PAPERS COMMUNICATED

87. Über die Fermatsche Vermutung, III.

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Satz: *Es sei*

$$x^p+y^p+z^p=0, \quad (x, y, z)=1, \quad xyz \not\equiv 0 \pmod{p}, \quad p>3.$$

Dann ist

$$\sum_{i=1}^{p-1}\frac{t^i}{i}\equiv 0 \pmod{p^2},$$

wo

$$t\equiv -\frac{y}{x}, \quad -\frac{x}{y}, \quad -\frac{z}{y}, \quad -\frac{y}{z}, \quad -\frac{x}{z}, \quad -\frac{z}{x} \pmod{p^2}.$$

Beweis: Es ist

$$\left[\sum_{i=1}^{p-1}\frac{t^i}{i}\right]^2=2\left[\sum_{i=2}^{p}\frac{t^i}{i}\sigma_i+\sum_{i=1}^{p-2}\frac{t^{p+i}}{p+i}(\sigma_p-\sigma_{i+1})\right],$$

wo

$$\sigma_i=1+\frac{1}{2}+\dots\dots+\frac{1}{i-1}, \quad \text{also wegen } \sigma_p\equiv 0 \pmod{p^2},$$

$$\left[\sum_{i=1}^{p-1}\frac{t^i}{i}\right]^2\equiv 2\left[\sum_{i=2}^{p-1}\frac{t^i}{i}\sigma_i-t\sum_{i=1}^{p-2}\frac{t^i}{i}\sigma_{i+1}\right]$$

$$\equiv 2\left[\sum_{i=2}^{p-1}\frac{t^i}{i}\sigma_i-t\sum_{i=2}^{p-1}\frac{t^i}{i}\sigma_i-t\sum_{i=1}^{p-1}\frac{t^i}{i^2}\right] \pmod{p},$$

also wegen* $\sum_{i=1}^{p-1}\frac{t^i}{i^e}\equiv 0, \quad 1\not\equiv t \pmod{p}, \quad e=1, 2$

$$\sum_{i=2}^{p-1}\frac{t^i}{i}\sigma_i\equiv 0 \pmod{p}. \quad (1)$$

Nun ist

$$(1-t)^p\equiv 1-t^p \pmod{p^3},$$

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* Das Kummersche Kriterium.

also

$$\sum_{i=1}^{p-1}(-1)^{i-1}\binom{p-1}{i-1}\frac{t^i}{i}\equiv\sum_{i=1}^{p-1}\frac{t^i}{i}-p\sum_{i=2}^{p-1}\frac{t^i}{i}\sigma^i\equiv 0 \qquad (\text{mod } p^2),$$

also nach (1)

$$\sum_{i=1}^{p-1}\frac{t^i}{i}\equiv 0 \qquad (\text{mod } p^2).$$