48. A Characterisation of Countably Compact Normal Space by AU-covering

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In my Note [1], I gave a characterisation of countably compact normal space by using the notion of AU-property.*) Following my Note, we shall define the AU-covering as follows: An open covering α of a topological space R is said to be an *AU-covering*, if there is a finite subfamily β of α such that the closure of the union of sets of β covers R. Then P. Alexandrov and P. Urysohn proved the following well-known proposition: *for a regular T_2-space, any open covering is AU-covering if and only if it is compact.*

In this Note, we shall show the following

Theorem. For a normal space R, any countable open covering is AU-covering if and only if R is countably compact.

Proof. If R is countably compact, since any countable open covering is a σ-discrete open covering, it is an AU-covering (see [1, Theorem 3]). To prove the converse, by the normality of R, it is sufficient to show that every continuous function on R is bounded. Let $f(x)$ be a continuous function on R. For each open interval $I_n=\left(n-\frac{1}{2}, n+\frac{1}{2}\right)$ $(n=0, \pm 1, \pm 2,\cdots)$, $O_n=f^{-1}(I_n)$ is an open set in R, and $\alpha=\{O_n \mid n=1, 2,\cdots\}$ is a countable open covering of R. We can find finite open sets $\{O_{n_i}\}$ $(i=1, 2,\cdots, k)$, such that $\bigcup_{i=1}^{k}\overline{O}_{n_i}=R$. On the other hand $f(\overline{O}_n)\subset\overline{I}$. Hence $f(R)\subset\bigcup_{i=1}^{k} I_{n_i}$ and $f(x)$ is bounded. Therefore the proof is complete.

It follows from the proof that:

Corollary. Any complete regular space is pseudo-compact, if any countable open covering is an AU-covering.

References

[1] K. Iséki: A remark on countably compact normal space, Proc. Japan Acad., **33**, 131 (1957).

[2] S. Mardešić et P. Papić: Sur les espaces dont toute transformation réelle continue est borné, Glasnik Mat.-Fiz. i Astr., **10**, 225–232 (1955).

*) In the preparation of this Note, I found a paper by S. Mardešić and P. Papić [2]. In their paper, they discussed the notion of AU-covering.