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ACTIONS OF FINITE GROUPS ON SELF-INJECTIVE RINGS[1]

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Let G be a finite group of automorphisms of a ring R (with 1), and suppose the order of G is not a zero diviser in R. We denote by R^G the subring of R consisting of elements fixed pointwise by each member of G. We consider, for a class of rings, the questions whether R viewed as a right (or left) R^G-module is finitely generated, and how the type classification of R and R^G relate when R is self-injective regular.

Even in the case of a commutative noetherian ring R, R need not be finitely generated over R^G, as shown by Chuang and Lee [1]. However, if R is a finite product of simple rings, more generally if R is biregular, then the finite generation does hold. The proof utilizes the skew group ring $R_S G$ and an elementary result from Morita theory; as a consequence, we obtain a short, easy proof of the theorem of Farkas and Snider [3; Theorem 1], for R semisimple artinian.

For self-injective rings R, the finite generation need not hold: nevertheless the techniques involved in the biregular case can be used to show that the type classifications are preserved. In otherwords, R and R^G are simultaneously of types I_f, I_∞, II_f, II_∞, or III. This completes work of the second author [14].

If R is self-injective regular, then R is injective as an R^G-module, and we show that R is projective if and only if it is finitely generated. This is done by showing that any nonsingular injective module over a self-injective regular ring that is also projective, must be finitely generated, or else the ring has an artinian ring direct summand.

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I. Biregular rings. A convenient tool for dealing with group actions, is the skew group ring. Let G be a group, with an action as automorphisms on R. Form the free left R-module with basis G, $R_s G$, equipped with multiplication extended R-linearly from

[1] In an earlier version, this paper was written in French, under the title, "Actions de Groupes".

$$rg = gr^g \quad \text{for } r \text{ in } R, g \text{ in } G .$$

See for example [7].

THEOREM 1. *Let R be a finite product of simple rings, and suppose G is a finite group acting as automorphisms of R, with the order of G invertible in R. Then R_sG, the skew group ring, is also a finite product of simple rings.*

REMARK. The proof below is due to the referee of the earlier version of this paper, viz, footnote 1. Other proofs were independently given by D. Passman, J. Fisher, and the authors.

Proof. Let A be a finite product of simple rings. The bimodule (two-sided ideal) structure of A is reflected in its semisimplicity as a left $A \otimes_Z A^{op}$-module, with the action $(\sum a_i \otimes b_j)(a) = \sum a_i a b_j$.

Setting $A = R_1 \times \cdots \times R_n = R$, with the R_i all simple, R_SG as an $R \otimes R^{op}$-module, is a direct sum of the simple $R \otimes R^{op}$-modules, R_ig, g varying over G, so R_SG is semisimple as an $R \otimes R^{op}$-module.

It suffices to show that every two-sided ideal I of R_SG is a retract of R_SG, as a bimodule, equivalently as an $R_SG \otimes (R_SG)^{op}$-bimodule.

Since R_SG is $R \otimes R^{op}$-semisimple, and I is a submodule, there is an $R \otimes R^{op}$-linear (i.e., an R-bimodule homomorphism) retraction $v: R_SG \to I$. Define the two-sided analogue of the usual averaging process,

$$V: R_SG \longrightarrow I$$

by setting

$$V(x) = \frac{1}{|G|^2} \sum_{g,h \in G} gv(g^{-1}xh^{-1})h .$$

One routinely checks that V is an R_SG-bimodule homomorphism, fixing I pointwise.

For A any ring, $Z(A)$ will denote its center.

COROLLARY 2. *Let R be a biregular self-injective ring, and G a finite group acting as automorphisms, with the order of G invertible in R. Then R_SG is also biregular and self-injective.*

REMARKS. "Self-injective" means right-injective unless otherwise specified. The corresponding result with self-injectivity deleted from the hypothesis and conclusion is false.

Proof. It is well-known that R_SG is self-injective (and regular), so by [13, Proposition 1.6], it suffices to show that every prime ideal of R_SG is maximal.

Let P be a prime ideal of R_SG, and observe that $(Z(P))^G/(P \cap (Z(R))^G)$ embeds naturally in the center of R_SG/P, so must be a field. Since $Z(R)$ is a commutative biregular ring and $Z(R)/(P \cap Z(R))$ is finitely generated over that field, $(P \cap (Z(R))^G)Z(R)$ is a finite intersection of maximal ideals of $Z(R)$. Since this ideal is contained in $Z(R) \cap P$, the latter is also such an intersection. From the biregularity of R, we deduce that $(Z(R) \cap P)R$ is a finite intersection of prime, hence maximal, ideals of R. As this ideal is contained in $R \cap P$, the latter is also a finite intersection.

Clearly, $R \cap P$ is a G-invariant ideal of R, and we may thus form the skew group ring $(R/(R \cap P))_sG$; there is a natural mapping of rings from this onto $(R_sG)/P$. By Theorem 1, the former is a finite product of simple rings, so the latter being prime, must be simple.

The following lemma is a standard result from Morita theory, and is a special case of [2; I, 4.1.3].

LEMMA 3. *Let A be a ring, and let P be a finitely generated projective A-module that is a generator for* Mod-A. *Set $B = \mathrm{End}_A P$. Then P_B is a finitely generated projective module.*

THEOREM 4. *Let R be a ring, and G a finite group of automorphisms of R, with the order of G invertible in R. If either* (1) *or* (2) *below hold,*

(1) *R is a finite product of simple rings*

(2) *R is a biregular, self-injective ring*

then R is a finitely generated projective R^G-module.

Proof. As in [4], consider the R_sG-R^G bimodule, R. As a left R_sG-module, R is projective and isomorphic to the principal left ideal R_sGe, where $e = |G|^{-1}\sum g$. Since R_sG is biregular (by the first two results), there exists a central idempotent F such that the ideal $Q = F \cdot R_sG$ is the left annihilator of R_sGe, hence of the left module R. So R is an (R_sG/Q)-R^G bimodule in a natural way; it is faithful, projective, and finitely generated over R_sG/Q.

Any finitely generated faithful projective module P over a biregular ring S is a generator: There exists an integer n so that $P \simeq eS^n$ (for some $e = e^2$ in M_nS); since M_nS is biregular, M_nSeM_nS is generated by a central idempotent E, but P will not be faithful if E is not the identity; hence $M_nSeM_nS = M_nS$, so $eS^n \simeq P$ is a generator.

Thus R as a left R_sG/Q-module is a generator, so if $E = \mathrm{End}_{R_sG/Q}R$, R_E is finitely generated projective. However, there is an isomorphism of E with R^G so that the action of E on R is translated to the usual action of P^G on R. Thus R is finitely generated projective as an R^G-module.

COROLLARY 5 [9]. *If R is a finite product of simple rings, and G is a finite group of automorphisms of R with the order of G invertible in R, then R^G is also a finite product of simple rings.*

Proof. As is implicit in the proof of Theorem 4, $R^G \simeq eR_sGe$ (same e), and Theorem 1 applies.

The theorem of Farkas and Snider [3; Theorem 1] asserts that R is a finitely generated R^G-module if R is semisimple artinian (and the usual condition on the order of G). This is of course a special case of Theorem 4, but is easier to prove as it only requires proving Theorem 1 for the special case, R semisimple artinian (which is just Maschke's theorem).

II. Self-injective regular rings. We may now complete the results of Renault [14] on the relation between the type classifications of R and R^G, when R is self-injective. Specifically, we show that R is type II (respectively type II_∞) if and only if R^G is, and since the corresponding result is known for type I_f (and type I_∞), it also holds for type III. For a review of the type classification for self-injective regular rings, see [5; § 7].[2]

PROPOSITION 6. *Let R be a regular self-injective ring, dnd G a finite group of automorphisms of R, with the order of G invertible in R. Then*

(1) *R is of type* II (*respectively* II_f)
if and only if
(2) *R^G is of type* II (*respectively type* II_f).

Proof. According to [14; Corollary 10], (1) implies (2). So it suffices to show (2) implies (1).

Let e be any finite idempotent in R^G; then $(eRe)^G = eR^Ge$, and we shall show e is finite as an idempotent in R.

Let M be a maximal two-sided ideal of $S = eRe$, and set $P = \cap M^g$. Now S/P is a finite prduct of simple rings, so by Theorem 4, S/P is finitely generated as a module over $(S/P)^G$, which equals

[2] A comprehensive treatment occurs in the very recently published von Neumann Regular Rings, by K. R. Goodearl, published by Pitman (1979).

$S^G/(P \cap S^G)$, and being projective as well, S/P embeds in a corner of a matrix ring over $S^G/(P \cap S^G)$. Since quotients, matrix rings, and corners of directly finite self-injective rings are also directly finite, S/P is directly finite. Since S/M is one of the simple ring direct summands of S/P, S/M is also thus directly finite.

So for all maximal two-sided ideals M of S, S/M is directly finite; it is easily checked that for self-injective rings, this implies S is directly finite (outline of proof: if not, there is a central idempotent E such that $T = ES$ satisfies, $T \oplus T \simeq T$ as T-modules; this property is inherited by all homomorphic images of T). Thus the idempotent e is finite in R.

If R^G were of type II_f, we may set $e = 1$, so that $R = S$ is of finite type, and it is easy to check that R^G having no artinian images implies the same for R; thus R is of type II_f. If R is merely of type II, there exists a faithful finite idempotent e in R^G, and to show R is of type II, it suffices to show that e is faithful in R.

If not, the right (and left) annihilator of ReR is ER for some central idempotent E. Since ReR is G-invariant, so is ER; since E^g must also be central, it follows that $E^g = E$, so E belongs to R^G. As e is a faithful idempotent in R^G, $E = 0$.

We can also show that for self-injective rings R, R^G is biregular if and only if R is. We first require a slight extension of [13; Proposition 1.6].

PROPOSITION 7. *A self-injective regular ring all of whose primitive images are simple, is biregular.*

Proof. Let M be a maximal ideal of the center, $Z(R)$ of R. The quotient ring $T = R/MR$ has its two-sided ideals totally ordered [12; Prop. 2.9]. Let N be a two-sided ideal of R properly containing MR; since T is regular, there is a maximal right ideal Q of R containing MR but not N. Inside Q is a primitive ideal containing MR, but not N; as Q must be maximal and the ideals of T are totally ordered, this is a contradiction. Hence, no such N exists, so MR is a maximal two-sided ideal. Thus all maximal ideals of R are of the form MR, so R is biregular [13; Prop. 1.1].

THEOREM 8. *Let R be regular and self-injective. Suppose G is a finite group of automorphisms of R, and the order of G is invertible in R. Then*

(1) *R is biregular*
if and only if
(2) *R^G is biregular.*

Proof. Since $R^G \simeq eR_sGe$ (as in the proofs of Theorem 4 and Corollary 5), (1) implies (2).

Assume (2) holds. Let P be a primitive ideal; by the preceding result, it suffices to show P is maximal. Form $Q = \cap P^g$, and the quotient ring $T = R/Q$. As Q is G-invariant, we may form T_sG, which is also, in a natural may, an image of R_sG.

Since R_sG is self-injective and regular, it satisfies general comparability: [5; Theorem 3.3].[2]

(*) For all idempotents e, f there exists a central idempotent E such that the right ideal generated by eE is subisomorphic to that generated by fE, and the right ideal generated by $f(1-E)$ is subisomorphic to that generated by $e(1-E)$.
Since subisomorphisms between idempotent-generated principal right ideals are equationally determined in a regular ring, (*) is inherited by all homomorphic images of R_sG; in particular T_sG satisfies (*).

Now consider the center of T_sG: routine computations (as in [7; 1.6 proof (1), (2)] — observe that since T_sG is regular, nonzero divisors are invertible), show that the center is contained in a finitely generated module over the center of T. However, T is a finite product of prime regular rings (T is a subdirect product of finitely many prime rings, but satisfies (*)), so $Z(T)$ is a finite product of fields; it easily follows that $Z(T_sG)$ is artinian. As T_sG is a regular ring satisfying (*), it follows that T_sG is also a finite product of prime rings.

As $T^G \simeq eT_sGe$, T^G is a finite product of prime rings. Since T^G is a homomorphic image of a biregular ring (R^G), T^G is thus a finite product of simple rings.

Thus T^G is a finite product of simple rings and T is a finite product of prime rings; by [4; 4.3], $R/Q = T$ is a finite product of simple rings, whence R/P is simple.

We note that in the course of the above proof, we have shown:

COROLLARY 9. *If R is a finite product of prime regular self-injective rings and G is a finite group of automorphisms with the order of G invertible in R, then both R^G, R_sG are finite products of prime regular self-injective rings.*

The main idea involved in the proofs of Corollary 2 and Theorem 8 is that prime ideals are maximal in R_sG if they are so in R. Now that the results of [11; 1.4] are available, shorter proofs can be given.

The following result, interesting in itself, is useful for determining the connection between the projectivity and the finite generation of R as an R^G-module, which we shall now be exploring.

THEOREM 10. *Let R be a right self-injective regular ring, and I a nonsingular injective right module. At least one of the following hold*:

(a) *I is finitely generated*;

(b) *There exists a strictly descending infinite sequence of central idempotents $F_1 > F_2 > \cdots$ such that*

$$\bigoplus F_i R \text{ is subisomorphic to } I\,;$$

(c) *There exists a nonzero central idempotent E such that $\aleph_0$ copies of the module ER is subisomorphic to I.*

REMARK. We adopt the notation nM or $n(M)$ to indicate a direct sum of n copies of M, when M is a module and n a positive integer or $\aleph_0$.

Proof. We repeatedly use general comparability, that is, for J a nonsingular injective over R, there exists a central idempotent E with $JE \lesssim ER$ and $(1-E)R \lesssim J(1-E)$ (all as right modules) [5; Theorem 3.3][2].

Assume neither (a) nor (c) hold. There is a central idempotent E_1 of R with $IE_1 \lesssim E_1R$ and $(1-E_1)R \lesssim I(1-E_1)$. As (a) fails, E_1 does not equal 1, and from the negation of (c), there exists a positive integer n_1 such that $n_1[(1-E_1)R] \lesssim I(1-E_1)$, but $(n_1+1)[(1-E_1)R] \not\lesssim I(1-E_1)R$. Since all the modules dealt with are injective, all these subisomorphisms split, so there exists an injective submodule K_1 with $I(1-E_1) \simeq K_1 \oplus n_1[(1-E_1)R]$.

Set $F_1 = 1 - E_1$, and view K_1 as a module over F_1R. There exists a central idempotent $E_2 < F_1$ such that $K_1E_2 \lesssim E_2R$ and $(F_1 - E_2)R \lesssim K_1(F - E_2)$. Now F_1R is not subisomorphic to K_1, so E_2 is not zero. On the other hand, there exists, by the negation of (c) a positive integer n_2 with $n_2(F_1 - E_2)R \lesssim K_1(F_1 - E_2)$ but no larger number of copies can be embedded in $K_1(F_1 - E_2)$. Write $K_1(F_1 - E_2) \simeq n_2(F - E_2)R \oplus K_2$ for some K_2. Set $F_2 = F_1 - E_2$; this process can obviously be continued inductively, and we obtain $\bigoplus n_iF_iR \lesssim I$, and the F_i are strictly descending. This verifies (b).

THEOREM 11. *Let R be a right self-injective regular ring, and I a projective injective right R-module. Then there is a decomposition*

$$I = J \oplus K$$

where J is finitely generated, $K = \mathrm{Socle}\,(K)$, and there is a central *idempotent E with $KE = K$, and ER is artinian.*

(For this proof only, we distinguish between the internal direct sum ($\oplus$), and the external direct sum ($\amalg$).)

Proof. We repeatedly employ the following idea: If $\{e_iR\}$ is an infinite collection of nonzero principal right ideals, and $\amalg e_iR \lesssim R$, then $\amalg e_iR$ cannot be injective. For, the image of $\amalg e_iR$ is an internal direct sum $\bigoplus f_iR \subseteq R$; being injective, it must be a direct summand of R, and hence would be principal; but this is impossible since the generator would have to appear in a finite direct summand.

Since I is projective, it (and all of its submodules) is nonsingular. Let K_1 be the injective hull of the socle of I; there is a direct summand J_1 so that $J_1 \oplus K_1 = I$, and of course $\mathrm{Socle}(J_1) = \{0\}$. We proceed to show that J_1 must be finitely generated.

Being projective (and P being regular), J_1 is isomorphic to a direct sum of principal right ideals of R, say $J \simeq \amalg e_iR$, where $e_i = e_i^2$ belong to R. Either 10(b) or 10(c) holds, and we show either leads to a contradiction, unless the index set is finite.

If 10(c) holds, we may find inside the index set infinitely many disjoint finite subsets $\{S_j\}$ such that $ER \lesssim \amalg_{i \in S_j} e_iR$ for some idempotent E, for all j. By passing to a direct summand of J_1, and multiplying by the central idempotent E, we may assume $E = 1$, and $\cup S_j$ is the entire index set, so that for all j,

$$R \lesssim \coprod_{i \in S_j} e_iR .$$

Since J_1 is a faithful ER-module and $\mathrm{Socle}(J_1) = \{0\}$, ER has zero socle, and in particular, is not artinian. So we may find an infinite orthogonal set of idempotents $\{f_j\}$ in R, in bijection with the set of S_j's. For each j, there is an i in S_j and a nonzero idempotent g_j in e_iR with $g_jR \lesssim f_jR$. There thus exists an idempotent h_j in f_jR with $g_jR \simeq h_jR$. As $\amalg g_jR$ is a direct summand of J_1, a contradiction arises unless J_1 is finitely generated.

Now suppose 10(b) holds. We may find infinitely many finite disjoint subsets $\{S_j\}$ of the index set, and an infinite sequence of descending central idempotents $\{F_j\}$ such that for all j, $F_jR \lesssim \amalg_{i \in S_j} e_iR$. Then $(F_j - F_{j+1})R \lesssim \amalg_{i \in S_j} e_iR$; set $E_j = F_j - F_{j+1}$, note that the E_j are orthogonal, and $\amalg E_jR$ is isomorphic to a direct summand of J_1, so is injective, and again the first paragraph applies to yield a contradiction. Hence J_1 must be finitely generated.

Write $K_1 = \amalg_{i \in T} h_iR$, $h_i = h_i^2 \in R$. Then for each i, $\mathrm{Socle}(h_iR)$ is essential in h_iR. Let S be a set of representatives of isomorphism classes of minimal right ideals of R that appear in $\mathrm{Socle}(K_1)$. Suppose there is an infinite subset T_0 of T, an infinite subset S_0 of S, and a bijection $f: T_0 \to S_0$ so that for t in T_0, $f(t) = g_tR \lesssim h_tR$. Then from the minimality of each of the g_tR (and their mutual nonisomorphism), $\amalg g_tR \cong \bigoplus g_tR \subseteq R$; but $\amalg g_tR$ is isomorphic to a direct summand of K_1 (each g_tR is a direct summand of h_tR), and again a contradiction occurs.

In particular, we deduce that for at most finitely many t in T can h_tR contain infinitely many elements of S. By absorbing these into the finitely generated J_1, we may assume all of the h_tR contain only finitely many members of S. Knowing this, we deduce from the previous paragraph that S must be finite, say $S = \{g_1R, g_2R, \cdots, g_nR\}$. Then if $E(\ )$ denotes injective hull, $K_1 = E(\bigoplus^n N_ig_iR) \cong \amalg E(N_ig_iR)$, N_i finite or infinite cardinals. Whichever of the $E(N_ig_iR)$ are finitely generated can be absorbed into the finitely generated part, and we are reduced to the situation, $K \simeq \amalg E(N_ig_iR)$, where each of the $E(N_ig_iR)$ is not finitely generated, and the g_iR are mutually nonisomorphic.

Let F_i be the central cover of g_iR. Then $E(N_ig_iR)$ is a projective injective module over $T_i = F_iR$. Since g_iR is a faithful irreducible T_i-module, T_i is primitive, and so 10(b) cannot apply to $E(N_ig_iR)$; as $E(N_ig_iR)$ is not a finitely generated R-module, it cannot be finitely generated as a T_i-module; thus 10(c) must apply. If T_i contained an infinite orthogonal set of idempotents, we could apply the process applied to J_1 to reach a contradictoin — thus T_i must be artinian, and so $E = \sum E_i$ generates an artinian corner of R. Since ER is artinian, K is completely reducible as an ER; hence as an R-module.

Now if R is regular self-injective, and G is a finite group of automorphisms of R with the usual order condition, then R is injective as a right R^G-module. If R were projective as well, modulo artinian direct summands (which can be dealt with separately), it would have to be finitely generated. On the other hand, if R were finitely generated as an R^G-module, the argument of the following lemma shows R must also be projective.

LEMMA 12. *Suppose A is a right self-injective regular ring, and P is a faithful finitely generated projective left A-module. Set $B = \mathrm{End}_A P$, and suppose P_B is finitely generated. Then ${}_AP$ is a projective generator for A-*Mod.

Proof. It is routine to check that P_B is nonsingular, and also that B is itself right self-injective, so being finitely generated, P_B must be projective ([5; Theorem I. 16]). There thus exists a split onto map of right B-modules,

$$nB_B \longrightarrow P_B \,.$$

Applying the contravariant functor, $\mathrm{Hom}\,(-, {}_AP_B)$, we obtain a split embedding, $n({}_AP) \leftarrow {}_A\mathrm{End}\,(P_B)$.

Now the natural map $A \to \mathrm{End}\,(P_B)$, $a \mapsto \hat{a}$, $\hat{a}(p) = ap$, has kernel the annihilator of ${}_AP$, and so is an embedding. Thus ${}_AA$ embeds in $n({}_AP)$. Since ${}_AP$, ${}_AA$ are projective modules over a regular ring, ${}_AA$ is a direct summand of $n({}_AP)$, and so ${}_AP$ is a generator.

Lorenz and Passman have given examples of type I_f self-injective regular rings with G of order 2 (and 1/2 belongs to R) such that R is not finitely generated over R^G. We now present complementary examples, with R prime (and necessarily not simple, by Theorem 4).

EXAMPLE. R prime regular self-injective, not simple, $G = \{1, g\}$ a group of automorphisms of order 2, with R neither finitely generated nor projective over R^G.

Take any prime, nonsimple self-injective regular ring with 2 invertible (examples of type I_∞ and III exist in profusion; examples of type II_∞ also exist, but require some subtlety to construct). Let M be the (unique, proper) maximal two-sided ideal of R, and pick a nonzero idempotent e in M.

Let g be the inner automorphism defined as conjugation by $1 - 2e$; so g^2 is the identity, and $R^G = eRe \times (1 - e)R(1 - e)$. If R were R^G-finitely generated on the right, multiplication by e yields that Re is finitely generated as a right eRe-module. Now Re is a faithful projective left R-module, so by Lemma 12, ${}_RRe$ would have to be a generator, and of course this implies $ReR = R$; but ReR is contained in M, a contradiction.

If R were R^G-projective, it would have to be finitely generated by Theorem 11.

The automorphism in the example was inner. Not surprisingly, when G consists of outer automorphisms, and R is prime regular self-injective, R is finitely generated over R^G. Here, 'outer' means not conjugation by an invertible element (the usual definition, as opposed to the ersatz definitions).

THEOREM 14. *Let R be a prime regular self-injective ring, and suppose G is a finite group of outer automorphisms of R, with the order of G invertible in R. Then R is finitely generated and projective as a right R^G-module, and R^G is a prime regular right self-injective ring.*

Proof. We observe that R_sG is self-injective, regular, and computing the center as in [7; 1.6 proof of (2)] (noting that in a prime ring $rR = Rr \neq \{0\}$ implies r is not a zero-divisor, and non-zero divisors in regular rings are invertible), we find the center is a field, $Z(R)^G$. Since R_sG satisfies central comparability, it must be prime (also a special case of [15; 2.6 (ii)]); as R^G is isomorphic to a corner of R_sG, R^G is also prime (and regular, self-injective).

Now let M be the unique maximal two-sided ideal of R. Since M is the only maximal two-sided ideal, it must be G-invariant, and thus MR_sG is a two-sided ideal of R_sG. The natural isomorphism, $R_sG/MR_sG \simeq (R/M)_sG$ carries a prime ring (the ideals of R_sG are totally ordered, so all images are prime) to a finite product of simple rings (Theorem 1), so both are simple.

The idempotent $e = |G|^{-1}\sum g$ has nonzero image in the simple ring R_sG/MR_sG, and as MR_sG must be the unique maximal ideal of R_sG, it follows that $R_sGeR_sG = R_sG$. Thus R_sG is Morita equivalent to R^G via the bimodule R_sGe, and after translating R_sGe to R, we can apply Lemma 3, as in the proof of Theorem 4.

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