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RAMSEY QUANTIFIERS AND THE FINITE COVER PROPERTY

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The main results of this paper are the following Theorem A. The Magidor-Malitz quantifier (in the $\aleph_0$-interpretation) is eliminable from a countable stable theory T if and only if T does not have the finite cover property. Theorem B. There is an $\aleph_0$-categorical theory which is not finitely model complete.

We consider various extensions of a countable first-order logic obtained by adding n-ary quantifiers which assert the existence of of a "large" set of n-tuples satisfying the formula following the quantifier. Specifically, the Ramsey (or Magidor-Malitz) quantifier Q^n is defined in the $\aleph_0$-interpretation by $A \models_0 Q^n x_1, \cdots, x_n \varphi(x_1, \cdots, x_n, \bar{a})$ iff there is an infinite $Y \subset A$ which is homogeneous for φ, i.e., such that $A \models_0 \varphi(y_1, \cdots, y_n, \bar{a})$ for all $\langle y_1, \cdots, y_n \rangle \in Y^n$. We denote by L^* the language which adjoins all the quantifiers Q^n and by L_0^* the associated logic in the $\aleph_0$-interpretation. Our principal concern is to identify those complete first order theories T in L which remain complete as theories of L_0^*. A sufficient condition on T is

E_0: For every formula φ in L_0^*, there is a formula ψ in L such that $T \models_0 \varphi \leftrightarrow \psi$ (i.e., if $A \models_0 T$ then $A \models_0 \varphi \leftrightarrow \psi$).

If E_0 holds, we say Q is eliminable in T.

Earlier Vinner [16] has shown that for an $\aleph_1$-categorical theory, the quantifier "there exists infinitely many" is eliminable. Winkler [17] showed the eliminability of the quantifier "there are infinitely many sequences" in an $\aleph_1$-categorical theory. It is easy to see that either of these is eliminable in an $\aleph_0$-categorical theory. Cowles [4] showed that the Ramsey quantifier in the $\aleph_0$-interpretation was eliminable from the theory of algebraically closed fields. Keisler [8] introduced the notion of the finite cover property (f.c.p.) and showed that every $\aleph_1$-categorical theory fails to have the f.c.p. We generalize these elimination results by showing that if T does not have the f.c.p. then all the above quantifiers are eliminable from T. Moreover, we characterize those stable theories T which do not have the f.c.p. as exactly those which satisfy E_0. We also show that if T is $\aleph_1$-categorical then T admits elimination of these quantifiers in the equi-cardinal interpretation (again generalizing Cowles [4]).

It is natural to ask whether there is some sort of "first order property" of T which is equivalent to E_0. We show several candidates for such a condition do not work. One of the examples adduced for this purpose answers a question of Paillet [10]; it is $\aleph_0$-categorical but not finitely model complete.

We first observe E_0 is equivalent to a somewhat more concrete condition. For any formula $\varphi(x_1, \cdots, x_m, \bar{v})$ of L there is a first order formula $H_x^m\bar{x}\varphi(x, v)$ which holds iff there is a set of cardinality $\geqq n$ which is homogeneous for φ. (We will omit the superscript m in the future.) Now the concrete condition is

A_0: For every L-formula $\varphi(\bar{x}, \bar{v})$ there is an n such that $T \models_0 \forall \bar{v}(H_n\bar{x}\varphi(\bar{x}, \bar{v}) \rightarrow Q\bar{x}\varphi(\bar{x}, \bar{v}))$.

THEOREM 1. $E_0 \Leftrightarrow A_0$.

Proof. A_0 implies E_0 by induction on formulas of L_0^*. For the converse suppose E_0 but that A_0 fails for the formula $\varphi(\bar{x}, \bar{v})$. By E_0 there is a $\psi(\bar{v})$ of L such that $T \models_0 Q\bar{x}\varphi(\bar{x}, \bar{v}) \Leftrightarrow \psi(\bar{v})$. Add to L a unary predicate symbol U and constants $\bar{c}$. Since A_0 fails, by compactness, the following set Γ of sentences is consistent:

$$T \cup \{\forall x_1, \cdots, \forall x_n[\bigwedge_1^n U(x_i) \longrightarrow \varphi(\bar{x}, \bar{c})], \sim \psi(\bar{c})\} \cup \{\exists^{\geqq n} x U(x) \colon n \in \omega\} .$$

But any model of Γ contradicts E_0.

The next result was observed independently by Baudisch [2].

THEOREM 2. *If T is $\aleph_0$-categorical then E_0 holds of T.*

Proof. We show A_0 holds. For any formula $\varphi(\bar{x}, \bar{v})$, by the Ryll-Nardjewski theorem only finitely many of the formulas $H_n\bar{x}\varphi(\bar{x}, \bar{v})$ can be inequivalent. Hence for some N, $T \models \forall x(H_N\bar{x}\varphi(\bar{x}, \bar{v}) \rightarrow H_k\bar{x}\varphi(\bar{x}, \bar{v}))$ for all $k \geqq N$. If $T \not\models_0 H_N\bar{x}\varphi(\bar{x}, \bar{v}) \rightarrow Q\bar{x}\varphi(\bar{x}, \bar{v})$ then for some $\bar{a} \in A$. the countable model of T, $A \models_0 H_N\bar{x}\varphi(\bar{x}, \bar{a}) \wedge \sim Q\bar{x}\varphi(\bar{x}, \bar{a})$. Let $\psi(\bar{v})$ generate the principal type realized by $\bar{a}$. Then by a compactness argument as in Theorem 1, and $\aleph_0$-categoricity of T, there is a $\bar{b}$ in A such that $A \models_0 \psi(\bar{b}) \wedge Q\bar{x}\varphi(\bar{x}, \bar{b})$. But this contradicts the homogeneity of A.

We cannot improve this by assuming that T has only finitely many countable models since the Ehrenfeucht example T of a theory with 3 countable models (in its finite language version [15]) does not satisfy E_0. This example is connected to our later results since the archetypal example of an ω-stable theory with the f.c.p. (an equivalence relation with one class of each finite cardinality) is interpretable in T.

We now want to show that if T does not have the f.c.p. then T

satisfies E_0. Before dealing directly with the f.c.p. we introduce an intermediate condition M on the existence of maximal homogeneous sets. Note that for any formula $\varphi(\bar{x}, \bar{v})$ there is a first order formula $\bar{H}_n\bar{x}\varphi(\bar{x}, \bar{v})$ which is true of $\bar{a}$ just if there is a set X of cardinality n which is homogeneous for $\varphi(\bar{x}, \bar{a})$ but no superset of X is homogeneous for $\varphi(\bar{x}, \bar{a})$.

> M: For every L-formula $\varphi(\bar{x}, \bar{v})$ there is an N such that if $m > N$, $T \models \forall \bar{v} \sim \bar{H}_m\bar{x}\varphi(\bar{x}, \bar{v})$.

LEMMA 3. *M implies E_0.*

Proof. If E_0 (i.e., A_0) is false then for every n there exists a model A_n and a sequence $\bar{a}_n$ such that $A_n \models_0 H_n\bar{x}\varphi(\bar{x}, \bar{a}_n) \wedge \sim Q\bar{x}\varphi(\bar{x}, \bar{a}_n)$. Then for some m, $n < m < \omega$ $A_n \models_0 \bar{H}_m\bar{x}\varphi(\bar{x}, \bar{a}_n)$ so M fails.

Surprisingly, as we will show later, the converse to this lemma is false.

DEFINITION. A formula $\varphi(\bar{x}, \bar{y})$ has the finite cover property in T if in some model of T for arbitrarily large n there exist $a_0, \cdots, a_{n-1}$ such that

$$A \models \bigwedge_{j<n} \Big(\exists \bar{x} \bigwedge_{i \neq j} \varphi(\bar{x}, \bar{a}_i)\Big) \wedge \sim \exists \bar{x} \bigwedge_{i<n} \varphi(\bar{x}, {}_i\bar{a}) .$$

The theory T has the finite cover property if some formula $\varphi(\bar{x}, \bar{y})$ has the finite cover property in T.

LEMMA 4. *If T does not have the* f.c.p. *then T satisfies M.*

Proof. If M fails then for arbitrarily large n, say $n \in J$, there exist $A \models T$ and $\bar{a}_n$ in A such that for a fixed formula $\varphi(x_0, \cdots, x_{m-1}, \bar{v})$, $A \models \bar{H}_n\bar{x}\varphi(\bar{x}, \bar{a}_n)$. By adding a dummy variable if necessary we may assume $m \geqq 2$. We may assume also that φ has the following property: if $x_i^* \in \{x_0, \cdots, x_{m-1}\}$ for all $i < m$, then $\models \varphi(\bar{x}, \bar{v}) \rightarrow \varphi(\bar{x}^*, \bar{v})$. We will show the formula $\psi(x_0, \bar{u}) = \varphi(x_0, x_1, \cdots, x_{m-1}, \bar{v}) \wedge x_0 \neq x_1$ has the f.c.p. Let for each $n \in J$, $H_n = \{c_0, \cdots, c_{n-1}\}$ be a maximal homogeneous set for $\varphi(\bar{x}, \bar{a}_n)$ and let $Z_n = \{\bar{s}_0, \cdots, \bar{s}_{k-1}\}$ enumerate the $m-1$ tuples from H_n. Set, for $i < k$, $\bar{b}_i = \bar{s}_i \cap \bar{a}_n$. For any $Y \subset k$ we write $P(Y)$ if $A \models \exists x_0 \bigwedge_{j \in Y} \psi(x_0, \bar{b}_j)$. Then $P(k)$ holds since $A \models \forall x_0[\bigwedge_{j<k} \varphi(x_0, \bar{b}_j) \rightarrow \bigvee_{l<n} x_0 = c_l]$.

Let $V \subset k$ be a minimal set such that $P(V)$ holds. Then V has at least n elements in it, since if $Y \subset k$ has fewer than n elements then $A \models \exists x_0 \bigwedge_{j \in Y} \psi(x_0, \bar{b}_j)$ (simply choose $x_0 \in H_n$ but different from the first term of $\bar{b}_j$ for all $j \in Y$). So listing $V = \{\bar{d}_0, \cdots, \bar{d}_l\}$ where

$l \geqq n-1$ we see that $A \models \neg \exists x_0 \bigwedge_{i<l} \psi(x_0, \bar{d}_i)$ since $P(V)$ holds, but $A \models \bigwedge_{j<l} \exists x_0 \bigwedge_{i \neq j} \psi(x_0, \bar{d}_i)$ by the minimality of V. Thus $\psi(x_0, \bar{u})$ has the finite cover property.

We collect the preceeding results in:

THEOREM 5. (a) *If the complete theory T does not have* f.c.p. *then T satisfies E_0.*

(b) *In particular, if T is $\aleph_1$-categorical then T satisfies E_0.*

Proof. Part (a) follows immediately from the lemmas. For (b) we need only recall Keisler's theorem [8] that an $\aleph_1$-categorical theory does not have f.c.p.

Now we show that for stable theories E_0 exactly captures the notion of the f.c.p. We rely on: Theorem A (Shelah [13, II, §4.4]). If T is stable and has the f.c.p. then there is a formula $\varphi(x_1, x_2, \bar{v})$ such that for every $\bar{a}$, $\varphi(x_1, x_2, \bar{a})$ is an equivalence relation and for arbitrarily large n there exist $\bar{a}_n$ and k, $n \leqq k < \omega$, such that $\varphi(x_1, x_2, \bar{a}_n)$ has exactly k equivalence classes. (This result was obtained independently but later by G. Cooper.)

THEOREM 6. *If T is stable, the following properties of T are equivalent*: E_0, M, $\sim$f.c.p.

Proof. By the lemmas above, it suffices to show that if T has the f.c.p. then T does not have E_0. Choose $\varphi(x_1, x_2, \bar{v})$ to satify Theorem A, and consider

$$\varphi'(x_1, x_2, \bar{v}) \colon x_1 \neq x_2 \longrightarrow \sim \varphi(x_1, x_2, \bar{v}) .$$

If E_0 holds, for some n:

$$T \models_0 H_n x_1 x_2 \varphi'(x_1, x_2, \bar{v}) \longrightarrow Q x_1, x_2 \varphi'(x_1, x_2, \bar{v}) .$$

But this contradicts the conclusion of Theorem A.

Note that in Theorem 6 we are able to apply E_0 to a formula involving Q^2. Thus for stable theories the eliminability of the Q^2 quantifier in the $\aleph_0$-interpretation implies the eliminability of all the Q^n. In contrast the language $L(Q^{n+1})$ is strictly stronger than the language $L(Q^n)$ (due in the $\aleph_0$-interpretation to Shelah [12] and in the $\aleph_\alpha$-interpretation for $\alpha > 0$ to Shelah [12], Garavaglia [6] and (assuming $\diamondsuit$) to Baudisch [2]). Presumably, some hypothesis on the theory T is necessary since Cowles [5] pointed out the theory of real closed field eliminates the quantifier "there exists infinitely many", but does not satisfy E_0.

We now consider the extent that these results apply to other formalizations of the notion, "for many sequences $\bar{x}$, $\varphi(\bar{x})$ holds."

Sticking first to the $\aleph_0$-interpretation we introduce the Ramsey quantifier on sequences.

DEFINITION. Let $\bar{x}_1, \cdots, \bar{x}_m$ be an m-tuple of n-tuples. Form the logic $L(Q_0^*)$ where $A \models_0 Q^{*,m,n}\bar{x}_1, \cdots, \bar{x}_m\ \varphi(\bar{x}_1, \cdots, \bar{x}_m)$ just if there is an infinite set Y of n-tuples from A such that $\bar{y}_1, \cdots, \bar{y}_m \in Y$ implies $A \models_0 \varphi(\bar{y}_1, \cdots, \bar{y}_m)$.

By applying our earlier arguments to sequences rather than elements one obtains the following.

THEOREM 7. (i) *If T is $\aleph_0$-categorical or does not have the* f.c.p. *then T admits elimination of quantifiers in $L(Q_0^*)$.*

(ii) *If T is stable then T admits elimination of quantifiers in $L(Q_0^*)$ if and only if T does not have the* f.c.p.

The quantifiers Q_0^* generalize two notions in the literature other than the Ramsey quantifiers.

DEFINITION. (i) $A \models_0 I^m x_1, \cdots, x_m \varphi(x_1, \cdots, x_m)$ if there are infinitely many pairwise distinct sequences $\langle \bar{a}_i : i < \omega \rangle$ such that $A \models_0 \varphi(\bar{x}_i)$.

(ii) $A \models_0 I^{*,m} x_1, \cdots, x_m \varphi(x_1, \cdots, x_m)$ if there exist infinitely many m-ary sequences $\langle \bar{a}_i : i < \omega \rangle$ such that $A \models_0 \varphi(\bar{a}_i)$ and if $i \neq j$ then $\operatorname{rng} \bar{a}_i \cap \operatorname{rng} \bar{a}_j = \emptyset$.

Clearly I^m is just the quantifier $Q^{*,m,1}$, while $I^{*,m}\varphi(x)$ is equivalent to

$$Q^{*,2,m}\bar{x}_1, \bar{x}_2\left[\varphi(\bar{x}_1) \wedge \varphi(\bar{x}_2) \wedge \left[\bigwedge_{j<m} x_1^j = x_2^j \vee \bigwedge_{i,j<m} x_1^i \neq x_2^j\right]\right]$$

where $\bar{x}_1 = \langle x_1^0, \cdots, x_1^{m-1} \rangle$ and $\bar{x}_2 = \langle x_2^0, \cdots, x_2^{m-1} \rangle$.

Schmerl [11] considers a variant of I^* and remarks that it is eliminable in a theory which is $\aleph_0$-categorical. Winkler [17] proves that I^* is eliminable in any theory which is either $\aleph_0$ or $\aleph_1$-categorical. All of these results follow from Theorem 7.

One sense in which I and I^* are weaker than the Ramsey quantifier is that there exists a stable theory T in which both I and I^* are eliminable but T does not have the f.c.p.

For this, consider a language with infinitely many constant symbols c_k and one ternary relation symbol $E(x, y, z)$. Partition an infinite set X into infinitely many infinite classes X_i for $i \in \omega$ and each X_i into $i + 1$ classes X_{ij} for $j \leq i$ with each X_{ij} infinite. Now let $E(a, b, c)$ hold just if for some i a, b, c are all in X_i and for some j both a and b are in X_{ij}. Let the constants name one member of

X_{ij}, for each i and j. It is easy to see that the formula $\varphi(x; y)$: $E(x, x, y) \wedge \sim E(x, y, y)$ has the finite cover property. On any saturated model of T the maximal quantifier-free types are first order complete so T is quantifier eliminable in L. Thus, to show that T admits elimination of the quantifier I^*, it suffices to find for any quantifier-free L-formula $\varphi(\bar{x}, \bar{y})$, an L-formula equivalent in T to $I^*\bar{x}\varphi(\bar{x}, \bar{y})$. Now if $A \models T$ and $A \models \exists \bar{x}\varphi(\bar{x}, \bar{a})$ then $A \models_0 I^*x\varphi(\bar{x}, \bar{a})$ unless $\varphi(\bar{x}, \bar{y})$ logically implies that some x_i is equal to some y_j or to constant c_k (necessarily occuring in φ). But then $I^*\bar{x}\varphi(\bar{x}, \bar{a})$ is equivalent to $\exists x_1(x_1 \neq x_1)$. The quantifier I is handled similarly, replacing "some x_i" by "each x_i".

Our results partially extend to the logic L_c^* where $A \models_c Qx\varphi(x)$ is interpreted as: there is a set X with $|X| = |A|$ which is homogeneous for φ. In this logic we assume A is infinite. In particular if we write A_c, E_c as the obvious analogs of A_0, E_0 we get by the same proof as Theorem 1. Theorem 1′: $E_c \Leftrightarrow A_c$.

We want to extend Theorem 5b to the equicardinality interpretation. The required lemma is

LEMMA 8. *If $\varphi(x)$ is an L-formula, A is $|A|^+$-universal and $A \models_0 Qx\varphi(x)$ then $A \models_c Qx\varphi(x)$.*

Proof. Since $A \models_0 Qx\varphi(x)$, by compactness and downward Lowenheim-Skolem there is a model B of T with $|B| = |A|$ and $B \models_c Qx\varphi(x)$. But since A is $|A|^+$ universal it follows that $A \models_c Qx\varphi(x)$.

THEOREM 9. *If T is $\aleph_1$-categorical then T satisfies E_c.*

Proof. By Theorem 5b, E_0 and hence A_0 holds of T; we show A_c holds of T. It suffices to show for any A T and L-formula φ that $A \models_0 Q\bar{x}\varphi(\bar{x}, \bar{a})$ implies $A \models_c Q\bar{x}\varphi(\bar{x}, \bar{a})$. But this is tautological if A is countable and follows immediately from $\aleph_1$-categoricity and Lemma 8 if A is uncountable.

Note that we did not prove that if A is saturated and φ is an L^*-formula then $A \models_0 \varphi$ implies $A \models_c \varphi$. In fact, that assertion is false as can be seen by examining the saturated model of cardinality $\aleph_1$ of the theory of an equivalence relation E with one equivalence class of size n for each finite n. The relevant formula is $Qx \sim QyE(x, y)$.

Is there some "first order property" of T which is equivalent to E_0? Clearly, $\sim$f.c.p. is not equivalent to E_0 in general since any $\aleph_0$-categorical unstable theory has the f.c.p. [14] but also has E_0 by Theorem 2. A more likely candidate is the condition M. The following example dashes this hope.

THEOREM 10. *There is an $\aleph_0$-categorical theory T_0 which does not satisfy M.*

Proof. Let L_0 be a language containing one binary relation R and for all $n < \omega$, $n+1$-ary relations P_n and Q_n. We let $\alpha_n(x_0, \cdots, x_n)$ denote the formula asserting all the x_i are distinct and $R(x_i, x_j)$ holds for all $i \leqq j \leqq n$. Let T_0 be axiomatized by the universal closure of

(i) $R(x, x) \wedge (R(z, y) \rightarrow R(y, z))$

(ii) $P_n(\bar{x}) \rightarrow [\alpha_n(\bar{x}) \wedge \forall y(\bigwedge_{i \leqq n} y \neq x_i \rightarrow \approx \bigwedge_{i \leqq n} R(y, x_i))]$

(iii) $Q_n(\bar{x}) \rightarrow \alpha_n(\bar{x})$

(iv) $\alpha_n(\bar{x}) \rightarrow (P_n(\bar{x}) \Leftrightarrow \sim Q_n(\bar{x}))$.

Now T_0 is a universal theory with the joint embedding and amalgamation properties (the union of any two models is a model), so T_0 has a countable ultrahomogeneous (i.e., homogeneous in the sense of Jonsson [7]) and universal model A. The symbols P_n and Q_n are trivial on sequences of less than n elements so for each n the number of nonisomorphic substructures of A with cardinality n is finite. Whence by ultrahomogeneity and the Svenonious characterization of $\aleph_0$-categoricity, $T = Th(A)$ is $\aleph_0$-categorical. Moreover T admits elimination of quantifiers because in the only countable model of T the quantifier free types are complete. Let A^* be the reduct of A to the language whose only relational symbol is R. The P_n and Q_n are definable from R in T. In particular, $T \models P_n(\bar{x}) \Leftrightarrow (\alpha_n(\bar{x}) \wedge \forall y[\bigwedge_{i \leqq n} y \neq x_i \rightarrow \bigvee_{i \leqq n} \sim R(y, x_i)])$. This follows from the universality of A. Thus $T^* = Th(A^*)$ is also $\aleph_0$-categorical. But it is easy to see that A^* contains arbitrarily large maximal finite homogeneous set for $R(x, y)$, namely sets $\{x_0, \cdots, x_n\}$ such that $A \models P_n(\bar{x})$. Hence T^* does not satisfy M.

A first order theory T is said to be finitely model complete if there is an extension of T by adding a finite number of definable predicates which is model complete. J. L. Paillet [10] asked whether every $\aleph_0$-categorical theory is finitely model complete.

THEOREM 11. *There is an $\aleph_0$-categorical theory which is not finitely model complete.*

Proof. The theory T^* defined in the previous theorem provides an example. Since T is quantifier eliminable (and a foritori, model complete), if any finite definitional extension of T^* is model complete, for some n, the theory T_n obtained by adding the symbols P_m, Q_m $m \leqq n$ and their definitions to T^* must be model complete. But no such T_n is model complete. Indeed, $P_{n+1}(\bar{x})$ is equivalent in T_n to a universal but not to an existential formula.

When the second author suggested the example for Theorem 9, the first author recalled its similarity (virtual identity, as it turns out) to the example of $2^{\aleph_0}$ $\aleph_0$-categorical theories due to Ash [1]. Our verification of the $\aleph_0$-categoricity of T is by Ash's method.

There are several further questions suggested by this work.

1. Find a "purely first order" property equivalent to E_0. One notion of "first order property" is suggested by Cooper in [3]. It may be too restrictive for our purposes. At the other extreme one can ask if E_0 is an absolute property of T.

2. For T a theory in a finite language, if T is L_0^*-complete must it have E_0? This seems more likely if, in addition, T is stable. Matt Kaufman has shown the assumption that L is finite is essential here.

3. Does the theory of differentially closed fields of characteristic p, $p \geqq 0$, satisfy E_0? Equivalently, does each such theory fail to satisfy the f.c.p.?

4. For T a theory in a finite language, if T admits E_c must T be $\aleph_1$-categorical? This is false if we don't assume the language is finite. Since E_c implies T has no two cardinal models the question reduces to, "if T, in a finite language, satisfies E_c must it be ω-stable?".

Added in Proof. (1) Recall that Shelah [14] has proved that every unstable theory has the f.c.p. Thus Theorem 6 could be rephrased as

THEOREM 6′. *T has* f.c.p. *iff T is stable and E_0 holds. We used the formulation in Theorem* 6 *because the goal in this paper is the characterisation of E_0.*

(2) The quantifier elimination given by our proofs is not effective (Theorem 2 and Lemma 4 are the relevant ones). In each case we know (either by Ryll-Nardzewski or the failure of f.c.p.) that a certain finite number exists for each formula $\psi(\bar{u})$, but we do not know how to compute it effectively. H. Kierstead and Jeff Remmel have exhibited a complete decidable theory in L which is not decidable in L^* even though it is ω-categorical.

(3) P. Tuschik and P. Rothmaler have shown question 4 has a negative answer. Subsequently we verified that the theory of an infinite binary tree with two successor functions is such an example

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