ERRATA

Corrections to

Multipliers of Type (p, p)
Kelly McKennon Volume 43 (1972), 429-436
Multipliers of Type (p, p) and Multipliers of the Group L_p-Algebras
Kelly McKennon Volume 45 (1973), 297-302
Multipliers and the Group L_p-Algebras
John Griffin and Kelly McKennon Volume 49 (1973), 365-370

The series of papers [2], [3], and [4] contains a number of errors, most of which deriving from a mis-statement in [5]. The subalgebra A of B in [5] 1.4.ii must be in addition a left ideal. Thus, assertion (4) and Theorem 6 of [2] are only valid in the case that A is a left ideal as well.

A second major problem is Theorem 1 of [1]. The proof to this theorem is false and the author has not been able to rectify it. For compact groups, Theorem 1 is trivial and, for unimodular groups G with $p \leqq 2$, easy to prove. For this reason, and because we shall need the following theorems and corrollary, we shall make the additional assumption *that G has equivalent left and right uniform structures and $p \leqq 2$.*

THEOREM 1. *The closed right ideal in $L_1(G)$ generated by the center $L_1(G)^z$ of $L_1(G)$ is just $L_1(G)$.*

Proof. Note that G, being a SIN group, is unimodular. Let Q be the family of compact neighborhoods of the identity invariant under inner automorphisms. Then, for $V \in Q$ and $t, x \in G$,

$$t^{-1}x \in V \text{ if and only if } xt^{-1} = x(t^{-1}x)x^{-1} \in xVx^{-1} = V$$

so that $\xi_V(t^{-1}x) = \xi_V(xt^{-1})$, ξ_V being the set-theoretic characteristic function of V. Thus, for all $V \in Q$, $x \in G$, and $f \in L_1(G)$,

$$f*\xi_V(x) = \int f(t)\xi_V(t^{-1}x)dt = \int f(t)\xi_V(xt^{-1})dt = \xi_V*f(x) .$$

Thus, $\{\xi_V \colon V \in Q\} \subset L_1(G)^z$.

Assume that A is not dense in $L_1(G)$. Then there exists some nonzero function $h \in L_\infty(G)$ such that

$$\int hgd\lambda = 0 \text{ for all } g \in A . \tag{1}$$

Choose a compact subset E of G and a nonzero complex number α such that $\lambda(E) > 0$ and

$$|h(x) - \alpha| < \frac{|\alpha|}{3} \text{ for all } x \in E .$$

The net $\{\xi_V/\lambda(V)\}_{V\in Q}$, Q directed by inclusion, is an approximate identity for $L_1(G)$. Hence, there exists a sequence $\{V(n)\}_{n=1}^{\infty}$ in Q such that

$$\lim_n \| \xi_{(E^{-1})} * \xi_{V(n)}/\lambda(V(n)) - \xi_{(E^{-1})} \|_1 = 0 .$$

By choosing a subsequence is necessary, it may also be assumed that $\xi_{(E^{-1})} * \xi_{V(n)}(x)/\lambda(V(n))$ converges to $\xi_{(E^{-1})}(x)$ for λ-almost all x. If $x^{-1} \in E^{-1}$ is a point of convergence, then

$$\begin{aligned} 1 &= \xi_{(E^{-1})}(x^{-1}) = \lim_n \xi_{(E^{-1})} * \xi_{V(n)}(x)/\lambda(V(n)) \\ &= \lim_n \frac{1}{\lambda(V(n))} \int \xi_{(E^{-1})}(t)\xi_{V(n)}(t^{-1}x^{-1})dt \\ &= \lim_n \frac{1}{\lambda(V(n))} \lambda(E^{-1} \cap x^{-1}V(n)^{-1}) = \lim_n \frac{\lambda(E \cap V(n)x)}{\lambda(V(n))} . \end{aligned}$$

Thus, there exists $V \in Q$ such that

$$\lambda(V) < \left(1 + \frac{|\alpha|}{3\|h\|_\infty}\right)\lambda(E \cap Vx) .$$

We have $\xi_{Vx} \in A$ and

$$\begin{aligned} \left|\int h\xi_{Vx}d\lambda\right| &\geqq \left|\int_{Vx\cap E} hd\lambda\right| - \left|\int_{Vx\cap E'} hd\lambda\right. \\ &\geqq \left|\int_{Vx\cap E} \alpha d\lambda\right| - \left|\int_{Vx\cap E} \frac{|\alpha|}{3} d\lambda\right| - \lambda(Vx \cap E')\|h\|_\infty \\ &\geqq |\alpha|\lambda(Vx \cap E) - \lambda(Vx \cap E)\frac{|\alpha|}{3} - \lambda(Vx \cap E)\frac{|\alpha|}{3\|h\|_\infty}\|h\|_\infty \\ &= \lambda(Vx \cap E)\frac{|\alpha|}{3} > 0 . \end{aligned}$$

This contradicts (1). Hence A is dense in $L_1(G)$.

THEOREM 1'. *Let G be a locally compact* SIN *group. Then the closed left ideal in $L_1(G)$ generated by $L_1(G)^z$ is $L_1(G)$.*

Proof. The proof is analogous to that of Theorem 1.

THEOREM 2. *Let G be a* SIN *group and $p \in [1, \infty[$. Let W_p be the unit ball of the Banach algebra of right multipliers of type (p, p), and B_p be the unit ball, of $L_p(G)$. Then, if $\{h_\beta\}$ is a bounded right approximate identity for $L_1(G)$ and $h \in L_1(G)$,*

$$\lim_{\beta} \sup \{\|T(g*h)*h_\beta - T(g*h)\|_p : g \in B_p,\ T \in W_p\} = 0 .$$

Proof. Let b be a bound for the numbers $\|h_\beta\|_1$ and d a positive number. Choose $f, q \in C_\infty(G)$ such that

$$\|f*q - h\|_1 < \frac{d}{3(1+b)} .$$

Theorem 1′ implies that there exist $\{f_j\}_{j=1}^n \subset L_1(G)^z$ and $\{x_j\}_{j=1}^n \subset G$ such that

$$\left\|f - \sum_{j=1}^{n} {}_{x_j}f_j\right\|_1 < \frac{d}{3(1+b)\|q\|_1} .$$

Choose an index B_0 such that, for all $\beta > \beta_0$ and $j = 1, 2, \cdots, n$,

$$\|f_j - f_j*h_\beta\|_1 < \frac{d}{3n\|q\|_1} .$$

For all $\beta > \beta_0$, $g \in B_p$, and $T \in W_p$, we have

$$\begin{aligned}
&\|T(g*h)*h_\beta - T(g*h)\|_p \\
&\quad \leqq \|g*f*T(q)*h_\beta - g*f*T(q)\|_p + (1+b)\frac{d}{3(1+b)} \\
&\quad \leqq \left\|\sum_{j=1}^{n} g*{}_{x_j}f_j*T(q)*h_\beta - g*{}_{x_j}f_j*T(q)\right\|_p \\
&\qquad + (1+b)\|q\|_1 d/[3(1+b)\|q\|_1] + \frac{d}{3} \\
&\quad \leqq \sum_{j=1}^{n} \|g*{}_{x_j}T(q)*f_j*h_\beta - g*{}_{x_j}T(q)*f_j\|_p + \frac{d}{3} + \frac{d}{3} \\
&\quad \leqq \sum_{j=1}^{n} \|g\|_p\|q\|_1\|f_j*h_\beta - f_j\|_1 + \frac{d}{3} + \frac{d}{3} \leqq \frac{d}{3} + \frac{d}{3} + \frac{d}{3} = d .
\end{aligned}$$

COROLLARY. *Let T be a right multiplier of type (p, p) and let $\{h_\beta\}$ be as in Theorem 2. Then the nets $\{W_{h_\beta} \circ T \circ W_{h_\beta}\}$, $\{W_{h_\beta} \circ T\}$, and $\{T \circ W_{h_\beta}\}$ all converge to T in the topology $\mathfrak{R}(\mathfrak{M}_p, \mathfrak{A}_p)$.*

Proof. That $\{T \circ W_{h_\beta}\}$ converges to T follows from Theorem 3 of [1]. That $\{W_{h_\beta} \circ T\}$ converges to T follows from Theorem 2. Thus, for all $V \in \mathfrak{A}_p$,

$$\begin{aligned}
&\overline{\lim_{\beta}} \|(W_{h_\beta} \circ T \circ W_{h_\beta} - T) \circ V\| \\
&\quad \leqq \lim(\|W_{h_\beta}\| \, \|(T \circ W_{h_\beta} - T) \circ V\| + \|(W_{h_\beta} \circ T - T) \circ V\|) = 0 .
\end{aligned}$$

We now return to the papers [1], [2], and [3]. Evidently, under our new assumptions, Theorems 1, 2, 3, 4, 5, the corollary to Theorem

4, and Lemma 2 of [1] are still true as stated. The same holds true for all the lemmas, propositions, and theorems of [2], and for Lemma 2 of [3].

Lemma 1 of [1], which depends on (4) can be repaired by applying to the corollary to Theorem 2 above instead. The assertion (6) of [3] is now a special case of the corollary of Theorem 2 above. Lemmas 1 and 2, Theorems 1 and 2, and Corollaries 1 and 2 of [3] all depend, either directly or derivatively on (6). Theorem 7 of [1] depends on the corollary to Theorem 2 above and Theorem 6 of [1]; but Corollary 1 of [3] implies that $\mathfrak{A}_p$ is a left ideal in $\mathfrak{M}_p$ and so Theorem 6 of [1] may be validly applied. Proposition 1 of [3] depends on Theorem 7 of [1] and Theorem 3 of [3] on Proposition 1.

The one last correction we note here is that f and h should be interchanged in the right side of the equation defining convolution at the beginning of [2].

REFERENCES

1. K. McKennon, *Multipliers of type* (p, p), Pacific J. Math., **43** (1972), 429-436.

2. ———, *Multipliers of type* (p, p) *and multipliers of the group* L_p*-algebras*, Pacific J. Math., **45** (1973), 297-302.

3. J. Griffin and K. McKennon, *Multipliers and the group* L_p*-algebras*, Pacific J. Math., **49** (1973), 365-370.

4. K. McKennon, *Multipliers, positive functionals, positive-definite functions, and Fourier-Stieltjes transforms*, Memoirs of the Amer. Math. Soc., **111** (1971).

Correction to

"a^*-CLOSURES TO COMPLETELY DISTRIBUTIVE LATTICE-ORDERED GROUPS"

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Volume 59 (1975), 43-67

The converse direction of Theorem 5.1 (see Pacific Journal of Mathematics, 59 (No. 1), 1975) is easily seen to be false. The proof is complete nonsense as Proposition 5.2 does not apply. However, the converse direction of Theorem 5.1 is true under the added assumption that (H, T) also has closed stabilizers. Moreover, wherever this direction of Theorem 5.1 has been used in the rest of the paper, the extra hypothesis is available (often courtesy of Proposition 5.2), so the remainder of the paper requires no change.