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ON GROUP ACTIONS WITH NONZERO FIXED POINTS

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Let G be a locally compact group that acts continuously by linear transformations on a locally convex space E and let K be a compact convex subset of E that is invariant under this action. In order to conclude that K has a nonzero fixed point, it is necessary that both G and K satisfy certain conditions. With these assumptions on K, it is shown that the existence of nonzero fixed points is equivalent to polynomial growth on G, provided G is connected or discrete, finitely generated and solvable.

A locally compact group G is said to have the fixed point property if whenever G acts continuously and linearly on a compact, convex subset K of a locally convex space E there is an element of K invariant under the action of G. A well known result states that the fixed point property is equivalent to amenability (see [1]). If zero is an element of K, then, by the linearity of the action on G, it will be a fixed point, and amenability of G does not gurantee the existence of any other fixed point. Of course, there may not be a nonzero fixed point. Consider, for example, $\boldsymbol{R}$ acting on $\boldsymbol{C}$ by multiplication by e^{it}. The unit disk is a compact, convex, invariant subset and 0 is the only fixed point. Thus, in order to conclude the existence of a nonzero fixed point for a group action, it is clearly necessary to restrict the nature of the action. One such restriction, that exclude the example just cited, is to require that some half-space be invariant under the action. This is the essence of the first condition in Definition 1. This restriction alone, however, is still not sufficient to imply the existence of a nonzero fixed point. We consider another example. Let $C_\infty(\boldsymbol{R})$ denote the space of continuous functions on $\boldsymbol{R}$ that vanish at infinity equipped with the sup norm topology, and let E be its dual space with the w^*-topology. Then $\boldsymbol{R}$ acts on E by the contragradient to translation on $C_\infty(\boldsymbol{R})$. K, the set of positive linear functionals in E of norm less than or equal one is compact, convex and lies in an invariant half-space. However, K has no nonzero fixed point. In fact, one easily sees that for any $p \in K$

$$w^*\text{-}\lim_{t\to\pm\infty} t\cdot p = 0 ,$$

i.e., 0 is in the closure of every orbit in K. If we were to ignore this fact for a moment and try to construct a fixed point by the usual scheme, we would fix a nonzero element p of K. Define a sequence of elements p_n in K by

$$p_n = \frac{1}{2n}\int_{-n}^{n} t\cdot p dt ,$$

note that the p_n's are "nearly invariant", and use the compactness of K to get an invariant cluster point of $\{p_n\}$. The difficulty, of course, is that w^*-lim $p_n = 0$. But even more critical is the fact that the entire orbits of the p_n's are "w^*-convergent to zero", i.e., for all $\varphi \in E^*$,

$$\lim_{n\to\infty} \sup_{t \in R} |\langle \varphi, t\cdot p_n\rangle| = 0 .$$

Hence, arbitrarily small neighborhoods of zero contain orbits of elements in K. The second condition in the following definition excludes such actions.

DEFINITION 1. A locally compact group G is said to have the nonzero fixed point property if whenever G acts continuously and linearly on a locally convex space E having an invariant, compact, convex subset K, and K satisfies

(i) there is a $\varphi \in E^*$ so that $\langle \varphi, x\rangle > 0$ for all $0 \neq x \in K$, and

(ii) for all $0 \neq x \in K$, $x/\sup_G \langle \varphi, s\cdot x\rangle \in K$,

then K has a nonzero fixed point.

The main result of this paper is the following characterization theorem.

THEOREM 2. *Suppose G is a connected, locally compact group or a discrete, finitely generated, solvable group. Then G has the nonzero fixed point property if, and only if, G has polynomial growth.*

The proof will be given in a series of propositions, but first we recall some facts about groups with polynomial growth.

Given a locally compact group G, and measurable subset U, denote by $|U|$ the left Haar measure of U, and for a positive integer n set $U^n = \{u_1u_2 \cdots u_n \mid u_i \in U, 1 \leqq i \leqq n\}$. G is said to have polynomial growth if for any compact neighborhood U of the identity in G, there is a polynomial p such that $|U^n| \leqq p(n)$ for all $n = 1, 2, \cdots$.

Milnor [7] and Wolf [9] showed that a discrete, finitely generated, solvable group G has polynomial growth if, and only if, it contains a nilpotent subgroup with finite index. Rosenblatt [8] added to the characterization by showing that for such G, polynomial growth is equivalent to G not having a free noncommutative subsemigroup on two generators.

For a Lie group G, let $t \to \mathrm{Ad}\, t$ denote the adjoint representation of G on its Lie algebra. G is said to be type R if the eigenvalues of $\mathrm{Ad}\, t$ are of modulus one for all $t \in G$. A connected, locally compact group G is said to be type R if for some compact, normal subgroup K, G/K is a type R Lie group.

A subset S of a locally compact group G is said to be uniformly discrete if there is a neighborhood U of the identity in G such that $sU \cap tU = \emptyset$ for $s, t \in S$, $s \neq t$.

It is shown in Jenkins [5] that for a connected group, G, polynomial growth is equivalent to G being type R, and also equivalent to G not having a uniformly discrete free subsemigroup on two generators. (The first equivalence was also proven by Guivarc'h [2].)

The following lemma was first proved in Guivarc'h [2]. We include a proof here for the convenience of the reader.

LEMMA 3. *Suppose G is a conncted, locally compact group or a discrete, finitely generated, solvable group. If G has polynomial growth, then there is a normal series $\{e\} = G_0 \subset G_1 \subset \cdots \subset G_n = G$ such that every element of G_{j+1}/G_j is contained in a compact neighborhood of the identity that is invariant under the inner-automorphisms from G/G_j.*

Proof. If G contains a normal, nilpotent subgroup of finite index, N, then the lower central series of N will give the desired normal series of G.

Suppose G is a connected Lie group with polynomial growth. Let S be the solv-radical of G, LS its Lie algebra, and LS_C, the complexification of LS. By Lie's theorem, there is an ordered basis for LS_C, $\{X_1, \cdots, X_m\}$ so that the matrix representation for $\mathrm{Ad}\, s$ with respect to this basis is upper triangular for all $s \in S$. Let V_j be the subspace spanned by $\{X_1, \cdots, X_j\}$. Then, because of polynomial growth, S is type R, and the action of $\mathrm{Ad}\, s$ on V_{j+1}/V_j is multiplication by a complex number of modulus one, i.e., $\mathrm{Ad}\, s$ acts by rotation on V_{j+1}/V_j. Thus we can find subspaces $\{0\} = W_0 \subset W_1 \subset \cdots \subset W_n = LS$, the real Lie algebra of S, each invariant under $\mathrm{Ad}\, s$, for all $s \in S$, with $\dim (W_{j+1}/W_j) \leqq 2$, and with $\mathrm{Ad}\, s$ actaing by rotation of W_{j+1}/W_j. Thus W_j is an ideal in LS and if S_j is the corresponding closed, normal subgroup of S, each element of S_{j+1}/S_j is in a compact neighborhood that is invariant under the inner-automorphisms from S/S_j. Since G has polynomial growth, the semisimple part of G, G/S, is compact. Hence, the S_j's give the desired normal series for G.

If G is a connected group with polynomial growth, there is a compact, normal subgroup K so that G/K is a Lie group with polynomial growth. The above argument applied to G/K completes the proof.

The following proposition, which is the keystone in the proof of Theorem 2, was inspired by Ludwig [6].

PROPOSITION 4. *Let G be a locally compact group with polynomial growth, and suppose G acts continuously and linearly on a locally convex space E. Let K be a compact, convex subset of E that satisfies the conditions in Definition* 1. *Let L, H be normal subgroups of G with $L \subset H$, and suppose every element of H/L is contained in a compact, G/L invariant neighborhood. If K has a nonzero point fixed by L, it has a nonzero point fixed by H.*

Proof. Let $\varepsilon > 0$ and U, a compact, symmetric neighborhood of the identity in H/L that is invariant under the inner-automorphisms from G/L, be given. We define a function $f_{\varepsilon,\dot U}$ on H/L by $f_{\varepsilon,\dot U}(\dot s) = (1+\varepsilon)^{-1}$ if $\dot s \in \dot U$, $f_{\varepsilon,\dot U}(\dot s) = (1+\varepsilon)^{-n}$ if $\dot s \in \dot U^n \sim \dot U^{n-1}$ for $n \geqq 2$, and $f_{\varepsilon,\dot U}(\dot s) = 0$ if $\dot s \notin \langle \dot U \rangle$, the subgroup generated by $\dot U$. Since H/L has polynomial growth

$$\| f_{\varepsilon,\dot U} \|_1 = (1+\varepsilon)^{-1} | \dot U | + \sum_{n=2}^{\infty} (1+\varepsilon)^{-n} | \dot U^n \sim \dot U^{n-1} | < \infty .$$

Also note that for $\dot s \in \dot U$ and $\dot t \in \langle \dot U \rangle$

$$| f_{\varepsilon,\dot U}(\dot s \dot t) - f_{\varepsilon,\dot U}(\dot t) | \leqq \varepsilon f_{\varepsilon,\dot U}(\dot t) .$$

Let $0 \neq x_0 \in K$ such that $s \cdot x_0 = x_0$ for all $s \in L$. Then H/L acts on x_0 and we can define $x'_{\varepsilon,\dot U} \in E^{**}$ by setting

$$\langle x'_{\varepsilon,\dot U}, \psi \rangle = \int_{H/L} f_{\varepsilon,\dot U}(\dot s) \langle \dot s \cdot x_0, \psi \rangle ds ,$$

for each $\psi \in E^*$. Since K is compact and convex, $x'_{\varepsilon,\dot U}$ is in the canonical image of K in E^{**}. We denote its preimage in K also by $x'_{\varepsilon,\dot U}$.

Let $\varphi \in E^*$ as in Definition 1, i.e., $\langle \varphi, x \rangle > 0$ and $x/\sup_{s \in G} \langle \varphi, s \cdot x \rangle \in K$ for all $0 \neq x \in K$. Define

$$\alpha_{\varepsilon,\dot U} = \sup_{s \in G} \langle \varphi, s \cdot x'_{\varepsilon,\dot U} \rangle ,$$

and pick $a \in G$ so that

$$\langle \varphi, a \cdot x'_{\varepsilon,\dot U} \rangle \geqq (1-\varepsilon) \alpha_{\varepsilon,\dot U} .$$

Finally, define

$$x_{\varepsilon,\dot U} = \alpha_{\varepsilon,\dot U}^{-1} a \cdot x'_{\varepsilon,\dot U} .$$

Note that by condition (ii) of Definition 1, $x_{\varepsilon,\dot{U}} \in K$, since a real multiple of $x'_{\varepsilon,\dot{U}} \in K$.

Let $\psi \in E^*$ and $\dot{b} \in \dot{U}$.

$$\begin{aligned}|\langle\psi, b\cdot x_{\varepsilon,\dot{U}} - x_{\varepsilon,\dot{U}}\rangle| &= \alpha_{\varepsilon,\dot{U}}^{-1}\left|\int f_{\varepsilon,\dot{U}}(\dot{s})(\langle\psi, bas\cdot x_0\rangle - \langle\psi, as\cdot x_0\rangle)d\dot{s}\right| \\ &\leqq \alpha_{\varepsilon,\dot{U}}^{-1}\int|f_{\varepsilon,\dot{U}}(\dot{a}^{-1}\dot{b}^{-1}\dot{a}\dot{s}) - f_{\varepsilon,\dot{U}}(\dot{s})||\langle\psi, \dot{a}\dot{s}\cdot x_0\rangle|d\dot{s} \\ &\leqq \varepsilon\alpha_{\varepsilon,\dot{U}}^{-1}\int f_{\varepsilon,\dot{U}}(\dot{s})|\langle\psi, \dot{a}\dot{s}\cdot x_0\rangle|d\dot{s}\ .\end{aligned}$$

Thus, in particular,

$$|\langle\varphi, \dot{b}\cdot x_{\varepsilon,\dot{U}} - x_{\varepsilon,\dot{U}}\rangle| \leqq \varepsilon\ .$$

We consider the net $\{x_{\varepsilon,\dot{U}}\}$ ordered by $(\varepsilon, \dot{U}) \leqq (\varepsilon', \dot{U}')$ if $\varepsilon < \varepsilon'$ and $\dot{U}' \subseteqq \dot{U}$, and pick a cluster point x_∞. Note that $x_\infty \in K$, that $s\cdot x_\infty = x_\infty$ for all $s \in L$, and that $\langle\varphi, x\cdot x_\infty\rangle = 1$ for all $s \in L$. Thus we may assume that the original x_0 had its orbit in the hyperplane $\{y|\langle\varphi, y\rangle = 1\}$. But then $\alpha_{\varepsilon,\dot{U}} \geqq \|f_{\varepsilon,\dot{U}}\|_1$, and we have that for all $\psi \in E^*$ and $\dot{b} \in \dot{U}$

$$|\langle\psi, \dot{b}\cdot x_{\varepsilon,\dot{U}} - x_{\varepsilon,\dot{U}}\rangle| \leqq \varepsilon \sup_{s\in G}|\langle\psi, s\cdot x_0\rangle|\ .$$

Thus, for all $s \in H$ and $\psi \in E^*$

$$\langle\psi, s\cdot x_\infty\rangle = \langle\psi, x_\infty\rangle\ .$$

Hence $s\cdot x_\infty = x_\infty$ for all $s \in H$.

Lemma 3 and Proposition 4 combine to prove that polynomial growth is sufficient to give the nonzero fixed point property for the class of groups given in Theorem 2. In order to prove the converse we need the following proposition which has some interesting consequences of its own.

PROPOSITION 5. *Let G be a locally compact, σ-compact group that has the nonzero fixed point property. Let $\theta \in BC(G)$, the bounded, continuous functions on G, with $0 \neq \theta \geqq 0$ and $\theta \leqq f_0 * \theta$ for some $f_0 \in L^1(G)$.*

Let

$$L_\theta = \{\psi \in L^\infty(G)\,|\,|\psi| \leqq f*\theta \text{ some } f \in L^1(G)\}$$

and set

$$\|\psi\|_\theta = \inf\{\|f\|_1\,|\,|\psi| \leqq f*\theta\}\ .$$

Then there is a continuous, positive linear functional on L_θ, p, such that $\langle p, s\cdot\psi\rangle = \langle p, \psi\rangle$ all $s \in G$ and $\psi \in L_\theta$ and such that $\langle p, \theta\rangle = 1$.

Proof. First note that we may assume that $\theta \leqq f_0 * \theta$ where $f_0(s) > 0$ for almost all $s \in G$.

Let

$$L_\theta^0 = \{f * \theta \mid f \in L^1(G)\}$$

and let E denote the dual of L_θ^0 with the w^*-topology. G acts on E by

$$\langle s \cdot p, f * \theta \rangle = \langle p, sf * \theta \rangle$$

where $sf(t) = f(st)$ for all $s, t \in G$ and $f \in L^1(G)$. This action is continuous since

$$\begin{aligned} |\langle s \cdot p, f * \theta \rangle - \langle p, f * \theta \rangle| &\leqq |\langle p, (sf - f) * \theta \rangle| \\ &\leqq \|p\| \|sf - f\|_1 , \end{aligned}$$

where $\|p\|$ is the norm of p induced by $\|\cdot\|_\theta$ restricted to L_θ^0.

Let

$$K = \{p \in E \mid p \geqq 0, \|p\| \leqq 1\} .$$

Then K is compact, convex and satisfies the two conditions of Definition 1.

The first condition is satisfied by the linear functional $f_0 * \theta$, for if $p \in K$ and $\langle p, f_0 * \theta \rangle = 0$ then for all $g \in L^1(G)$ with $0 \leqq g \leqq f_0$, $\langle p, g * \theta \rangle = 0$. Thus, for all bounded, compactly supported g in $L^1(G)$, $\langle p, g * \theta \rangle = 0$. By denseness of the set of all such $g * \theta$ in L_θ^0, $p = 0$.

For the second condition, let $\psi \in L_\theta^0$ and $p \in K$. Then, if $|\psi| \leqq f * \theta$

$$\begin{aligned} |\langle p, \psi \rangle| \leqq \langle p, |\psi| \rangle &\leqq \langle p, f * f_0 * \theta \rangle \\ &= \int f(s) \langle p, sf_0 * \theta \rangle ds . \end{aligned}$$

Thus, setting

$$\alpha = \sup_{s \in G} \langle p, sf_0 * \theta \rangle ,$$

we have that for $|\psi| \leqq f * \theta$,

$$|\langle p, \psi \rangle| \leqq \alpha \|f\|_1 .$$

Hence

$$\|p\| = \sup_{\|\psi\|_\theta = 1} |\langle p, \psi \rangle| \leqq \alpha ,$$

and so $p/\alpha \in K$.

An application of the nonzero fixed point property gives a $0 \neq p' \in K$ that is fixed by G.

Let $\mathscr{U}$ be a neighborhood basis of e in G consisting of open sets with compact closures. For each $U \in \mathscr{U}$, let φ_U denote the normalized characteristic function of U. Assume $\liminf_{\mathscr{U}} \langle p', \varphi_U * \theta\rangle = 0$. Then, by relabeling, we may assume $\lim_{\mathscr{U}} \langle p', \varphi_U * \theta\rangle = 0$. Fix a $U_0 \in \mathscr{U}$. Given $\varepsilon > 0$ there exists $s_1, \cdots, s_n \in G$, $U_1, \cdots, U_n \in \mathscr{U}$ such that

(i) $s_i U_i \cap s_j U_j = \varnothing$ for $i \neq j$.

(ii) $\chi_{U_0} \leqq \sum_{k=1}^{n} s_k \cdot \varphi_{U_k} | U_k |$.

(iii) $\| \chi_{U_0} - \sum s_k \cdot \varphi_{U_k} | U_k | \|_1 \leqq \varepsilon$.

(iv) $\langle p', \varphi_{U_i} * \theta \rangle \leqq \varepsilon$.

Thus,

$$0 \leqq \langle p', \chi_{U_0} * \theta\rangle \leqq \langle p', \sum s_k \cdot \varphi_{U_k} | U_k |\rangle \leqq \varepsilon \sum_{k=1}^{n} | U_k | \leqq \varepsilon(\varepsilon + | U_0 |) .$$

Since $\varepsilon > 0$ was arbitrary, $\langle p', \chi_{U_0} * \theta\rangle = 0$. It follows immediately that $p' = 0$. This contradiction implies that $\liminf_{\mathscr{U}} \langle p', \varphi_u * \theta\rangle = \delta > 0$, and from this, one easily sees that

$$\langle p', f * \theta\rangle = \delta \int f(s) ds$$

for all $f \in L^1(G)$.

To get the desired element in L_θ^*, we note first that by the nonzero fixed point property G is amenable. Pick a left invariant mean m on $L^\infty(G)$, extend p' to a continuous, positive linear functional $\bar{p}$ on L_θ by the Krein Extension Theorem, and define p on L_θ by $\langle p, \psi\rangle = \langle m, F_\psi\rangle$ where $F_\psi(s) = \langle \bar{p}, s\psi\rangle$. One easily checks that, up to normalization, p is the desired functional.

REMARK. In [8], Rosenblatt defined a group G to be superamenable if for any measurable subset $A \subset G$ there is a translation invariant, positive linear functional p defined on the space spanned by the left-translates of χ_A with $\langle p, \chi_A\rangle = 1$. He showed that a discrete group with polynomial growth is superamenable. In [4], this was generalized by showing that for any locally compact group G with polynomial growth and $0 \neq \theta \in L^\infty(G)$, $\theta \geqq 0$, there is a translation invariant, positive linear functional p defined on the space spanned by the left-translates of θ with $\langle p, \theta\rangle = 1$. However, this functional p, in general, can not be extended to the larger space L_θ (see the remark after Corollary 6.).

A Banach *-algebra A is said to be symmetric if for all $a \in A$, $\mathrm{sp}(aa^*) \subseteq [0, \infty)$. Ludwig [6] proved that if G is a connected locally compact group or a discrete, finitely generated, solvable group and G has polynomial growth then $L^1(G)$ is symmetric. Essentially, the same proof gives

COROLLARY 6. *If G has the nonzero fixed point property then $L^1(G)$ is symmetric.*

Proof. Suppose $f_0 \in L(G)$ such that $-1 \in \operatorname{sp}(f_0 * f_0^*)$. Then there is an $0 \neq \varphi \in BC(G)$ such that $f_0 * f_0^* * \varphi = -\varphi$. Let

$$\theta = |\varphi|^2 .$$

Note that

$$\theta = |\varphi|^2 = |f_0 * f_0^* * \varphi|^2 \leqq \|f_0 * f_0^*\|_1 |f_0 * f_0^*| * \theta .$$

Hence, by Proposition 5, there is a positive continuous linear function p on L_θ that is invariant and has $\langle p, \theta\rangle = 1$.

Define a form B on $L^1(G)$ by

$$B(f, g) = \langle p, (f * \varphi)(\overline{g * \varphi})\rangle .$$

This makes sense, for if $f, g \in L_1(G)$ then

$$|(f * \varphi)(\overline{g * \varphi})| \leqq \frac{1}{2}\{|g * \varphi|^2 + |f * \varphi|^2\} \in L_\theta .$$

Clearly $B(f, f) \geqq 0$ for all $f \in L(G)$ and $B(f_0 * f_0^*, f_0 * f_0^*) = \langle p, |\varphi|^2\rangle = 1$. Also B is bounded, since for $g \in L^1(G)$

$$B(g, g) = \langle p, |g * \varphi|^2\rangle \leqq \|g\|_1 \langle p, |g| * \theta\rangle = \|g\|_1^2 .$$

Finally, by the invariance of p, $B(sf, sg) = B(f, g)$ for all $f, g \in L'(G)$ and $s \in G$. Thus $B(f * g, h) = B(g, f^* * h)$. Hence

$$0 \leqq B(f_0^*, f_0^*) = -B(f_0^* * f_0^* * f_0^*, f_0^*) = -1 .$$

This contradiction implies that $-1 \notin \operatorname{sp}(f_0 * f_0^*)$.

REMARK. Hulanicki [3] has shown that there is a discrete, solvable group (not finitely generated) with polynomial growth that has a nonsymmetric group algebra. Hence not every group with polynomial growth enjoys the nonzero fixed point property.

PROPOSITION 7. *Suppose G is a connected locally compact group or a discrete, finitely generated, solvable group. If G does not have polynomial growth it does not have the nonzero fixed point property.*

Proof. With the assumptions on G, failure of polynomial growth implies the existence of elements $a, b \in G$ and a compact neighborhood of the identity U such that the semigroup generated by a and b, S, is free and for $s, t \in S$, $s \neq t$, $sU \cap tU = \varnothing$. Let ψ be the characteristic function of $S \cdot U$. Let V be a compact neighborhood of the identity and $\alpha > 0$ such that $\chi_V \leqq \alpha\chi_V * \chi_V$. Then, with $f = \alpha\chi_V$ and $\theta = \chi_V * \psi$, we have $\theta \leqq f * \theta$. Hence, if G has the nonzero fixed point property, Proposition 5 gives a positive linear function $p \in L_\theta^*$ that is invariant and such that $\langle p, \theta\rangle = 1$. Now $\psi \in L_\theta$ and

$$\langle p, \theta\rangle = \int_V \langle ps, s\cdot\psi\rangle d = |V|\langle p, \psi\rangle > 0 .$$

Since S is free and uniformly discrete, $aSU \cap bSU = \varnothing$. Hence $\psi - a\cdot\psi - b\cdot\psi \geqq 0$. Thus, since $p \geqq 0$ and invariant, $\langle p, \psi\rangle \geqq 2\langle p, \psi\rangle$. This contradiction shows that G does not have the nonzero fixed point property.

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