49. *On Weakly Compact Topological Spaces*

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In my Note [1], I gave some characteristic properties of countably compact normal spaces by using AU-covering. Recently I have an opportunity of reading a paper [2] of S. Mardešić and P. Papić, and I found that a theorem of my Note [1] gives a characterisation of weakly compact spaces. In this Note, I shall give a detail of it.

Following S. Mardešić and P. Papić [2], we shall define a weakly compact space as follows:

Definition. A topological space S is said to be *weakly compact*, if any family of disjoint countable open sets of S has at least one cluster point.

If a topological space is completely regular, two notions of weakly compactness and pseudo-compactness are coincide (see [2]).

S. Mardešić and P. Papić [2] proved that *a regular space S is weakly compact, if and only if any countable open covering is an AU-covering in sense of present author* [1], i.e. for a countable open covering α of S, there is a finite subfamily β of it such that the union of closure of each open set of β is S.

From the statement of them, for a regular weakly compact space S, every point finite countable open covering of S has the AU-property. Conversely, for a regular space S, if every locally finite (point finite) countable open covering of S has the AU-property, S is weakly compact. To prove it, suppose that S is not weakly compact, then there is a family α of disjoint countable open sets U_n $(n=1, 2,\cdots)$ and α has no cluster point, i.e. for any point x of S, there is a neighbourhood $V(x)$ of x such that $V(x)$ meets only finite members of α. By the regularity of S, we can find open sets W_n such that $\overline{W}_n\subset U_n$ for each n. The union of $\overline{W}_n$ is a closed set.

We shall consider the open covering $\beta=\{S-\bigcup_{n=1}^{\infty}\overline{W}_n, U_1, U_2,\cdots\}$ of S. Then β is a point finite covering of S. The covering β has no AU-property. Therefore we have the following

Theorem. A regular space is weakly compact, if and only if every point finite countable open covering has an AU-covering.

References

[1] K. Iséki: A remark on countably compact normal space, Proc. Japan Acad., **33**, 131 (1957).
[2] S. Mardešić et P. Papić: Sur les espaces dont toute transformation réelle continue est borné, Glasnik Mat.-Fiz. i Astr., **10**, 225–232 (1955).