Nobusawa, Nobuo
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On the Imbedding Problem of Algebraic Number Fields

By Nobuo NOBUSAWA

The purpose of this note is to give another proof of Akagawa's theorem [1] on the imbedding problem of algebraic number fields: the imbedding problem for a Galois extension with a Galois group of order l^n (l is a prime) in case a relative Galois group is of order l can be solved if the corresponding local imbedding problem at each place of the ground field is solved; moreover we can find a solution such that its local completions coincide with the solutions given in advance of local problems at a finite number of places and that the mappings of local Galois groups to the global Galois group coincide with the canonical ones. The proof in this note, using Richter's "*Monodromiesatz*", seems to be a little simpler.

1. Local and global imbedding problems

Throughout this paper, let $\mathfrak{g}$ be a Galois group of order l^{n-1} of a Galois extension K over a finite algebraic number field k, let $\mathfrak{G}$ be a group of order l^n, and let $\mathfrak{N}$ be a normal subgroup of $\mathfrak{G}$ of order l such that $\mathfrak{G}$ is a central extension of $\mathfrak{N}$ by $\mathfrak{g}$:

$$(1) \qquad \mathfrak{G}/\mathfrak{N} \overset{\varphi}{\cong} \mathfrak{g}\,.$$

The imbedding problem $P(K/k, \mathfrak{G}/\mathfrak{N} \overset{\varphi}{\cong} \mathfrak{g})$ is then to find a larger field L containing K such that L/k is a Galois extension with the Galois group $\mathfrak{G}$ and that the homomorphism $\varphi^* = \varphi\lambda$ of $\mathfrak{G}$ onto $\mathfrak{g}$ (λ is the canonical homomorphism: $\mathfrak{G} \overset{\lambda}{\rightarrow} \mathfrak{G}/\mathfrak{N}$) coincides with the natural mapping of $\mathfrak{G}$ onto $\mathfrak{g}$ gained by the restriction of the Galois group $\mathfrak{G}$ of L to the subfield K.

A local imbedding problem corresponding to $P(K/k, \mathfrak{G}/\mathfrak{N} \overset{\varphi}{\cong} \mathfrak{g})$ at a place (a prime divisor) $\mathfrak{p}$ of k is defined as follows: Let $\mathfrak{P}$ be a place of K over $\mathfrak{p}$, and let $\mathfrak{z}_{\mathfrak{P}}$ be the decomposition group at $\mathfrak{P}$. $\mathfrak{z}_{\mathfrak{P}}$ is considered to be the Galois group of $K_{\mathfrak{P}}/k_{\mathfrak{p}}$. Let $\mathfrak{Z}_{\mathfrak{P}}$ be a group and let $\nu_{\mathfrak{P}}$ be an isomorphism of $\mathfrak{Z}_{\mathfrak{P}}$ into $\mathfrak{G}$ such that $\varphi^*\nu_{\mathfrak{P}}(\mathfrak{Z}_{\mathfrak{P}}) = \mathfrak{z}_{\mathfrak{P}}$. If the kernel of $\varphi^*\nu_{\mathfrak{P}}$ is $\mathfrak{N}_{\mathfrak{P}}$, we have

$$\mathfrak{Z}_{\mathfrak{P}}/\mathfrak{N}_{\mathfrak{P}} \overset{\psi}{\cong} \mathfrak{z}_{\mathfrak{P}}\,, \tag{2}$$

where ψ is gained naturally from $\varphi^*\nu_{\mathfrak{P}}$. An imbedding problem $P(K_{\mathfrak{P}}/k_{\mathfrak{p}},\ \mathfrak{Z}_{\mathfrak{P}}/\mathfrak{N}_{\mathfrak{P}} \overset{\psi}{\cong} \mathfrak{z}_{\mathfrak{P}})$ is called a local imbedding problem corresponding to $P(K/k,\ \mathfrak{G}/\mathfrak{N} \overset{\varphi}{\cong} \mathfrak{g})$ at a place $\mathfrak{p}$. This is of course not uniquely determined.

Now we recall the Brauer-Richter-Reichardt's theory. Let N be a generator of $\mathfrak{N}$. The group extension (1) is uniquely determined by a factor set $\{N^{a_{u,v}}\}$ since it is a central extension, where u and $v \in \mathfrak{g}$ and $a_{u,v}$ are rational integers. On the other hand, let ζ be a primitive l-th root of the unity. Assume $K \ni \zeta$ (and hence $k \ni \zeta$), and $\{\zeta^{a_{u,v}}\}$ is a factor set of K/k. Then we can make a crossed product A of K/k by $\mathfrak{g}$ with a factor set $\{\zeta^{a_{u,v}}\}$. The next theorem is a special case of Brauer's theorem [2].

Theorem 1. *Assume that the group extension* (1) *does not split. When $K \ni \zeta$, the imbedding problem $P(K/k,\ \mathfrak{G}/\mathfrak{N} \overset{\varphi}{\cong} \mathfrak{g})$ is solved if and only if the crossed product A splits.*

From this theorem, a special case of Richter's theorem [5] is gained:

Theorem 2. *Assume $K \ni \zeta$. $P(K/k,\ \mathfrak{G}/\mathfrak{N} \overset{\varphi}{\cong} \mathfrak{g})$ is solved if and only if at least a local imbedding problem $P(K_{\mathfrak{P}}/k_{\mathfrak{p}},\ \mathfrak{Z}_{\mathfrak{P}}/\mathfrak{N}_{\mathfrak{P}} \overset{\psi}{\cong} \mathfrak{z}_{\mathfrak{P}})$ corresponding to it at each place $\mathfrak{p}$ of k is solved.*

When $K \not\ni \zeta$, we put $\bar{k} = k(\zeta)$ and $\bar{K} = K(\zeta)$. Then $\bar{K}/\bar{k}$ is a Galois extension with the Galois group $\mathfrak{g}$. Denote by $\bar{A}$ a crossed product of $\bar{K}/\bar{k}$ by $\mathfrak{g}$ with the factor set $\{\zeta^{a_{u,v}}\}$. The proof of the next theorem is gained by Reichardt [4].

Theorem 3. *Assume* (1) *does not split. If $\bar{A}$ splits, then $P(K/k,\ \mathfrak{G}/\mathfrak{N} \overset{\varphi}{\cong} \mathfrak{g})$ is solved.*

The next theorem is a natural consequence of the above three theorems.

Theorem 4. *$P(K/k,\ \mathfrak{G}/\mathfrak{N} \overset{\varphi}{\cong} \mathfrak{g})$ is solved if and only if at least a local problem corresponding to it is solved at each place of k, whether K contains ζ or not.*

Proof. The theorem is clear when $K \ni \zeta$. If (1) splits, $P(K/k,\ \mathfrak{G}/\mathfrak{N} \overset{\varphi}{\cong} \mathfrak{g})$ is always solved. Hence assume that $K \not\ni \zeta$ and that (1) does not split. If $P(K_{\mathfrak{P}}/k_{\mathfrak{p}},\ \mathfrak{Z}_{\mathfrak{P}}/\mathfrak{N}_{\mathfrak{P}} \overset{\psi}{\cong} \mathfrak{z}_{\mathfrak{P}})$ is solved, then $P(\bar{K}_{\mathfrak{P}}/\bar{k}_{\mathfrak{p}},\ \mathfrak{Z}_{\mathfrak{P}}/\mathfrak{N}_{\mathfrak{P}} \overset{\psi}{\cong} \mathfrak{z}_{\mathfrak{P}})$ is solved, as is easily seen. By Theorem 2, $P(\bar{K}/\bar{k},\ \mathfrak{G}/\mathfrak{N} \overset{\varphi}{\cong} \mathfrak{g})$ is solved, and hence

$\bar{A}$ splits by Theorem 1, that is, $P(K/k,\ \mathfrak{G}/\mathfrak{N}\overset{\varphi}{\cong}\mathfrak{g})$ is solved by Theorem 3.

2. The exact imbedding problem

According to Akagawa [1], the exact imbedding problem is defined as follows: Let $P(K/k,\ \mathfrak{G}/\mathfrak{N}\overset{\varphi}{\cong}\mathfrak{g})$ be a global imbedding problem, and let S be a set of a finite number of places $\mathfrak{p}$ of k. We assume that a local imbedding problem $P(K_{\mathfrak{P}}/k_{\mathfrak{p}},\ \mathfrak{Z}_{\mathfrak{P}}/\mathfrak{N}_{\mathfrak{P}}\overset{\psi}{\cong}\mathfrak{z}_{\mathfrak{P}})$ is given and has a solution $F_{\mathfrak{P}}$ at each place $\mathfrak{p}$ of S. Then arises a problem to find a solution L of $P(K/k,\ \mathfrak{G}/\mathfrak{N}\overset{\varphi}{\cong}\mathfrak{g})$ such that its $\mathfrak{P}$-completion $L_{\mathfrak{P}}$ is isomorphic to $F_{\mathfrak{P}}$ and that $\nu_{\mathfrak{P}}$ coincides with the canonical mapping (the inclusion mapping as a decomposition group) of the Galois group $\mathfrak{Z}_{\mathfrak{P}}$ of $L_{\mathfrak{P}}/k_{\mathfrak{p}}$ (which we may identify with $F_{\mathfrak{P}}/k_{\mathfrak{p}}$) into the Galois group $\mathfrak{G}$ of L/k. This problem is called *an exact imbedding problem* and will be denoted by $P(\mathfrak{G}/\mathfrak{N}\overset{\varphi}{\cong}\mathfrak{g}\,;\ \mathfrak{Z}_{\mathfrak{P}},\ \nu_{\mathfrak{P}},\ F_{\mathfrak{P}})$. (We always fix K and a set S).

Corresponding to the definition of the product of group extensions (that is, the multiplication in 2-cohomology groups), we define a product of two exact imbedding problems $P(\mathfrak{G}^{(1)}/\mathfrak{N}\overset{\varphi_1}{\cong}\mathfrak{g}\,;\ \mathfrak{Z}_{\mathfrak{P}}^{(1)},\ \nu_{\mathfrak{P}}^{(1)},\ F_{\mathfrak{P}}^{(1)})$ and $P(\mathfrak{G}^{(2)}/\mathfrak{N}\overset{\varphi_2}{\cong}\mathfrak{g}\,;\ \mathfrak{Z}_{\mathfrak{P}}^{(2)},\ \nu_{\mathfrak{P}}^{(2)},\ F_{\mathfrak{P}}^{(2)})$ as follows: Put $(\mathfrak{G}^{(1)}\times\mathfrak{G}^{(2)})_{\mathfrak{g}}=\{(A^{(1)},\ A^{(2)})$ with $A^{(1)}\in\mathfrak{G}^{(1)}$ and $A^{(2)}\in\mathfrak{G}^{(2)}$ such that $\varphi_1^*(A^{(1)})=\varphi_2^*(A^{(2)})\}$. Then $(\mathfrak{G}^{(1)}\times\mathfrak{G}^{(2)})_{\mathfrak{g}}$ contains a normal subgroup $\tilde{\mathfrak{N}}$ which is generated with $(N,\ N)$. Put $\mathfrak{G}^{(3)}=(\mathfrak{G}^{(1)}\times\mathfrak{G}^{(2)})_{\mathfrak{g}}/\tilde{\mathfrak{N}}$. $\mathfrak{G}^{(3)}$ contains $(\mathfrak{N}\times\mathfrak{N})/\tilde{\mathfrak{N}}$, which is isomorphic with $\mathfrak{N}$ by the natural way and we identify with $\mathfrak{N}$. Then we have

$$\mathfrak{G}^{(3)}/\mathfrak{N}\overset{\varphi_3}{\cong}\mathfrak{g}\,,$$

where φ_3 is defined such that $\varphi_3((A^{(1)},\ A^{(2)}) \bmod \mathfrak{N})=\varphi_1^*(A^{(1)})\ \ (=\varphi_2^*(A^{(2)}))$. We shall next determine $\mathfrak{Z}_{\mathfrak{P}}^{(3)}$, $\nu_{\mathfrak{P}}^{(3)}$ and $F_{\mathfrak{P}}^{(3)}$. Put $\bar{F}_{\mathfrak{P}}=F_{\mathfrak{P}}^{(1)}\cup F_{\mathfrak{P}}^{(2)}$ and $\bar{\mathfrak{Z}}_{\mathfrak{P}}=\mathfrak{G}(F_{\mathfrak{P}}^{(1)}\cup F_{\mathfrak{P}}^{(2)}/k_{\mathfrak{p}})$ (the Galois group of $F_{\mathfrak{P}}^{(1)}\cup F_{\mathfrak{P}}^{(2)}/k_{\mathfrak{p}}$). Let $\bar{\nu}_{\mathfrak{P}}$ be an isomorphism of $\bar{\mathfrak{Z}}_{\mathfrak{P}}$ into $(\mathfrak{G}^{(1)}\times\mathfrak{G}^{(2)})_{\mathfrak{g}}$ defined such that, for an element $\bar{A}$ of $\bar{\mathfrak{Z}}_{\mathfrak{P}}$, $\bar{\nu}_{\mathfrak{P}}(\bar{A})=(\nu_{\mathfrak{P}}^{(1)}(A^{(1)}),\ \nu_{\mathfrak{P}}^{(2)}(A^{(2)}))$ where $A^{(i)}$ are elements of $\mathfrak{Z}_{\mathfrak{P}}^{(i)}$ gained by the restriction of $\bar{A}$ to $F_{\mathfrak{P}}^{(i)}$. Now let $F_{\mathfrak{P}}^{(3)}$ be the fixed subfield of $F_{\mathfrak{P}}^{(1)}\cup F_{\mathfrak{P}}^{(2)}$ of $\bar{\nu}_{\mathfrak{P}}^{-1}(\bar{\nu}_{\mathfrak{P}}(\bar{\mathfrak{Z}}_{\mathfrak{P}})\cap\mathfrak{N})$, let $\mathfrak{Z}_{\mathfrak{P}}^{(3)}$ be the Galois group $\mathfrak{G}(F_{\mathfrak{P}}^{(3)}/k_{\mathfrak{p}})$, and let $\nu_{\mathfrak{P}}^{(3)}$ be the isomorphism of $\mathfrak{Z}_{\mathfrak{P}}^{(3)}$ into $\mathfrak{G}^{(3)}$ gained naturally from $\bar{\nu}_{\mathfrak{P}}$. In this case, we put

$$\begin{aligned}P(\mathfrak{G}^{(1)}/\mathfrak{N}\overset{\varphi_1}{\cong}\mathfrak{g}\,;\ \mathfrak{Z}_{\mathfrak{P}}^{(1)},\ \nu_{\mathfrak{P}}^{(1)},\ F_{\mathfrak{P}}^{(1)})&+P(\mathfrak{G}^{(2)}/\mathfrak{N}\overset{\varphi_2}{\cong}\mathfrak{g}\,;\ \mathfrak{Z}_{\mathfrak{P}}^{(2)},\ \nu_{\mathfrak{P}}^{(2)},\ F_{\mathfrak{P}}^{(2)})\\ &=P(\mathfrak{G}^{(3)}/\mathfrak{N}\overset{\varphi_3}{\cong}\mathfrak{g}\,;\ \mathfrak{Z}_{\mathfrak{P}}^{(3)},\ \nu_{\mathfrak{P}}^{(3)},\ F_{\mathfrak{P}}^{(3)})\,.\end{aligned}$$

It will be verified that all exact imbedding problems (for fixed K and S) form an additive group by this product.

Theorem 5. *When the group extension* (1) *splits, every exact imbedding problem* $P(\mathfrak{G}/\mathfrak{N}\overset{\varphi}{\cong}\mathfrak{g}\,;\,\mathfrak{Z}_{\mathfrak{P}},\nu_{\mathfrak{P}},F_{\mathfrak{P}})$ *has infinitely many solutions for arbitrary* $\mathfrak{Z}_{\mathfrak{P}}$, $\nu_{\mathfrak{P}}$, *and* $F_{\mathfrak{P}}$.

Proof. Since $\mathfrak{G}/\mathfrak{N}\overset{\varphi}{\cong}\mathfrak{g}$ splits, $\mathfrak{G}=\mathfrak{A}\times\mathfrak{N}$ with $\mathfrak{A}\cong\mathfrak{g}$. It suffices to find a field k_1 such that K and k_1 are independent over k, that k_1/k is a Galois extension with the Galois group $\mathfrak{N}$ and that Kk_1 suffices the condition of a solution. Then we may assume $\mathfrak{A}(\cong\mathfrak{g})$ is the Galois group of K/k. First, assume $\nu_{\mathfrak{P}}(\mathfrak{Z}_{\mathfrak{P}})\supset\mathfrak{N}$. Then $\mathfrak{Z}_{\mathfrak{P}}=\mathfrak{Z}'\times\mathfrak{N}'$ with $\nu_{\mathfrak{P}}(\mathfrak{Z}')\subset\mathfrak{A}$ and $\nu_{\mathfrak{P}}(\mathfrak{N}')=\mathfrak{N}$, and $F_{\mathfrak{P}}=K_{\mathfrak{P}}\times k'_{\mathfrak{p}}$ where $K_{\mathfrak{P}}$ is the invariant field of $\mathfrak{N}'$ and $k'_{\mathfrak{p}}$ of $\mathfrak{Z}'$. From the assumption, $\nu_{\mathfrak{P}}$ is uniquely determined for each element of $\mathfrak{Z}'$ (the canonical mapping). So the problem is reduced to find a field k_1 independent of K over k such that k_1/k is a Galois extension with the Galois group $\mathfrak{N}$, that its completion $k_{1\mathfrak{p}}\cong k'_{\mathfrak{p}}$, and that the mapping $\nu_{\mathfrak{P}}$ of the Galois groug $\mathfrak{N}'$ of $k_{1\mathfrak{p}}/k_{\mathfrak{p}}$ $(=k'_{\mathfrak{p}}/k_{\mathfrak{p}})$ onto $\mathfrak{N}$ coincides with the canonical one. Next, assume that $\nu_{\mathfrak{P}}(\mathfrak{Z}_{\mathfrak{P}})\not\supset\mathfrak{N}$. Then $\mathfrak{Z}_{\mathfrak{P}}\cong\mathfrak{z}_{\mathfrak{P}}$, and hence $F_{\mathfrak{P}}=K_{\mathfrak{P}}$. If we put $\nu_{\mathfrak{P}}(\mathfrak{Z}_{\mathfrak{P}})=\mathfrak{A}'$, $\mathfrak{A}'=\mathfrak{z}_{\mathfrak{P}}$ or $\mathfrak{z}_{\mathfrak{P}}\cap\mathfrak{A}'$ has index l in $\mathfrak{z}_{\mathfrak{P}}$. Let $k'_{\mathfrak{p}}$ be the invariant subfield of $\mathfrak{z}_{\mathfrak{P}}\cap\mathfrak{A}'$. We can define an isomorphism $\nu'_{\mathfrak{P}}$ of the Galois group $\mathfrak{G}(k'_{\mathfrak{p}}/k_{\mathfrak{p}})$ into $\mathfrak{N}$ such that, if A' is an element of $\mathfrak{G}(k'_{\mathfrak{p}}/k_{\mathfrak{p}})$ gained by the restriction of $\bar{A}\in\mathfrak{G}(F_{\mathfrak{P}}/k_{\mathfrak{p}})$, we put $\nu'_{\mathfrak{P}}(A')=N^a$ where $\nu_{\mathfrak{P}}(\bar{A})=(A,N^a)\in\mathfrak{A}\times\mathfrak{N}$. $\nu'_{\mathfrak{P}}$ is then defined independent of the choise of $\bar{A}$ as is seen from the definition of $k'_{\mathfrak{p}}$. (When $\mathfrak{z}_{\mathfrak{P}}=\mathfrak{A}'$, $\nu'_{\mathfrak{P}}$ is the trivial mapping of the unity.) In this case, the problem is reduced to find a field k_1 independent of K over k such that k_1/k is a Galois extension with the Galois group $\mathfrak{N}$, that its local completion $k_{1\mathfrak{p}}$ is isomorphic to $k'_{\mathfrak{p}}$, and that the mapping $\nu'_{\mathfrak{P}}$ coincides with the canonical one. To find k_1 of these properties at a finite number of places is always possible by infinitely many ways by Hasse [2]. (Cf. also [1].)

Theorem 6. *Let*

$$P(\mathfrak{G}^{(1)}/\mathfrak{N}\overset{\varphi_1}{\cong}\mathfrak{g}\,;\,\mathfrak{Z}^{(1)}_{\mathfrak{P}},\nu^{(1)}_{\mathfrak{P}},F^{(1)}_{\mathfrak{P}})+P(\mathfrak{G}^{(2)}/\mathfrak{N}\overset{\varphi_2}{\cong}\mathfrak{g}\,;\,\mathfrak{Z}^{(2)}_{\mathfrak{P}},\nu^{(2)}_{\mathfrak{P}},F^{(2)}_{\mathfrak{P}})$$
$$=P(\mathfrak{G}^{(3)}/\mathfrak{N}\overset{\varphi_3}{\cong}\mathfrak{g}\,;\,\mathfrak{Z}^{(3)}_{\mathfrak{P}},\nu^{(3)}_{\mathfrak{P}},F^{(3)}_{\mathfrak{P}})\,.$$

If $P(\mathfrak{G}^{(1)}/\mathfrak{N}\overset{\varphi_1}{\cong}\mathfrak{g}\,;\,\mathfrak{Z}^{(1)}_{\mathfrak{P}},\nu^{(1)}_{\mathfrak{P}},F^{(1)}_{\mathfrak{P}})$ *and* $P(\mathfrak{G}^{(2)}/\mathfrak{N}\overset{\varphi_2}{\cong}\mathfrak{g}\,;\,\mathfrak{Z}^{(2)}_{\mathfrak{P}},\nu^{(2)}_{\mathfrak{P}},F^{(2)}_{\mathfrak{P}})$ *have solutions independent of each other, then the third problem* $P(\mathfrak{G}^{(3)}/\mathfrak{N}\overset{\varphi_3}{\cong}\mathfrak{g}\,;\,\mathfrak{Z}^{(3)}_{\mathfrak{P}},\nu^{(3)}_{\mathfrak{P}},F^{(3)}_{\mathfrak{P}})$ *has a solution.*

Proof. If L_1 and L_2 are independent solutions of the first and second problems, then $L_1 \cup L_2$ is a Galois extension with the Galois group $(\mathfrak{G}^{(1)} \times \mathfrak{G}^{(2)})_{\mathfrak{g}}$. If L_3 is the invariant subfield of $\mathfrak{N}(\subset (\mathfrak{G}^{(1)} \times \mathfrak{G}^{(2)})_{\mathfrak{g}})$, then L_3 is a solution of the third problem.

Now we conclude this paper with Akagawa's theorem.

Theorem 7. *If S contains all ramified places at K/k, then every exact imbedding problem $P(\mathfrak{G}/\mathfrak{N} \overset{\varphi}{\cong} \mathfrak{g}\,;\, \mathfrak{Z}_{\mathfrak{P}}, \nu_{\mathfrak{P}}, F_{\mathfrak{P}})$ has infinitely many solutions.*

Proof. The existence of $\mathfrak{Z}_{\mathfrak{P}}$, $\nu_{\mathfrak{P}}$ and $F_{\mathfrak{P}}$ at all ramified places implies the solvability of a local imbedding problem at each place of k, since the solvability at unramified places is clear. Then by Theorem 4 the global imbedding problem $P(K/k,\ \mathfrak{G}/\mathfrak{N} \overset{\varphi}{\cong} \mathfrak{g})$ has a solution L. If we denote the decomposition group, the canonical mapping and the local completion of L by $\mathfrak{Z}_{\mathfrak{P}}(L)$, $\iota_{\mathfrak{P}}$ and $L_{\mathfrak{P}}$, then $P(\mathfrak{G}/\mathfrak{N} \overset{\varphi}{\cong} \mathfrak{g}\,;\, \mathfrak{Z}_{\mathfrak{P}}(L), \iota_{\mathfrak{P}}, L_{\mathfrak{P}})$ has a solution L. Put

$$P(\mathfrak{G}/\mathfrak{N} \overset{\varphi}{\cong} \mathfrak{g}\,;\, \mathfrak{Z}_{\mathfrak{P}}(L), \iota_{\mathfrak{P}}, L_{\mathfrak{P}}) - P(\mathfrak{G}/\mathfrak{N} \overset{\varphi}{\cong} \mathfrak{g}\,;\, \mathfrak{Z}_{\mathfrak{P}}, \nu_{\mathfrak{P}}, F_{\mathfrak{P}})$$
$$= P(\mathfrak{G}^{(0)}/\mathfrak{N} \overset{\varphi_0}{\cong} \mathfrak{g}\,;\, \mathfrak{Z}_{\mathfrak{P}}^{(0)}, \nu_{\mathfrak{P}}^{(0)}, F_{\mathfrak{P}}^{(0)})\,.$$

Then $\mathfrak{G}^{(0)}/\mathfrak{N} \overset{\varphi_0}{\cong} \mathfrak{g}$ splits, and hence by Theorem 5 and 6 we can find infinitely many solutions of $P(\mathfrak{G}^{(0)}/\mathfrak{N} \overset{\varphi_0}{\cong} \mathfrak{g}\,;\, \mathfrak{Z}_{\mathfrak{P}}^{(0)}, \nu_{\mathfrak{P}}^{(0)}, F_{\mathfrak{P}}^{(0)})$ and also infinitely many solutions of $P(\mathfrak{G}/\mathfrak{N} \overset{\varphi}{\cong} \mathfrak{g}\,;\, \mathfrak{Z}_{\mathfrak{P}}, \nu_{\mathfrak{P}}, F_{\mathfrak{P}})$.

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