k = \dim A(\beta + k\alpha) \\ &= (\textstyle\sum_k N_k)\beta(xz) + (\textstyle\sum_h k N_k)\alpha(xz) \\ &- (\textstyle\sum_k N_k)\beta(xz), \text{ since } \alpha(xz) = 0 . \end{aligned}$$

This equation and (2.10) imply $\beta(xz) = 0$. Thus for any weight $\beta$ and any $y \in A_\beta$ we have $yR(xz) = \beta(xz)y = 0$ which implies $R(xz) = 0$ and therefore $xz = 0$ i.e. $A(\alpha)A(-\alpha) = 0$. We use this fact to obtain a contradiction to $(A^2(\alpha), A(-\alpha)) \neq 0$. So let $x, y \in A(\alpha)$, $z \in A(-\alpha)$, then using (1.4) we have

$$\begin{aligned} (xy, z) &= (x, yz) + u(xy\cdot z + x\cdot yz) \\ &= u(xy\cdot z), \text{ using } yz \in A(\alpha)A(-\alpha) = 0 \\ &= 0, \text{ using } xy \in A(\alpha) \text{ and } A(\alpha)A(-\alpha) = 0 . \end{aligned}$$

This contradiction shows case $\beta = -\alpha$ does not exist and so from previous remarks we have for any weight $\alpha$, $A_\alpha \subset K$ which proves

THEOREM 2.11. *Let $A$ be a simple el-algebra satisfying the*

*following conditions*
(1) *there exists a Cartan subalgebra* $N$ *of* $A$ *so that* $R(N) = \{R_n: n \in N\}$ *acts diagonally in* $A$
(2) *if* $A = \sum_\alpha A(N, \alpha)$ *is the weight space decomposition of* $A$ *relative to* $R(N)$ *where* $N$ *is the subalgebra of* (1), *then* $A(N, \alpha)^2 = 0$ *or* $A(N, \alpha)^2 \subset A(N, \pi(\alpha))$ *for some weight* $\pi(\alpha)$.

*Then* $A$ *is a Lie or 7-dimensional Malcev algebra.*

3. **On derivations.** Again let $A$ be a simple el-algebra. To use the derivation algebra $D(A)$ in the analysis of $A$ we first locate the derivations of $A$ as follows.

THEOREM 3.1. *Every derivation of* $A$ *is inner, that is,* $D(A)$ *is contained in the Lie trasformation algebra* $L(A)$ *which is the smallest Lie algebra containing* $R(A) = \{R_x: x \in A\}$ [4].

*Proof.* Since $A$ is simple it contains no nontrivial $L(A)$-invariant subspaces and so $L(A)$ is irreducible in $A$. This implies $L(A) = C \oplus L(A)'$ where $C$ is the center of $L(A)$ and $L(A)' = [L(A), L(A)]$ is semi-simple [1; Th. 2.11]. Furthermore $C = 0$ or $C = FI$; for if $S$ is a linear transformation in $C$, then since $F$ is algebraically closed $S$ has a characteristic root $\lambda$ in $F$. Using the fact $[R(A), S] = 0$ we see $\{x \in A: xS = \lambda x\}$ is a nonzero ideal of $A$ and therefore equals $A$. From this the results concerning $C$ follow.

Now let $D \in D(A)$, then we have $[R_x, D] = R(xD)$ for all $x \in A$ and this together with the Jacobi identity imply $[L(A)', D] \subset L(A)'$. Thus the mapping

$$L(A)' \to L(A)' : X' \to [X', D] \qquad \text{all } X' \in L[A]'$$

is a derivation of $L(A)'$. Since $L(A)'$ is semi-simple every derivation of $L(A)'$ is inner and therefore there exists $D' \in L(A)'$ so that $[X', D] = [X', D']$ all $X' \in L(A)'$ [1; Th. 3.6]. But for any $X = aI + X' \in L(A)$ where $a \in F$ (if $C \neq 0$) we have $[X, D] = [X, D']$. Thus if $T = D - D'$ we have in particular that $[R(A), T] = 0$. Again since $F$ is algebraically closed $T$ has a characteristic root $\mu$ and we see that $\{x \in A : xT = \mu x\}$ is a nonzero ideal in $A$. This implies either $T = 0$ in which case $D = D'$ or $T = \mu I$ in which case $D = \mu I + D'$. Now in this latter case we note $D' \in L(A)'$ so that trace $D' = 0$ and since $(x, y) =$ trace $R_x R_y$ is nondegenerate we have from $[R_x, D] = R(xD)$ that $(xD, y) + (x, yD) = 0$ so that $D$ is skewsymmetric and also trace $D = 0$. From these facts on trace and $D = \mu I + D'$ we conclude $D = D' \in L(A)$ in both cases.

Even though we know all derivations of a simple el-algebra are inner, their exact form has not yet been determined. However the following is not too difficult to prove: If $A$ is a simple el-algebra, then $A$ is a Lie algebra if and only if there exists an element $x \in A$ so that $R_x$ is a nonzero derivation of $A$. Next we have

THEOREM 3.2. *If $A$ is a simple el-algebra, then $A$ is not a Lie or 7-dimensional Malcev algebra if and only if there exists a nonzero element $a \in A$ such that for every derivation $D$ of $A$ we have $aD = 0$.*

*Proof.* If $A$ is a Lie or 7-dimensional Malcev algebra then the conclusion is well known [2]. Conversely, if $A$ is not Lie or 7-dimensional Malcev, then since $(x, y) = \text{trace } R_x R_y$ is nondegenerate we use Lemma 1.1 to obtain a nonzero element $a \in A$ so that for all $x \in A$, $u(x) = (x, a)$. But for any derivation $D$ we have $R(xD) = [R_x, D]$ and $(xD, y) + (x, yD) = 0$ so that in particular we have for any $x \in A$, $(aD, x) = -(a, xD) = -u(xD) = -\text{trace } R(xD) = 0$. Thus since $(x, y)$ is nondegenerate $aD = 0$.

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