PACIFIC JOURNAL OF MATHEMATICS
Vol. 51, No. 1, 1974

ON SEPARABLE POLYNOMIALS OVER A COMMUTATIVE RING

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Separable polynomials over an arbitrary commutative ring are studied. Given any separable polynomial $p(X)$ over the commutative ring R one can find a "splitting ring" for $p(X)$ which is a finitely generated normal separable extension of R generated by roots of $p(X)$. A polynomial closure Λ of R generated by roots of separable polynomials is constructed. Any separable polynomial over Λ factors into linear factors in Λ. A Galois theory for such extensions is discussed. Applications to separable extensions of von Neumann regular rings and the Brauer group are given.

In [6] G. J. Janusz developed the fundamental properties of separable polynomials over a commutative ring with no idempotents other than 0 and 1. If R is a commutative ring a monic polynomial $p(X)$ in $R[X]$ is separable in case $R[X]/(p(X))$ is a separable R-algebra. Here we develop a corresponding theory for separable polynomials over an arbitrary commutative ring. Since the fundamental difficulty to be overcome is the presence of idempotents, the tool (due to R. S. Pierce [11]) of representing a commutative ring as a global cross section of a sheaf of rings with no idempotents other than 0 and 1 is employed throughout. Basic properties of separable algebras and Galois theory discovered by A. Magid [8], [9], [10] are also employed.

Our first section is devoted to introducing necessary terminology and refining some of Magid's results for our own use. The principal result of the section is an extension of the fundamental theorem of Galois theory (Theorem 2.10 of [9]) to include the correspondence between normal extensions and normal subgroupoids of the Galois groupoid.

In the second section we analyze separable polynomials. Let $p(X)$ be a separable polynomial over the commutative ring R. We find a finitely generated normal separable extension S of R generated by roots of $p(X)$ and so that $p(X)$ factors into linear factors over S. Also, S satisfies a "local projectivity" condition. Such an R-algebra S is a splitting ring for $p(X)$. Associated to the extension S of R and thus to $p(X)$ is a compact totally disconnected topological groupoid $G(S/R)$. The nonuniqueness of S and $G(S/R)$ are discussed. To each commutative ring R we associate an extension Λ called a polynomial closure of R and a compact totally disconnected topological groupoid $G(\Lambda/R)$. The extension Λ of R is generated by roots of separable polynomials and every separable polynomial over Λ factors into linear factors in Λ. We show that the Galois theory developed in [9] applies to Λ. The nonuniqueness of Λ is also discussed.

In the third section we give applications of our results to separable extensions of von Neumann regular rings, to the Brauer group of rings whose prime ideal spectrum is totally disconnected, and to the Brauer group of the polynomial closure of such rings.

1. **Preliminaries.** Let R be a commutative ring, $B(R)$ the Boolean algebra of idempotents of R, and $X(R)$ the maximal ideal spectrum of $B(R)$. Then $X(R)$ is a compact totally disconnect Hausdorff space in the hull-kernel topology. For each $x \in X(R)$ let Rx be the ideal in R generated by x and for each R-module M let $M_x = M/RxM$. Then $R_x = R/Rx$ and M_x is an R_x-module. The rings R_x are stalks in a sheaf over the base space $X(R)$ and R is naturally represented as a global cross section of this sheaf. There are several standard lines of argument which have been developed to lift information true at all stalks R_x to information about R. We will not usually go through the details of these arguments and refer the reader to [13], [3], [7], [12], among others for examples of this sort of reasoning.

In [9] A. Madgid calls a commutative R-algebra S a quasi sepapable cover of R in case for each $x \in X(R)$, every finite subset of S_x is contained in a finitely generated projective separable subalgeba of S_x (S_x is a locally strongly separable R_x-algebra). The algebra S is a separable cover of R if S is a quasi separable cover of R and S is separable over R. Corollary 2.7 of [9] asserts that every quasi separable cover S of R is a union of subalgebras which are separable covers of R. R is called separably closed if whenever S is any projective separable R-algebra then there is an R-algebra homomorphism from S to R. A separably closed quasi separable cover Γ of R is called a separable closure of R. If Γ is a separable closure of R and S is a projective separable R-algebra, then there is an R-algebra homomorphism from S to Γ induced from the one from $\Gamma \otimes_R S$ to Γ. This homomorphism need not be one-to-one and in general the separable closure need not be unique. The problem is in the existence of idempotents in $\Gamma \otimes_R S$ which are not in Γ.

If we call a ring R extra separably closed in case whenever S is a separable cover of R which is finitely generated as an R-module then there is an R-algebra homomorphism from S to R, then there is a unique minimal extra separable closure S of R [10]. The Boolean algebra $B(S)$ of idempotents of S is the completion of $B(R)$. However, in many cases (for example if $X(R)$ is countable or R is a uniform ring [9]) a separable closure of R exists with no new idempotents. It is simpler to deal with the situation where no new idempotents are added so we will do this whenever possible (see Theorem 1.1).

A quasi separable cover S of R is called normal in case for each $x \in X(R)$, all R_x-homomorphism from S_x into the separable closure Γ_x of R_x have the same image. Since the composite of locally strongly separable R_x-subalgebras of an R_x algebra Γ with no idempotents other than 0 and 1 is locally strongly separable, this definition is equivalent to the one in [9]. Also, as observed in [8], S is a normal quasi separable cover of R if and only if whenever $x \in X(R)$ and $a, b \in X(S)$ with a and b lying over x then $S_a \cong S_b$ as R_x-algebras and S_a is a normal extension of R_x.

Our next step is to restate and extend the fundamental theorem of Galois theory presented in [9]. Let S be a normal quasi separable cover of R. Let $x \in X(R)$ and let $a, b \in X(S)$ lying over x. Let h be an R_x-algebra isomorphism from S_b to S_a. Let σ be the set of idempotents in $S \otimes_R S$ contained in the kernel of the map $S \otimes_R S \to S_a$ by $st \to s_a h(t_b)$. Then it is shown in [9] that the above correspondence gives a bijection between the points σ of $X(S \otimes_R S)$ and the four tuples (x, h, a, b) where $x \in X(R)$; $a, b \in X(S)$ over x and h is an R_x-algebra isomorphism from S_b to S_a.

The Galois groupoid G of S over R is the set of four tuples (x, g, a, b) with the topology corresponding to that in $X(S \otimes S)$. The partial multiplication is defined between pairs of four tuples of the following form,

$$(x, g, a, b)(x, h, b, c) = (x, gh, a, c) .$$

The identities of the groupoid are the four tuples $(x, 1, a, a)$ and a subgroupoid is a subset containing all identities and closed under multiplication and inversion.

Let H be a subgroupoid of G. Let

$$S^H = \{s \in S \mid g(s_b) = s_a \text{ for all } (x, g, a, b) \in H\} .$$

Let T be an R-subalgebra of S. Let

$$G(S/T) = \{(x, g, a, b) \in G \mid t_a = g(t_b) \text{ for all } t \in T\} .$$

Then Theorem 2.10 of [9] asserts the usual Galois correspondence between the set of all quasi separable covers of R in S and all closed subgroupoids of G. If G and H are groupoids and $h: G \to H$ is a homomorphism of G onto H then the inverse image of the identities of H is a normal subgroupoid K of G and the natural multiplication on the quotient structure turns G/K into a groupoid isomorphic to H. If G and H are compact Hausdorff topological groupoids and h is continuous, then K is closed in G and the natural isomorphism from G/K to H is a homeomorphism. (See page. 16 of [1].)

THEOREM 1.1. (Fundamental theorem of Galois theory.) *Let S*

be a normal quasi separable cover of R with Galois groupoid G. Then there is a one-to-one correspondence between the quasi separable covers T of R in S and the closed subgroupoids K of G by

$$K \longrightarrow S^K , \qquad T \longrightarrow G(S/T) .$$

Moreover, if T is a normal extension of R then the corresponding subgroupoid K is normal in G and G/K is the Galois groupoid of T over R. If every idempotent of S belongs to R and K is normal in G, then S^K is a normal extension of R.

Proof. All except the moreover statement are proved in [9]. First we state a result which is a consequence of the Galois theory in Chapter III, § 3 of [5].

PROPOSITION 1.2. *Let R be a commutative ring whose only idempotents are 0 and 1 and let Γ be the separable closure of R. Let S be a normal locally strongly separable R-algebra whose only idempotents are 0 and 1 and let T be a locally strongly separable R-subalgebra of S. Let h be any R-homomorphism from T to Γ, then h can be extended to an R-monomorphism from S to Γ. Any two extension of h to S have the same image in Γ. If S is a subalgebra of Γ then any extension of h to S is an automorphism of S.*

Now we can prove the theorem. Assume T is a normal extension of R with Galois groupoid K. Define a groupoid homomorphism $h: G \rightarrow K$ by the rule

$$h((x, g, a, b)) = (x, g^*, a^*, b^*)$$

where $x \in X(R)$; $a, b \in X(S)$ lying over x, and a^*, b^* are points in $X(T)$ so that a lies over a^* and b lies over b^*. Also $g^*: T_{b^*} \rightarrow T_{a^*}$ by $g^*(t_{b^*}) = u_{a^*}$ in case $g(t_b) = u_a$ where $t, u \in T$. There is a natural R_x-homomorphism from T_{a^*} into S_a by $t_{a^*} \rightarrow t_a$. By Proposition 1.2 this correspondence is a monomorphism so if $t, v \in T$ then $t_a = v_a$ if and only if $t_{a^*} = v_{a^*}$. To see that g^* is well defined assume $t, v \in T$ and $t_{b^*} = v_{b^*}$ in T_{b^*}. Then $t_b = v_b$ in S_b so if $g(t_b) = u_a$ and $g(v_b) = w_a$ with $u, w \in T$ then $u_a = w_a$ since g is well defined. Since T is normal over R; u, w can be chosen from T. Thus $u_{a^*} = w_{a^*}$ so g^* is well defined. Now T_{a^*} and T_{b^*} are isomorphic locally strongly separable normal R_x-algebras with no idempotents other than 0 and 1. By Proposition 1.2 g^* is an isomorphism. It is now routine to see that h is a continuous homomorphism. Let (x, g^*, a^*, b^*) be an element of K. Let a and b be elements of $X(R)$ lying over a^* and b^* respectively. Then T_{a^*} is a locally strongly separable R_x-subalgebra of S_a and T_{b^*} is a locally strongly separable R_x-subalgebra of S_b. Also, S_a and S_b are isomorphic R_x-algebras which are normal over R_x. So by Proposition 1.2. g^* extends to an isomorphism g from S_b to S_a. Thus, $(x, g^*, a^*, b^*) = h((x, g, a, b))$ and h is onto. The kernel of h is the subgroupoid H of G which fixes T and $G/H = K$.

Now assume every idempotent of S belongs to R and let T be a locally quasi separable cover of R in S. Let K be the closed subgroupoid of G with $S^K = T$ and assume K is a normal subgroupoid of G. Since every idempotent of S belongs to R, each element of G is of the form (x, g, x, x) where g is an automorphism of S_x. Thus, G is the disjoint union of the Galois groups of each S_x over R_x as x ranges through $X(R)$ and multiplication is not defined between different terms in the union. One can show that K must be the union of the normal subgroups which fix T_x over R_x in S_x. Thus, T_x is a normal extension of R_x for each $x \in X(R)$ so T is a normal extension of R. This completes the proof.

If we do not assume that every idempotent in S belongs to R then for a^* and b^* in $X(T)$ lying over $x \in X(R)$ one may have T_{a^*} and T_{b^*} both normal over R_x but not isomorphic. In this case $G(S/T)$ will be normal in $G(S/R)$ but T will not be normal over R. If Γ is a separable closure of R and $a, b \in X(\Gamma)$ lying over $x \in X(R)$ then both Γ_a and Γ_b are separable closures of R_x. Any R_x-algebra homomorphism from Γ_a and Γ_b to a separable closure Ω of R_x have image Ω (as is shown in the discussion on page 105 of [5]) so Γ is normal over R.

2. Separable polynomials. Let $p(X)$ be a separable polynomial over R. A normal quasi separable cover S of R is called a splitting ring for $p(X)$ in case

(1) S is generated over R by roots of $p(X)$.

(2) $p(X)$ factors into linear factors in $S[X]$.

PROPOSITION 2.1. *Let $p(X) \in R[X]$ be a separable polynomial, then a splitting ring S for $p(X)$ exists. Moreover, S can be chosen to be finitely generated over R.*

Proof. Let Γ be a separable closure of R. Then $p(X)$ is separable over Γ and there is an Γ-algebra homomorphism from $\Gamma[X]/(p(X))$ to Γ. The image of X is a root α of $p(X)$ in Γ. Now $R_x(\alpha_x)$ is a finitely generated separable R_x-subalgebra of Γ_x for each $x \in X(R)$ so by Lemma 1.2 of [8] $R(\alpha)$ is a finitely generated separable cover of R in Γ. Let $T = R(\alpha)$. The transitivity properties of separability, projectivity, and finite generation insure that Γ is a separable closure of T. Also, $p(X) = (X - \alpha)q(X)$ in $T[X]$ and $T[X]/(q(X))$ is a homomorphic image of $T[X]/p(X)$ by the natural map. Thus, $q(X)$ is a separable polynomial over T and we can apply the first step in the proof to $q(X)$ over T. After a finite number of steps we come to an extension S of R which is a finitely generated separable cover of R, generated by roots of $p(X)$, and so that $p(X)$ factors into linear factors in S. Now let $x \in X(R)$ and $a, b \in X(S)$ lying over x. Then both S_a and S_b are splitting rings of the separable polynomial $p(X)$. By the normality and uniqueness of splitting rings in case of no nontrivial idempotents we have S_a as a normal extension of R_x and all homomorphisms from S_a to S_b to be a separable closure of R_x have the same image. Thus, S is normal over R.

With respect to uniqueness of splitting rings, the same situation holds as for the separable closure. Let Γ be the separable closure of R with the property that $B(\Gamma)$ is the completion of $B(R)$. Let $p(X)$ be a separable polynomial in $R[X]$ and let S be the R-subalgebra of Γ generated by all the roots of $p(X)$ in Γ. Then S is a normal quasi separable cover of R and any R-splitting ring N of $p(X)$ with $B(N) \subseteq B(S)$ will be isomorphic to an R-subalgebra of S. If $B(R)$ is countable than S can be chosen so that every idempotent in S is in R (see [9]). If R is a uniform ring (see [4]) then in addition, S can be chosen to be projective over T. Consider the following example. Let $X = \{1, 1/2, 1/3, 1/4, \cdots 1/n, \cdots\} \cup \{0\}$ with the topology inherited from the reals. Let R be the ring of complex valued continuous f on X so that $f(0)$ is real with the discrete topology on the complex numbers. Let S be the ring of continuous complex valued functions on X. Then $B(R) = B(S)$ and S is a normal finitely generated separable cover of R. The Galois groupoid G of S over R is the cyclic group of order 2 whose nonidentity element is complex conjugation at $f(0)$ for each $f \in S$. One observes that there are no nontrivial R automorphisms of S, and there are no projective finitely generated separable covers of R in S containing a root of the separable polynomial $X^2 + 1$ in $R[X]$.

We next construct a normal quasi-separable cover Λ of R so that every separable polynomial over R factors completely in Λ and every finite subset of Λ is contained in an extension $R(\alpha_1, \cdots, \alpha_n)$ of R in Λ with α_i the root of a separable polynomial in $R(\alpha_1, \cdots, \alpha_{i-1})$. Such an extension Λ is called a polynomial closure of R (see [4]).

THEOREM 2.3. *Let R be a commutative ring, then a polynomial closure of R exists and is a normal extension of R.*

Proof. Let Γ be a separable closure of R, consider the set $\mathscr{S}$ of all extensions of R in Γ of the form $R(\alpha_1, \cdots, \alpha_n)$ with α_i the root of a separable polynomial in $R(\alpha_1, \cdots, \alpha_{i-1})$. The property of being a separable cover is transitive (2.3 of [9]) and as in the proof of Proposition 2.1 each $R(\alpha_1, \cdots, \alpha_i)$ is a separable cover of $R(\alpha_1, \cdots, \alpha_{i-1})$ so $R(\alpha_1, \cdots, \alpha_n)$ is a separable cover of R. If $R(\beta_1, \cdots, \beta_m)$ is another element of $\mathscr{S}$, then the separable polynomial over R having β_1 as a root is also separable over $R(\alpha_1, \cdots, \alpha_n)$ so $R(\alpha_1, \cdots, \alpha_n, \beta_1, \cdots \beta_m)$ is in $\mathscr{S}$. By a finite induction $R(\alpha_1, \cdots, \alpha_n, \beta_1, \cdots, \beta_m)$ is in $\mathscr{S}$ so $\mathscr{S}$ is a directed set under inclusion. Let Λ be the union of all the elements in $\mathscr{S}$. Then Λ is a quasi separable cover of R in Γ. Let $p(X) \in \Lambda[X]$ be a separable polynomial and let $(t(Y^i Y^j))$ be the $n \times n$ matrix whose $i+1, j+1$ entry is $Y^i Y^j$ where $Y = X + (p(X))$ in $\Lambda[X]/p(X)$, $n =$ degree $(p(X))$, and t is the trace of the free Λ-module $\Lambda[X]/(p(X))$. Exactly as in the proof of Theorem 4.4 page 111 of [5] one can show $p(X)$ is separable over Λ if and only if $\det(t(Y^i Y^j))$ is a unit in Λ. Find an element $S \in \mathscr{S}$ containing the coefficients of $p(X)$, the elements $t(Y^i Y^j)$ and $\det(t(Y^i Y^j))^{\pm 1}$. Then $p(X)$ is separable over S, and Γ contains a root β of $p(X)$ as proved in Proposition 2.1. Then $S(\beta) \in \mathscr{S}$ so $S(\beta) \subseteqq \Lambda$ and Λ contains a root β of $p(X)$. Then in $\Lambda[X]$, $p(X) = (X - \beta)q(X)$ and $q(X)$ is separable over Λ. Continue the process until $p(X)$ factors completely in Λ. For normality first observe that $p(X) = X^2 - X$ is separable over R so every idempotent in Γ is in Λ. Let $x \in X(R)$ and let $a, b \in X(\Lambda) = X(\Gamma)$ be elements lying over x. Then Λ_a and Λ_b are polynomial closures of R_x in the separable closures Γ_a and Γ_b of R_x. Therefore by Theorem 1.1 of [4], Λ_a and Λ_b are isomorphic normal extension of R_x so Λ is a normal extension of R.

We can take $B(\Lambda)$ to be the completion of $B(R)$ since every idempotent in Γ is in Λ and $B(\Gamma)$ can be the completion of $B(R)$. Also a compact totally disconnected groupoid $G(\Lambda/R)$ can be associated to the extension Λ over R which puts one in the context of the fundamental theorem of Galois theory, (Theorem 1.1).

3. Applications.

A ring R is von Neumann regular if and only if R_x is a field for each $x \in X(R)$. On account of the primitive element theorem in the Galois theory of fields one has an extension of Theorem 2.7 of [4].

THEOREM 3.1. *Let R be a von Neumann regular ring and let S be a finitely generated separable cover of R, then $S = R(\alpha_1, \cdots, \alpha_n)$ with α_i the root of a separable polynomial over $R(\alpha_1, \cdots, \alpha_{i-1})$.*

Proof. R is regular so for each $x \in X(R)$, R_x is a field. Therefore, S_x is a finite direct sum of separable field extensions of R_x, say $S_x = F_1 \oplus \cdots \oplus F_n$. By the primitive element theorem, $F_1 = R_x(\alpha_x^1)$, $F_2 = R(\alpha_x^2), \cdots, F_n = R(\alpha_x^n)$ with α_x^i satisfying separable polynomials

$p_x^i(X)$ in $R_x[X]$ of the same degree (see [14]). Then $S_x = R_x(\alpha_x^1, \cdots, \alpha_x^n)$ and α_x^n satisfies the separable polynomial $p_x^i(X)$ in $R_x(\alpha_x^1, \cdots, \alpha_x^{i-1})$. Lift $p_x^i(X)$ and α_x^i to monic polynomials $p^i(X) \in R[X]$ and $\alpha^i \in S$. Since $S_x = R_x(\alpha_x^1, \cdots, \alpha_x^n)$ and S is finitely generated, (2.11) of [13] implies there is a neighborhood U of x so that for each $y \in U$, $S_y = R_y(\alpha_y^1, \cdots, \alpha_y^n)$. Also, $p_y^i(X)$ is separable for each y in a neighborhood of x. Therefore, there is an idempotent $e \in R$ so that $Se = Re\,(\alpha_n^1, \cdots, \alpha_n^n)$ and $\alpha^i e$ satisfies the monic separable polynomial $p^i(X)e$ in $Re\,[X]$. Employing the compactness of $X(R)$ one can decompose R by a finite number of such idempotents e thereby obtaining the result.

COROLLARY 3.2. *If R is a von Neumann regular ring then every separable closure of R is a polynomial closure and conversely.*

Let Spec (R) be the maximal ideal spectrum of R, than Spec R is totally disconnected if and only if R_x is semi-local (finite number of maximal ideals) for each $x \in X(R)$. For such rings we have the following result.

THEOREM 3.3. *Let R be a ring with Spec (R) totally disconnected. If A is a central separable R-algebra, then A is split in the Brauer group of R by a normal finitely generated separable cover $R(\alpha_1, \cdots, \alpha_n)$ of R with α_i the root of a separable polynomial over $R(\alpha_1, \cdots, \alpha_{i-1})$. If Λ is a polynomial closure of R, we can assume $R(\alpha_1, \cdots, \alpha_n)$ is in Λ.*

Proof. Modifying the proof of Theorem 1 in [2] by using Henselization instead of completion one has for each $x \in X(R)$ a strongly separable extension S_x over R_x of the form $R_x(\alpha_x)$ with α_x the root of a separable polynomial $p_x(X)$ over $R_x[X]$ which splits A_x. Now S_x is contained in a normal separable extension N_x of R_x generated by the roots of $p_x(X)$ by Theorems 3.4.2 and 3.2.9 or [5]. Arguing as in Theorem 2.1 of [4] one can construct a finite set of orthogonal idempotents $e_1, \cdots, e_n \in R$ with $1 = e_1 + \cdots + e_n$ and Ne_i a normal extra separable extension of Re_i in Λe_i generated by roots of the separable polynomial $p_i(X)$ and Ne_i splits Ae_i. The extension $N = Ne_1 \oplus \cdots \oplus Ne_n$ is the one we seek.

LEMMA 3.4. *Let S be a locally extra separable R-algebra and P a finitely generated projective S-module. Assume Spec (R) is totally disconnected, then there is a finite set $e_1, \cdots, e_n$ of orthogonal idempotents in S summing to 1 with Pe_i free over Se_i.*

Proof. Let $x \in X(R)$ and let $a \in X(S)$ lying over x. Then S_a is a locally strongly separable R_x-algebra with no idempotents other than 0 and 1 and R_x is a semi-local ring so the proof of Proposition 3 of

[2] shows P_a is a free S_a-module. Let $[P_a: S_a]$ be the rank of P_a over S_a. The function $a \to [P_a: S_a]$ is continuous and bounded from $X(S)$ to the nonnegative integers so there exists orthogonal idempotents $e_1, \cdots, e_n$ in S summing to 1 with the rank of Pe_i over Se_i defined. Let $P = Pe_i$ and $S = Se_i$ with $[P: S] = n$. Let $a \in X(S)$ and let $y_a^1, \cdots, y_a^n$ be a free basis for P_a over S_a. Lift the y_a^i to elements y^i in P, and let E be the submodule of P generated by y^i. Then $E_a = P_a$ so $E_b = P_b$ for all b in a neighborhood of a (2.11 of [13]). This neighborhood determines an idempotent e of S and ey^1 generate Pe over Re. The natural map from Pe onto the free Se-module $Se^{(n)}$ by assigning the ey^i to basis elements of $Se^{(n)}$ has a kernel K which is a direct summand of P. Now $K_a = 0$ so in a neighborhood V of a, $K_y = 0$ for each $y \in V$. This gives an idempotent $e_1 \in Se$ with Pe_1 free over Se_1. Using the compactness of $X(S)$ gives the result.

COROLLARY 3.5. *Let Λ be a polynomial closure or a separable closure of R. If Spec (R) is totally disconnected, then the Brauer group of Λ is trivial.*

Proof. Let A be a central separable Λ-algebra. We can write $A = Ae_1 \oplus \cdots \oplus Ae_n$ over $\Lambda e_1 \oplus \cdots \oplus \Lambda e_n$ with Ae_i a free Λe_i-module. Let $a_1^i, \cdots, a_n^i$ be a free Λe_i basis for Ae_i. Let c_{lj}^{ki} be the multiplication constants for the algebra A with respect to the basis $a_1^i, \cdots, a_n^i$, that is $a_l^i a_m^i = \Sigma_k c_{lm}^{ki} a_k^i$ with $c_{lm}^{ki} \in \Lambda$. Let $S = R(\alpha_1, \cdots, \alpha_n)$ be a finitely generated separable cover of R in Λ containing $\{c_{lm}^{ki}, e_j\}$. Define the central separable S-algebra A_S by letting $A_S e_j$ be the free Se_j-module $Se_j a_1^i + \cdots + Se_j a_n^i$ with multiplication constants $\{c_{lm}^{ki}\}$.

Now Spec (R) is totally disconnected and since S is a finitely generated separable cover of R, Spec (S) is totally disconnected. By Theorem 3.3 there is a finitely generated separable over $S(\beta_1, \cdots, \beta_m)$ of S in Λ which splits A. Let $T = S(\beta_1, \cdots, \beta_m)$ then $A \cong \Lambda \otimes_T (T \otimes_S A_S)$ so A is in the zero class of the Brauer group of Λ. The argument in case Λ is a separable closure of R is completely analogous.

As an alternate proof of the last result, we can use Theorem 1.1 of [4]. One can show as in Proposition 3.3 of [9] that a quasi separable cover Λ of R is a polynomial closure of R if and only if for each $x \in X(R)$ and each $a \in X(\Lambda)$ lying over x that Λ_a is a polynomial closure *of* R_x. Corollary 1.11 of [7] asserts that the Brauer group of Λ is trivial if and only if the Brauer group of Λ_a it trivial for each $a \in X(\Lambda)$. Thus, an alternate proof of Corollary 3.5 would be provided by showing the Brauer group of the polynomial closure of a semi-local ring whose only idempotents are 0 and 1 is trivial. This is proved in Theorem 1.6 of [12].

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Received November 8, 1972.

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