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CORRECTIONS AND SUPPLEMENTS TO "ON TIGHT 4-DESIGNS"

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The paper in the title, which we quote as (T), contains numerous errors of careless "typographical" nature. Two of them, however, which occur in (105) and (170), are very serious.

The purpose of this note is to salvage a portion of (T) with necessary corrections and to have a few more formulas.

(T) may be divided into two parts. The first part handles the case where $b_2 \geqq b_1$ (for the definition of b_1 and b_2 see (T), p. 495), and this is the case for two Witt tight 4-designs. There is no trouble in this part. All the troubles lie in the second part which handles the case where $b_1 > b_2$.

The author is thankful to Professor Hikoe Enomoto for his kind comments and suggestions.

REMARK. H. Enomoto, R. Noda and the author are now jointly preparing a paper which shows that (105) does not hold and that there exist finitely many possibilities for v and k as parameters of tight 4-designs.

1. Corrections

Page 493 Line 2. Replace to by of.
Line. 18. Replace 27 by 23.
Line 22. Replace s by k.

Page 494 (5). Replace N_2 and N_1 by N_1 and N_2 respectively.

Page 497 (36). Insert (between $\frac{1}{2}$ and v.

Page 498. The first assertion in §3 becomes obvious if we consider the Diophantine equation $2X - Y = Y^2 - X^2$ or $2X = Y^2 - X^2$.

Page 501 Remark. Replace 8 by 18.

Page 503 Line 23. Replace e_3 by c_3.

Page 504 (73), Page 505 Lines 4 and 8. Replace $2e^2$ by $4e^2$. Then the given argument does not hold. But we can argue as follows: We may assume

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that $k\leqq\frac{1}{2}v$. By (21) $2ae=2k-2m_2<v$. By (70) $2e<2C+6e-v+3$. By (71) $2e<(c_1+4)e+c_0+2-D\leqq e^2+4e+1-D$. Thus $D=e^2+2e$, and hence $c_1=c_0=e-1$. Furthermore, $v=4e^2+4e$, $2C+6e-v+3=2e+1$ and $a=2e+1$. Now by (67) $k^2-4e(e+1)k+(2e+1)^2(2e^2+2e-1)=0$. The discriminant of this equation is negative.

Page 506 Line 8. Replace 5 by 4. Then we can argue as in the case $a=4$.

Page 509 Line 26. Replace 1 by 3.

Page 511 Line 13. Replace 8 by 12. Then the given argument does not hold. But we can argue as follows: We have that $v=9a^2-1$. By (30) $2k=3a^2+1+x$. Thus from (1) we obtain that

$$\{x-(6a^2-2)\}^2=(3a^2-1)(9a^2-1).$$

Since a is odd, we may put $3a^2-1=2A^2$ and $9a^2-1=2B^2$, where A and B are positive integers. Then $B^2=3A^2+1$. In particular, A and B are relatively prime. Furthermore, $x=4A^2\pm 2AB$. By (47) we have that $3yA=(3A^2\pm AB+1)\times(2A\pm B)$, which implies that $A=1$.

Page 512 Line 2. Replace $e>4d_0$ by $e<4d_0$. Then the given argument does not hold, and there remain two cases which we fail to repair. Anyway we argue as follows: We may assume that $d_1=e-1$, $c_1=2$ and $c_0d_0=me$ with m positive. Then from (103) we obtain that $c_0\equiv 2d_0+m+2 \pmod e$. So we distinguish three cases.

Case 1. $c_0+2e=2d_0+m+2$. In this case from (103) we obtain that $3d_0=3e-3-m$ and $3c_0=m$. Since $c_0d_0=me$, we get a contradiction that $d_0=3e$.

Case 2. $c_0+e=2d_0+m+2$. In this case from (103) we obtain that $3d_0=2e-m-2$ and $3c_0=e+m+2$. From (100) we obtain that $v=4e^2+4e+1$. From (102) we obtain that $3(v+1)=2a(10e+5+m)$. So we have that

$$\text{(i)}\qquad 3(2e^2+2e+1)=a(10e+5+m)$$

and

$$\text{(ii)}\qquad 9me=(e+2+m)(2e-2-m).$$

Eliminating m from (i) and (ii), we obtain that

$$\text{(iii)}\qquad a^2-4ae+2e^2-2a+2e+1=0.$$

(iii) can be rewritten as a Pellian equation

$$\text{(iv)}\qquad (2e-2a+1)^2-2a^2=-1.$$

We notice that $(X, Y)=(1, 1)$ is the fundamental solution of $X^2-2Y^2=-1$ and other positive solutions are obtained by multiplying positive powers of

$$\begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix} \text{ to } (1, 1).$$

Case 3. $c_0=2d_0+m+2$. In this case from (103) we obtain that $3d_0=e-m-1$ and $3c_0=2e+m+4$. From (100) $v=4e^2+4e+2$. From (102) $3(v+1)=a(22e+11+2m)$. So we have that

$$3(4e^2+4e+3) = a(22e+11+2m) \tag{i}$$

and

$$9me = (2e+4+m)(e-1-m)\,. \tag{ii}$$

Eliminating m from (i) and (ii), we obtain that

$$a^2-8ae+4e^2-4a+4e+3 = 0\,. \tag{iii}$$

(iii) can be rewritten as a Pellian equation

$$(4e+2-a)^2-3(2e+1)^2 = -2\,. \tag{iv}$$

We notice that $(X, Y)=(1, 1)$ is the fundamental solution of $X^2-3Y^2=-2$ and other positive solutions are obtained by multiplying positive powers of

$$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \text{ to } (1, 1)\,.$$

So we have to leave Cases 2 and 3 incomplete. It is, however, not so difficult to invoke Thue-Siegel type theorems to secure that only finitely many pairs of a and e come into the possibilities for the parameters of tight 4 designs.

Henceforth we assume that $d_1<e-1$, namely (105). But we notice that most relevant equalities in (T) are independent from this assumption.

Page 512 Lines 12, 15, 21 and 29. Page 513 Line 4. Replace d_0 by $2d_0$.

Page 512 Line 13. Replace $(d_1+7)e^2+1$ by $(d_1+7)e^2-e+2$ if $c_0=0$; by $(d_1+6)e^2+2$ if $d_0=0$; by $(d_1+7)e^2-4$ if $c_0d_0\neq 0$.

Page 512 Line 18. Replace 7 and, by 6 and, respectively.

Drop Page 512 Lines 19–22 and the first sentence of Line 23.

Page 512 Line 24. Replace 6 by 5.

Drop from "Since" of Page 512 Line 25 to the second sentence of Page 513 Line 1.

Page 513 Line 5. Replace $\leqq e^3+2e^2+(7+d_0)e+1$ by $<e^3+2e^2+(8+d_0)e$.

Page 513 Line 16. Replace $a=4$ by $18a=73$. Hence we can drop the next few lines.

Page 514 Lines 16, 21 and 26. Replace $k_0(c_0+d_0+1)$ by $2k_0(c_0+d_0+1)$.

Page 514 Line 25. Replace e_1 by c_1.

Page 515, (120). We notice that (120) holds without the assumption that $d_1<e-1$.

Page 516 Line. 6 Replace 3 by 13.

Page 518 Lines 1 and 5. Replace $a(d_0+1)$ by $d_0(a+1)$.

Page 518 Line 26. Replace $a+1$ by $a-1$.

Page 520 (164). Replace $4e^2$ by $2e^2$.

Page 521 (170). Replace $a(e+1)v$ by $e(a+1)v$. Then the given argument does not hold, and the proof is incomplete.

2. Main results of (T)

We may summarize main results of (T) in the following two propositions. We use the notation in (T).

Proposition 1. *Witt* 4-(23, 7, 1) *design is the only non-trivial tight* 4-*design with k a prime.*

Proof. (T), Lemma 1.

Proposition 2. *Witt designs are only non-trivial tight* 4-*designs with* $b_2 \geqq b_1$.

Proof. By (59) and (17) of (T) we have that $2b_1=C(v-3)+2C+2e$ and $2b_2=(v-3)(v-C)+2(v-C)-2e-2$. Then $b_2 \geqq b_1$ if and only if $v \geqq 2C+\frac{4e+2}{v-1}$. Now if $d_2=0$ ((T), Page 510), then by (100) and (104) of (T) we have that $v=4e^2+(j+2)e+c_0+d_0+1 \leqq 4e^2+(j+4)e$. Moreover, by (101) and (111) of (T) we have that $2C=4e^2+2c_1e+2c_0 \geqq 4e^2+(2j+4)e$. So the proof is complete if $d_1<e-1$. If $d_1=e-1$, then we have that $v \leqq 4e^2+4e+2$ and $C=2e^2+2e+2d_0+m+2$.

3. Formulas

We would like to add a few more formulas. By (164) and (166) of (T) we have that

$$(1) \qquad (a^2-1)J = 4\{e^2-(2a-1)e-(a-1)\}\,.$$

By (112) and (138) of (T) (1) implies that

$$(2) \qquad 1 \leqq J \leqq 2(e-a-1)\,.$$

Hence by (155) of (T) we have that

$$(3) \qquad (a^2-1)(v-4e^2-4e+1) = 4(e-a)(e-a+1)$$

and

$$(4) \qquad 4e^2+4e+4 \leqq v < 4e^2+6e\,, \qquad \text{(provided that } d_1<e-1)\,.$$

Moreover by (110) of (T) we obtain that

$$(a^2-1)k^2-\{4a^2e^2+4a(a-2)e+(3a-1)(a-1)\}k \\ +2a^2e(e+1)(2ae+a-3)=0\,. \tag{5}$$

We notice that the discriminant of (5) must be a square.

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