96. *Note on Orientable Surfaces in 4-Space*

By Reibun TAKASE
Waseda University

(Comm. by Kinjirô KUNUGI, M.J.A., Sept. 12, 1963)

In the preceding paper [1], the *Stiefel-Whitney number* w is defined for a closed (orientable) oriented surface M embedded in an (orientable) oriented 4-manifold W without boundary. In the note we will prove the following;

Theorem. *The Stiefel-Whitney number w is the self-intersection number of the fundamental homology class of M in W* [2].

In consequence, the Stiefel-Whitney number w of M in W is an invariance under the iso-neighboring relation, and in particular if W is euclidean 4-space R, then $w=0$, because the 2-dimensional homology group of R vanishes.

Proof of Theorem. Using the notation of the paper [1], the longitudes b_j $(j\neq 0)$ may be chosen so that $\sum_{j\neq 0} b_j$ is on T_0, and b_j is the intersection of $\widetilde{F}_j$ and T_j, where $\widetilde{F}_j$ is an oriented surface in 3-sphere $\partial\square_j$ with boundary $\partial\nabla_j$. Let $F_j(j\neq 0)$ be $Cl(\widetilde{F}_j-U_j)$ with the orientation satisfying $\partial F_j=b_j$. Let F_0 be an oriented surface in $\partial\square_0$ with boundary $\sum_{j\neq 0} b_j$. Then the intersection number $KI(F_0, \partial\nabla_0)$ is equal to the looping coefficient $LC(\sum_{j\neq 0} b_j, \partial\nabla_0)$ by the definitions of intersection number and looping coefficient. Since $\sum_{j\neq 0} b_j$ is homologous to wa_0-b_0 on T_0, $LC(\sum_{j\neq 0} b_j, \partial\nabla_0)=LC(wa_0-b_0, \partial\nabla_0)=wLC(a_0, \partial\nabla_0)-LC(b_0, \partial\nabla_0)$ in $\partial\square_0$. Since $LC(a_0, \partial\nabla_0)=1$ and $LC(b_0, \partial\nabla_0)=0$ in $\partial\square_0$, $KI(F_0, \partial\nabla_0)=w$. Since $F_0\cap M=\partial\nabla_0$, M and $\partial\nabla_0$ meet F_0 at the same number of points, so by the definition of intersection number $KI(F_0, M)=KI(F_0, \partial\nabla_0)=w$. For each j, F_j is homologous to ∇_j in $\square_j$, and hence the 2-cycle $\sum_j F_j$ is homologous to $\sum_j \nabla_j(=M)$ in W. Since $F_j\cap M$ is empty $(j\neq 0)$, $KI(\sum_j F_j, M)=KI(F_0, M)=w$, completing the proof.

References

[1] H. Noguchi: A classification of orientable surfaces in 4-space, Proc. Japan Acad., **39**, 422–423 (1963).
[2] We owe the formulation to J. Milnor.