60. *On Axiom Systems of Ontology. I*

By Shôtarô TANAKA

(Comm. by Kinjirô KUNUGI, M. J. A., March 12, 1970)

It is well known that Leśniewski's original system of ontology has the form of the following single axiom [1], [2]:

T. $a \varepsilon b \equiv [\exists c]\{c \varepsilon a\} \wedge [c]\{c \varepsilon a \supset c \varepsilon b\} \wedge [cd]\{c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d\}$.

It is mentioned that the following expression can act as the single axiom of Ontology by C. Lejewski [1]:

A. $a \varepsilon b \equiv [\exists c]\{c \varepsilon a \wedge c \varepsilon b\} \wedge [c]\{c \varepsilon a \supset a \varepsilon c\}$.

In this paper, we shall prove that T and A are equivalent. The proofs of theorems will be given in the form of suppositional proofs [1], [2].

Lemma 1. T *implies* A.

Proof.

T 1. $a \varepsilon b \wedge b \varepsilon c \supset a \varepsilon c$

Proof.

1	$a \varepsilon b$	(premise)
2	$b \varepsilon c \supset$	(premise)
3	$[d]\{d \varepsilon b \supset d \varepsilon c\}$	(T, 2)
4	$a \varepsilon b \supset a \varepsilon c$	(OΠ : 3)
	$a \varepsilon c$	(4, 1)

T 2. $[\exists c]\{c \varepsilon a\} \wedge [cd]\{c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d\} \wedge [c]\{c \varepsilon a \supset c \varepsilon b\} \supset a \varepsilon b$ (T)

T 3. $a \varepsilon b \supset a \varepsilon a$

Proof.

1	$a \varepsilon b \supset$	(premise)
2	$[\exists c]\{c \varepsilon a\} \wedge [cd]\{c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d\} \wedge [c]\{c \varepsilon a \supset c \varepsilon b\}$	(T, 1)
3	$[c]\{c \varepsilon a \supset c \varepsilon a\}$	($p \supset p$)
	$a \varepsilon a$	(T 2, 2, 3)

T 4. $a \varepsilon b \wedge c \varepsilon a \supset a \varepsilon c$

Proof.

1	$a \varepsilon b$	(premise)
2	$c \varepsilon a \supset$	(premise)
3	$[de]\{d \varepsilon a \wedge e \varepsilon a \supset d \varepsilon e\}$	(T, 1)
4	$a \varepsilon a \wedge c \varepsilon a \supset a \varepsilon c$	(OΠ : 3)
5	$a \varepsilon a$	(T 3, 1)
	$a \varepsilon c$	(4, 5, 2)

T 5. $a \varepsilon b \supset [c]\{c \varepsilon a \supset a \varepsilon c\}$

Proof.

1	$a \varepsilon b \supset$	(premise)
2	$c \varepsilon a \supset a \varepsilon c$	(T 4, 1)
	$[c]\{c \varepsilon a \supset a \varepsilon c\}$	(DΠ : 3)

T 6.		$a \varepsilon b \supset [\exists c]\{c \varepsilon a \wedge c \varepsilon b\}$	
Proof.	1	$a \varepsilon b \supset$	(premise)
	2	$[\exists c]\{c \varepsilon a\}$	(T, 1)
	3	$c_1 \varepsilon a$	(OΣ : 2)
	4	$[c]\{c \varepsilon a \supset c \varepsilon b\}$	(T, 1)
	5	$c_1 \varepsilon a \supset c_1 \varepsilon b$	(OΠ : 4)
	6	$c_1 \varepsilon b$	(5, 3)
	7	$c_1 \varepsilon a \wedge c_1 \varepsilon b$	(3, 6)
		$[\exists c]\{c \varepsilon a \wedge c \varepsilon b\}$	(DΣ : 7)
T 7.		$a \varepsilon b \supset [\exists c]\{c \varepsilon a \wedge c \varepsilon b\} \wedge [c]\{c \varepsilon a \supset a \varepsilon c\}$	(T 6, T 5)
T 8.		$[\exists c]\{c \varepsilon a \wedge c \varepsilon b\} \wedge [c]\{c \varepsilon a \supset a \varepsilon c\} \supset a \varepsilon b$	
Proof.	1	$[\exists c]\{c \varepsilon a \wedge c \varepsilon b\}$	(premise)
	2	$[c]\{c \varepsilon a \supset a \varepsilon c\} \supset$	(premise)
	3	$c_1 \varepsilon a \wedge c_1 \varepsilon b$	(OΣ : 1)
	4	$c_1 \varepsilon a \supset a \varepsilon c_1$	(OΠ : 2)
	5	$a \varepsilon c_1$	(4, 3)
		$a \varepsilon b$	(T 1, 5, 3)
T 9 = A		$a \varepsilon b \equiv [\exists c]\{c \varepsilon a \wedge c \varepsilon b\} \wedge [c]\{c \varepsilon a \supset a \varepsilon c\}$	(T 7, T 8)

Lemma 2. A *implies* T.

Proof.

A 1		$a \varepsilon b \wedge c \varepsilon a \supset a \varepsilon c$	
Proof.	1	$a \varepsilon b$	(premise)
	2	$c \varepsilon a \supset$	(premise)
	3	$[c]\{c \varepsilon a \supset a \varepsilon c\}$	(A, 1)
	4	$c \varepsilon a \supset a \varepsilon c$	(OΠ : 3)
		$a \varepsilon c$	(4, 2)
A 2.		$a \varepsilon b \wedge c \varepsilon a \supset c \varepsilon b$	
Proof.	1	$a \varepsilon b$	(premise)
	2	$c \varepsilon a \supset$	(premise)
	3	$a \varepsilon c$	(A 1, 1, 2)
	4	$a \varepsilon c \wedge a \varepsilon b$	(3, 1)
	5	$[\exists d]\{d \varepsilon c \wedge d \varepsilon b\}$	(DΣ : 4)
	6	$[d]\{d \varepsilon c \supset c \varepsilon d\}$	(A, 2)
		$c \varepsilon b$	(A, 5, 6)
A 3.		$a \varepsilon b \wedge c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d$	
Proof.	1	$a \varepsilon b$	(premise)
	2	$c \varepsilon a$	(premise)
	3	$d \varepsilon a \supset$	(premise)
	4	$a \varepsilon c$	(A 1, 1, 2)
	5	$a \varepsilon d$	(A, 1, 3)
	6	$[\exists e]\{e \varepsilon c \wedge e \varepsilon d\}$	(4, 5, DΣ)
	7	$[e]\{e \varepsilon c \supset c \varepsilon e\}$	(A, 2)
		$c \varepsilon d$	(A, 6, 7)

A 4. $a \varepsilon b \supset [cd]\{c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d\}$

Proof. 1 $a \varepsilon b \supset$ (premise)

2 $c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d$ (A 3, 1)

$[cd]\{c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d\}$ (DΠ: 2)

A 5. $a \varepsilon b \supset [c]\{c \varepsilon a \supset c \varepsilon b\}$

Proof. 1 $a \varepsilon b \supset$ (premise)

2 $c \varepsilon a \supset c \varepsilon b$ (A 2, 1)

$[c]\{c \varepsilon a \supset c \varepsilon b\}$ (DΠ: 2)

A 6. $a \varepsilon b \supset [\exists c]\{c \varepsilon a\} \wedge [cd]\{c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d\} \wedge [c]\{c \varepsilon a \supset c \varepsilon b\}$ (A, A4, A5)

Now we use the rule of extensionality:

ER 1. $[x]\{x \varepsilon X \equiv x \varepsilon Y\} \supset [\varphi]\{\varphi(X) \equiv \varphi(Y)\}$

Let D 1 be the definition:

D 1. $\rho\langle X\rangle(x) \equiv x \varepsilon X$

A 7. $[\exists c]\{c \varepsilon a\} \wedge [cd]\{c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d\} \wedge [c]\{c \varepsilon a \supset c \varepsilon b\} \supset a \varepsilon b$

Proof. 1 $[\exists c]\{c \varepsilon a\}$

2 $[cd]\{c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d\}$ (premise)

3 $[c]\{c \varepsilon a \supset c \varepsilon b\} \supset$

4 $c_1 \varepsilon a$ (OΣ: 1)

5 $c_1 \varepsilon a \supset c_1 \varepsilon b$ (OΠ: 3)

6 $c_1 \varepsilon b$ (5, 4)

7 $c \varepsilon c_1 \supset c \varepsilon a$ (A 2, 4)

8 $c \varepsilon a \wedge c_1 \varepsilon a \supset c \varepsilon c_1$ (OΠ: 2)

9 $c \varepsilon a \supset c \varepsilon c_1$ (8, 4)

10 $c \varepsilon c_1 \equiv c \varepsilon a$ (7, 9)

11 $[c]\{c \in c_1 \equiv c \varepsilon a\}$ (DΠ: 10)

12 $[\varphi]\{\varphi(c_1) \equiv \varphi(a)\}$ (ER 1, 11)

13 $\rho\langle b\rangle(c_1) \equiv \rho\langle b\rangle(a)$ (OΠ: 12)

14 $\rho\langle b\rangle(c_1)$ (D 1, 6)

15 $\rho\langle b\rangle(a)$ (13, 14)

$a \varepsilon b$ (D 1, 15)

A 8=T $a \varepsilon b \equiv [\exists c]\{c \varepsilon a\} \wedge [cd]\{c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d\} \wedge [c]\{c \varepsilon a \supset c \varepsilon b\}$ (A 6, A 7)

Theorem. T *is equivalent to* A. A *can act as a single axiom of ontology.*

The former of this theorem is true from Lemma 1 and Lemma 2. The latter of this theorem is true from the fact that T is a single axiom.

References

[1] C. Lejewski: On Leśniewski's ontology. Ratio, **1**, 150–176 (1958).
[2] J. Slupecki: S. Leśniewski's calculus of names. Studia Logica, **3** (1955).