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SYLLOGISTIC WITH COMPLEX TERMS

A. J. BAKER

1. *Simple Terms* In studies of classical syllogistic logic it has been shown how the theses of that logic can be axiomatized in accordance with the principles of modern logic. Thus J. Łukasiewicz[1] and I. M. Bocheński[2] have axiomatized the ordinary system of positive terms—i.e., the system containing the 24 syllogistic moods and the 18 rules for conversion and opposition—by introducing four axioms and deducing the remaining theses as theorems. If we adopt Łukasiewicz's symbolism for expressing the A, E, I, O propositions of syllogistic (so that Aab is to be read "All a are b", etc.) and if, for clarity, we use Russellian-type symbolism for the propositional calculus, we can, for example, express Bocheński's axioms as follows.

System **CS**

S1 Aaa
S2 Iaa
S3 $Acb \,\&\, Aac \supset Aab$ (Barbara)
S4 $Ecb \,\&\, Iac \supset Oab$ (Ferio)

When the theses for negative terms are added—i.e., when the rules for obversion and consequent rules for contraposition, etc. are introduced—the complete system of simple terms can be axiomatized.

Thomas, in demonstrating this[3], used the following axioms.

System **CS**(n)

S1 Aaa
S2 Iaa
S4 $Ecb \,\&\, Iac \supset Oab$ (Ferio)

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1. *Aristotle's Syllogistic, from the standpoint of modern logic*, 2nd ed. (1957).
2. "On the Categorical Syllogism," in *Logico-Philosophical Studies*, A. Menne, ed. (1962).
3. "CS (n): an extension of CS," *op. cit.*, Menne, ed.

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Łukasiewicz, Bocheński and Thomas employ as axioms the non-contingent propositional forms *Aaa* and *Iaa*. But if, perhaps more in the spirit of Aristotle and other traditional logicians, we confine syllogistic propositions to *contingent* forms, a different set of axioms is needed. An illustration of such a set, which fits in well with various traditional preoccupations, is the following.

System **A**(n)

S5 *Iab* ⊃ *Iba* (*I* conversion)
S6 *Aab* ⊃ *Iab* (*I* subalternation)
S7 *Ecb* & *Aac* ⊃ *Eab* (Celarent)

There remains, however, one considerable branch of traditional logic which has not been incorporated in the axiomatized system of syllogistic. This is the branch dealing with complex terms, i.e., with conjunctive terms of the form "both *a* and *b*" (symbolized by k*ab*) and with disjunctive terms of the form "either *a* or *b*" (symbolized by v*ab*). This subject, first mentioned in some detail by de Morgan[4], has been virtually untouched since J. N. Keynes' full, but by modern standards informal and unsystematized, account[5].

What I want to do is to show how the complete set of theses involving conjunctive terms can be deduced, and to go on to point out the consequent effects on the whole system of syllogistic. For when complex terms are introduced, it is not simply a matter of enlarging the system by addition; as will emerge, a new set of axioms, essentially different from axiom-sets so far presented in the literature, can be used to deduce all the theorems concerning both simple and complex terms.

2. *Introducing Complex Terms* When complex terms are added to the system of simple terms, a difficulty arises which may appear to preclude from the outset the possibility of a traditional system containing complex terms. Thus, one well-known form of complex inference allows us to move from *Aac* to *A*k*abc*. But when this rule is applied to *Aaa* it gives us *A*k*aba* and applied to *Abb* it gives us *A*k*abb*. But from *A*k*abb* and *A*k*aba* it follows syllogistically (Darapti) that *Iab*. There is thus the extraordinary and unworkable consequence that any *I* proposition is true[6]. Again, suppose we have the rule for the conversion of *I* propositions and add the rule for a form of complex inference, *I*a*kbc* ⊃ *Iab* & *Iac*. If we now take the trivial truth, *I*k*aba*, it follows once more that *Iab*[7]. Or, as a more pointed version of this case, suppose we take the analogous trivial truth, *A*k*a*n*aa* (where n*a*

4. *Formal Logic* (1847).
5. *Studies and Exercises in Formal Logic,* 4th ed. (1906).
6. This consequence was pointed out by C. A. Meredith in an unpublished note drawn to my attention by A. N. Prior.
7. *Cf.* K. R. Popper, "The trivialization of Mathematical Logic," *Proceedings of the Tenth International Congress of Philosophy* (1949), p. 727.

is to be read as "non-a"). From this we obtain $Iakana$ (conversion), which implies Iaa and $Iana$. Hence, whatever a may be, $Iana$ is true[8]!

These examples show that *unrestricted* admission of complex terms into systems which admit propositions like Aaa would vitiate those systems. It by no means follows, however, that complex terms cannot be introduced into such systems. It is hardly surprising that if the term kab is formable, Iab will be inferable in the system, and to accommodate this inference while avoiding unwelcome consequences, we have only to apply to complex terms the principle of existential import assumed in the classical system of simple terms, *viz.*, that all terms be non-empty and non-universal. In this case, the formation of propositional forms like $Akaba$ will be subject to the condition that kab be non-empty, so that terms like $kana$ will be ill-formed and have no place at all in the system, and Iab will not be derivable unless it is in fact true.

In the case of systems like **A**(n), which exclude forms like Aaa, and when extended to complex terms will also exclude forms like $Akaba$ and $Ikaba$, the cited unwelcome inferences cannot even appear to arise. But these systems will also restrict the formation of complex terms, in accordance with the principle that all terms be non-empty. Thus, given two terms a and b and their complements na and nb, there are four possible conjunctive terms, kab, $knab$, $kanb$, $knanb$; but not all of these terms can be automatically introduced in the system; if, say, Eab is taken to be true within the system, kab is an empty, inadmissible term[9].

3. *Rules of Deduction* In addition to syllogistic axioms, axiomatization employs various other rules and definitions. These rules and definitions, as relating to the full system of simple and complex terms, are set out below, though the deduction of theorems concerning simple terms will not be repeated here.

3.1 *Rules from the Propositional Calculus*

I	$p \supset p$	(propositional identity)
II	$(p \supset \sim q) \supset (q \supset \sim p)$	(first rule of transposition)
III	$(p \supset q) \supset (\sim q \supset \sim p)$	(second rule of transposition)
IV	$(p \supset q) \supset ((q \supset r) \supset (p \supset r))$	(rule of hypothetical syllogism)
V	$\sim\sim p \supset p$	(double negation)

8. *Cf.* J. C. Shepherdson, "On the Interpretation of Aristotelian Syllogistic," *The Journal of Symbolic Logic*, vol. 21 (1956), pp. 143-144.
9. For a fuller discussion of this question, see my article "Non-empty Complex Terms," *Notre Dame Journal of Formal Logic*, vol. VII (1966), esp. pp. 49-50. On p. 53 of that article there is an inaccuracy concerning relations of opposition, a correction of which is contained in Section 4.3 of the present article.

VI	$(p \,\&\, q \supset r) \supset (q \,\&\, p \supset r)$	(commutation of premises)
VII	$(p \,\&\, q \supset r) \supset (p \supset (q \supset r))$	(first rule of exportation)
VIII	$(p \,\&\, q \supset r) \supset (q \supset (p \supset r))$	(second rule of exportation)
IX	$(p \,\&\, q \supset r) \supset (\sim r \,\&\, q \supset \sim p)$	(first rule of indirect reduction)
X	$(p \,\&\, q \supset r) \supset (p \,\&\, \sim r \supset \sim q)$	(second rule of indirect reduction)
XI	$(p \,\&\, q \supset r) \supset ((s \supset p) \supset (s \,\&\, q \supset r))$	(first rule of direct reduction)
XII	$(p \,\&\, q \supset r) \supset ((s \supset q) \supset (p \,\&\, s \supset r))$	(second rule of direct reduction)
XIII	$p \,\&\, q \supset p$	(simplification)
XIV	$(p \,\&\, q \supset r) \supset [(p \,\&\, q \supset s) \supset (p \,\&\, q \supset r \,\&\, s)]$	(conjunction)[10]

3.2 *Rule of Detachment*: *Modus Ponens*

3.3 *Substitution Rules*

3.31 Syllogistic forms may be substituted for p, q, r and s in the rules from the propositional calculus, providing this is done uniformly throughout an expression.

3.32 Term-variables, or term-ingredient-variables, may be substituted for one another in a thesis providing this is done uniformly throughout the thesis.

3.33 With definitions, the definiens and the definiendum may be substituted for one another non-uniformly in any part of a thesis.

3.4 *Primitive Expressions for Terms* In the formulations which follow, the letters a, b, and c, and combinations derived from them, should, precisely speaking, be described as "ingredient-variables" and "term-variables", but for brevity they will be referred to as "ingredients" and "terms".

3.41 A simple, positive, ingredient is any of the letters, a, b, c; a simple, negative, ingredient is any positive ingredient prefaced by n. A positive ingredient and its corresponding negative ingredient are called "complements" of one another[11].

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10. The rules cited follow Bocheński, I to XII being the ones used by him in setting out the deduction of System **CS**. But other, more concise, sets of rules can be specified; *cf.* A. Menne, "Some results of investigation of the Syllogism and their philosophical consequences," *op. cit.*, Menne, ed., in which the total list of rules for the system of simple terms, including axioms, definitions, rules from the propositional calculus and other rules, is reduced to 20.

11. To minimize complications of detail, only three positive ingredients are introduced. If more were introduced we should have to distinguish between complex terms and complex ingredients in order to allow for forms like kkbcd which

3.42 A simple term is any simple ingredient.

3.43 A complex conjunctive term consists of any pair of simple ingredients prefaced by k or nk, except that no pair may consist of an ingredient and its complement. The ingredients of a conjunctive term are called "conjuncts"[12].

3.5 *Special Rules for Terms and Ingredients* Representing ingredients by x and y, the following forms are interchangeable: kxy and kyx (commutation); kxx and x (redundancy); nn$x$ and x (double negation).

3.6 *Syllogistic Operators* The undefined syllogistic operator is A.

Definitions

$$Eab =_{Df} Aa\mathrm{n}b$$
$$Iab =_{Df} \sim Aa\mathrm{n}b$$
$$Oab =_{Df} \sim Aab$$

3.7 *Formation Rules for Propositions* Rules for admissible syllogistic propositions need to be specified in two alternative ways in order to allow (1) for systems which admit both contingent and non-contingent A, E, I, O propositions, and (2) for systems which admit only contingent A, E, I, O propositions.

3.71 (Contingent and non-contingent) A syllogistic operator followed by two terms is a well-formed formula.

3.72 (Contingent only) (a) A syllogistic operator followed by two terms, both of which are simple terms, or one of which is a simple term and the other a conjunctive term, is a well-formed formula, providing that each term consists of or contains a simple ingredient such that neither this ingredient nor its complement appears in the other term.

(b) A syllogistic operator followed by two conjunctive terms is a well-formed formula, providing that the conditions stated in (a) are satisfied, and providing further that if a simple ingredient appears in one term its complement does not appear in the other term.

Here (a) precludes the formation of non-contingent forms like Aaa, Akaba. It also precludes the two contingent forms Aakab and Oakab, which will be mentioned later. The effect of (b) is to preclude forms like Akabknac, which while they satisfy (a) are nonetheless non-contingent[13].

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could occur as terms in propositions like Aakkbcd. De Morgan's pioneer account of complex terms, *op. cit.*, is unnecessarily complicated because he worked with four ingredients.

12. Complex disjunctive terms are not introduced. This is because disjunctive terms add much detail but little of logical interest to axiomatization. For given de Morgan's Laws (which were incidentally introduced by him with reference to *complex terms, op. cit*, p. 116), *viz.*, that kab and vn$a$nb, and vab and kn$a$nb, are pairs of complements, every rule concerning disjunctive terms can be easily deduced from a corresponding rule concerning conjunctive terms.

13. When the system of complex terms is enlarged to take in disjunctive terms, further conditions need to be specified in order to exclude certain additional non-contingent forms.

4. *Rules for Complex Terms* Set out below are deductions of the complete set of primary rules for conjunctive terms. (Secondary cases involving subaltern inference, commutation, etc., will not be listed.) The rules for simple terms are assumed, together with certain axioms for complex terms which are introduced as the deductions proceed. To avoid complications of detail, (a) rules for simple terms will not be listed and their use will be indicated briefly as cases of "conversion", "obversion", etc., and (b) when a particular type of deduction is introduced it will be explained in full, but further deductions of the same type will be indicated in outline only.

4.1 *Addition and Omission of Conjuncts*

4.11 Ordinary Conjuncts

C1	$Aac \supset Akabc$	axiom	
C2	$Aakbc \supset Aac$	axiom	
C3	$Eac \supset Ekabc$	(i) $Aanc \supset Akabnc$	C1, c/nc
		(ii) $Eac \supset Aanc$	obversion of Eac
		(iii) $Eac \supset Akabnc$	(ii), (i), **H.S., M.P.**
		(iv) $Akabnc \supset Ekabc$	obversion of $Akabnc$
		(v) $Eac \supset Ekabc$	(iii), (iv), **H.S., M.P.**
C4	$Eac \supset Eakbc$	(i) $Eca \supset Ekcba$	C3, a/c, c/a
		(ii) $Ekcba \supset Ekbca$	commutation of terms
		(iii) $Eac \supset Eca$	conversion
		(iv) $Ekbca \supset Eakbc$	conversion
		(v) $Eac \supset Eakbc$	from (i) to (iv) by successive applications of **H.S.** and **M.P.**
C5	$Ikabc \supset Iac$	(i) $\sim Ekabc \supset \sim Eac$	C3, transposition
		(ii) $Ikabc \supset Iac$	definition of E and I
C6	$Iakbc \supset Iac$		C4, transposition, definition of E and I
C7	$Okabc \supset Oac$		C1, transposition, definition of O
C8	$Oac \supset Oakbc$		C2, transposition, definition of O.

4.12 Addition and Omission of Superfluous Ingredients In everyday life there are certain obvious inferences such as "All black swans are Australian, therefore All black swans are Australian swans", "No tame lions are tame tigers, therefore No tame lions are tigers", in which an ingredient may indifferently occur or not occur a second time. It is worth noting that propositions of these forms are contingent.

C9	$Akabc \supset Akabkac$	axiom	
C10	$Ekabkac \supset Ekabc$	axiom	
C11	$Ikabc \supset Ikabkac$		C10, transposition, definition of E and I
C12	$Okabkac \supset Okabc$		C9, transposition, definition of O

When these are taken together with the rules for adding and omitting conjuncts we derive the following co-implications.

C9.1	$Akabc \equiv Akabkac$	C9 + C2
C10.1	$Ekabc \equiv Ekabkac$	C10 + C4

C11.1 $Ikabc \equiv Ikabkac$	C11 + C6
C12.1 $Okabc \equiv Okabkac$	C12 + C8

4.13 Double Addition; Omission Textbooks of traditional logic often refer to a special form of inference known as "Immediate Inference by Added Determinants", which is illustrated by the argument "All comets are material bodies, therefore All visible comets are visible material bodies". But this type of argument, it will be seen, can be analysed into two parts, involving rules expressed in 4.11 and 4.12.

C13 $Abc \supset Akabkac$	C1 (addition of conjuncts) a/b, b/a, commutation of terms, C9 (addition of superfluous conjuncts), **H.S., M.P.**
C14 $Ebc \supset Ekabkac$	C3, a/b, b/a, commutation of terms, C4, (i.e., addition of conjuncts twice), **H.S., M.P.**
C15 $Ikabkac \supset Ibc$	C14, transposition, definition of E and I
C16 $Okabkac \supset Obc$	C13, transposition, definition of O.

4.2 *The Transference of Conjuncts* The ordinary rules for immediate inference, including the rules for conversion, obversion and contraposition, apply, of course, to complex as well as to simple propositions, but there are in addition various other rules that arise only with complex terms, such as the rule $Ekabc \supset Ekacb$, which I call "partial conversion".

4.21 Partial Conversion

C17 $Ekabc \supset Ekacb$	(i)	$Ekabc \supset Ekabkac$	C4, a/kab, b/a—addition of conjuncts
	(ii)	$Ekabkac \supset Ekackab$	conversion
	(iii)	$Ekackab \supset Ekacb$	C10, kab/kac, kac/kab, c/b—omission of superfluous conjuncts
	(iv)	$Ekabc \supset Ekacb$	(i) to (iii), **H.S., M.P.**
C18 $Ikabc \supset Ikacb$			C17, b/c, c/b, transposition, definition of E and I

C17.1 $Ekabc \equiv Ekacb$	$Ekacb \supset Ekabc$	C17, b/c, c/b + C17
C18.1 $Ikabc \equiv Ikacb$	$Ikacb \supset Ikabc$	C18, b/c, c/b + C18

In C17.1 and C18.1 two equivalents are specified, but in each case there are, in fact, six equivalents, deducible by ingredient-substitution or simple conversion:

C17.11 $Ekabc \equiv Ekacb \equiv Ekbca \equiv Eakbc \equiv Ebkac \equiv Eckab$

C18.11 $Ikabc \equiv Ikacb \equiv Ikbca \equiv Iakbc \equiv Ibkac \equiv Ickab$

There are also several primary non-equivalent forms of partial conversion.

C19 $Aa\mathrm{k}bc \supset A\mathrm{k}abc$	C2, C1, **H.S., M.P.** (i.e., omission and addition of b)
C20 $O\mathrm{k}abc \supset Oa\mathrm{k}bc$	C7, C8, **H.S., M.P.** (i.e., omission and addition of b)
C21 $A\mathrm{k}abc \supset I\mathrm{k}acb$	Subaltern inference, C18, **H.S., M.P.**

Given partial conversion, various complex forms can be obtained which are analogues of obverses and contrapositives. To illustrate, let us take a concrete example:

(i) All precious metals are scarce.
(ii) No precious metals are non-scarce. (obversion)
(iii) No metals are both precious and non-scarce. (partial conversion)
(iv) All metals are either non-precious or scarce. (obversion)
(v) No non-scarce metals are precious. (From (ii), partial conversion)
(vi) All non-scarce metals are non-precious. (From (v), obversion)

Of these, (iii) may be called a "partial obverse" of (i), while (iv), (v) and (vi) are all forms in which (i) has been partly contraposed. If we take for comparison the ordinary contrapositive of (i), expressed in accordance with de Morgan's Laws, "All non-scarce things are either non-precious or non-metals", it will be seen that this contains transposed complements of all three original ingredients, whereas (iv) and (v) each has one transposed complement, and (vi) contains two transposed complements. We thus have degrees of transposition, and since the term "partial contrapositive" is already employed in traditional logic, the term "transpositive" will be used to refer to these special forms[14].

For simplicity, relevant deductions are here first set out *en bloc* in the following tables in which, parallel to the deduction of analogous rules for simple terms, (a) in tables 1 and 2 the processes of obversion and partial conversion are alternately employed, and (b) in tables 3 and 4 the processes of partial conversion and obversion are alternately employed.

Table 1

$A\mathrm{k}abc$	
$E\mathrm{k}ab\mathrm{n}c$	(obversion)
$E\mathrm{k}a\mathrm{n}cb$	(partial conversion)
$E\mathrm{k}a\mathrm{n}c\mathrm{n}b$	(obversion)

Table 2

$A\mathrm{k}abc$	
$E\mathrm{k}ab\mathrm{n}c$	(obversion)
$Ea\mathrm{k}b\mathrm{n}c$	(partial conversion)
$Aa\mathrm{vn}bc$	(obversion)

Table 3

$E\mathrm{k}abc$	
$E\mathrm{k}acb$	(partial conversion)
$A\mathrm{k}ac\mathrm{n}b$	(obversion)

Table 4

$Ea\mathrm{k}bc$	
$E\mathrm{k}abc$	(partial conversion)
$A\mathrm{k}ab\mathrm{n}c$	(obversion)

14. The term "transposition" was used by de Morgan, *op. cit.*, p. 120, to refer to *any* transference of ingredients within a complex proposition.

4.22 Partial Obversion

C22	$Akabc \equiv Eakbnc$	Table 2
C23	$Eakbc \equiv Akabnc$	Table 4
C24	$Iakbc \equiv Okabnc$	C23, transposition, definition of E, I, O
C25	$Okabc \equiv Iakbnc$	C22, transposition, definition of E, I, O
C26	$Aakbc \supset Okabnc$	From $Aakbc$, parallel to Table 4
C27	$Ekabc \supset Iakbnc$	From $Ekabc$, parallel to Table 2.

4.23 Partial Transposition

C28	$Akabc \equiv Ekancb$	Table 1
C29	$Ekabc \equiv Akacnb$	Table 3
C30	$Ikabc \equiv Okacnb$	C29, transposition, definition of E, I, O
C31	$Okabc \equiv Ikancb$	C28, transposition, definition of E, I, O
C32	$Akabc \supset Okacnb$	From $Akabc$, parallel to Table 3
C33	$Ekabc \supset Okancnb$	From $Ekabc$, parallel to Table 1.

4.24 Transposition

C34	$Akabc \equiv Akancnb$	Table 1
C35	$Ekabc \supset Okancnb$	From $Ekabc$, parallel to Table 1
C36	$Okabc \equiv Okancnb$	C34, transposition, definition of O

4.3 *Rules of Opposition* Various new relations of opposition arise with complex propositions. The number of cases would be large if we also listed forms which contain negative ingredients, but these merely introduce complications of detail so the rules given will be confined to positive simple and conjunctive terms. Here there are two main cases to consider, the relations between simple and complex propositions, and special relations between complex and complex propositions.

4.31 Contrary Relation

Simple and Complex Propositions:

C37	$Aac \supset \sim Ekabc$	(i) $Aac \supset Akabc$	C1—addition of conjuncts
		(ii) $Akabc \supset \sim Ekabc$	contraries
		(iii) $Aac \supset \sim Ekabc$	M.P., H.S.
C38	$Aac \supset \sim Okabc$	(i) $Aac \supset Akabc$	C1—addition of conjuncts
		(ii) $Akabc \supset \sim Okabc$	contradictories
		(iii) $Aac \supset \sim Okabc$	H.S., M.P.

C39	$Eac \supset {\sim}Akabc$	Addition, contraries, **H.S., M.P.**
C40	$Eac \supset {\sim}Ikabc$	Addition, contradictories, **H.S., M.P.**
C41	$Eac \supset {\sim}Aakbc$	Addition (to predicate), contraries, **H.S., M.P.**
C42	$Eac \supset {\sim}Iakbc$	Addition (to predicate), contradictories, **H.S., M.P.**
C43	$Oac \supset {\sim}Aakbc$	Addition (to predicate), contradictories, **H.S., M.P.**

To each of the rules C37 to 43 there corresponds a reverse rule provable in an analogous way. However, each of these rules is also provable in a more simple way by transposition of the corresponding rule and *modus ponens*. As a result, these reverse rules will be listed wholesale, without special proofs.

C37.1 $Ekabc \supset {\sim}Aac$
C38.1 $Okabc \supset {\sim}Aac$
C39.1 $Akabc \supset {\sim}Eac$
C40.1 $Ikabc \supset {\sim}Eac$
C41.1 $Aakbc \supset {\sim}Eac$
C42.1 $Iakbc \supset {\sim}Eac$
C43.1 $Akabc \supset {\sim}Oac$

An exception to the above rules and reverse rules is provided by the relation between Aac and $Eakbc$. Here a proof of contrary relation would require one or other of the moves, $Aac \supset Akabc$, $Eakbc \supset Ekabc$, but in contrast with the proofs of C37-43 in each of which any term formed by addition is already present as a permissible term, in the case under consideration there is no guarantee that kab is a permissible term, so that the needed moves cannot be made. Hence the relation of Aac and $Eakbc$, being that of indifference, does not appear in the list of rules.

Complex and Complex Propositions:

C44	$Akabc \supset {\sim}Eakbc$	(i) $Akabc \supset {\sim}Ekabc$	contraries
		(ii) ${\sim}Ekabc \supset {\sim}Eakbc$	17.11, partial conversion
		(iii) $Akabc \supset {\sim}Eakbc$	**H.S., M.P.**
C45	$Ekabc \supset {\sim}Aakbc$		partial conversion, contraries, **H.S., M.P.**
C46	$Okabc \supset {\sim}Aakbc$		partial conversion (C20), contradictories, **H.S., M.P.**

Corresponding rules are derivable by transposition and *modus ponens*:

C44.1 $Eakbc \supset {\sim}Akabc$
C45.1 $Aakbc \supset {\sim}Ekabc$
C46.1 $Aakbc \supset {\sim}Okabc$

4.32 Subcontrary Relation

Simple and Complex Propositions:

C47	$\sim Iac \supset Ekabc$	(i) $\sim Iac \supset Eac$	contradictories
		(ii) $Eac \supset Ekabc$	C3—addition of conjuncts
		(iii) $\sim Iac \supset Ekabc$	**H.S., M.P.**
C48	$\sim Iac \supset Okabc$	(i) $\sim Iac \supset Eac$	contradictories
		(ii) $Eac \supset Ekabc$	C3—addition of conjuncts
		(iii) $Ekabc \supset Okabc$	subaltern inference
		(iv) $\sim Iac \supset Okabc$	**H.S., M.P.**
C49	$\sim Oac \supset Akabc$		contradictories, addition, **H.S., M.P.**
C50	$\sim Oac \supset Ikabc$		contradictories, addition, subaltern inference, **H.S., M.P.**
C51	$\sim Aac \supset Oakbc$		contradictories, addition, **H.S., M.P.**
C52	$\sim Iac \supset Eakbc$		contradictories, addition, **H.S., M.P.**
C53	$\sim Iac \supset Oakbc$		contradictories, addition, subaltern inference, **H.S., M.P.**

Corresponding rules are derivable by transposition and *modus ponens*:

C47.1 $\sim Ekabc \supset Iac$
C48.1 $\sim Okabc \supset Iac$
C49.1 $\sim Akabc \supset Oac$
C50.1 $\sim Ikabc \supset Oac$
C51.1 $\sim Oakbc \supset Aac$
C52.1 $\sim Eakbc \supset Iac$
C53.1 $\sim Oakbc \supset Iac$

A rule for *Oac* and *Iakbc* does not appear since their relation is indifference.

Complex and Complex Propositions:

C54	$\sim Akabc \supset Oakbc$	contradictories, partial conversion (C20), **H.S., M.P.**
C55	$\sim Ikabc \supset Oakbc$	contradictories, partial conversion (C17.11), subaltern inference, **H.S., M.P.**
C56	$\sim Okabc \supset Iakbc$	contradictories, subaltern inference, partial conversion (C17.11), **H.S., M.P.**

Three corresponding rules are derivable by transposition and *modus ponens*:

C54.1 $\sim Oakbc \supset Akabc$
C55.1 $\sim Oakbc \supset Ikabc$
C56.1 $\sim Iakbc \supset Okabc$

4.33 Contradictory Relation Contradictory relation arises only between complex and complex propositions, when one is an E, and the other an I, proposition.

C57 $Ekabc \supset \sim Iakbc$	partial conversion, contradictories, **H.S., M.P.**
C58 $\sim Ekabc \supset Iakbc$	contradictories, partial conversion, **H.S., M.P.**
C59 $Ikabc \supset \sim Eakbc$	partial conversion, contradictories, **H.S., M.P.**
C60 $\sim Ikabc \supset Eakbc$	contradictories, partial conversion, **H.S., M.P.**

Corresponding rules are derivable by transposition and *modus ponens*:

C57.1 $Iakbc \supset \sim Ekabc$
C58.1 $\sim Iakbc \supset Ekabc$
C59.1 $Eakbc \supset \sim Ikabc$
C60.1 $\sim Eakbc \supset Ikabc$

4.4 *Mediate Rules* Commentators on subject-predicate logic often mistakenly assume that syllogisms and sorites are the only types of mediate inference involving A, E, I, O propositions. Even upholders of traditional logic sometimes made this assumption, as when, in attempting to "reduce" to syllogistic forms relational arguments like "A is greater than B, B is greater than C, therefore A is greater than C", they did so in the interests of showing that the syllogism had logical or metaphysical primacy. But they failed to note that irreducible arguments are already present within the subject-predicate system in the form of mediate arguments involving complex terms, since these cannot be deduced from syllogisms.

Of these mediate arguments, the most notable is the argument in which predicates are conjoined, for example, "All halogens are chemical elements, All halogens are artificially-produced, therefore All halogens are chemical elements which are artificially-produced". This form is the one used here as an axiom.

4.41 Conjunction of Predicates

C61 $Aab \,\&\, Aac \supset Aakbc$ Axiom	
C61.1 $Aab \,\&\, Aac \equiv Aakbc$	C61 + $Akabc \supset Aab$ (C2, c/b), and $Akabc \supset Aac$ (C2)
C62 $Iab \,\&\, Aac \supset Iakbc$	

The proof given of this thesis depends on the use of implications like $\sim Okabc \,\&\, Iab \supset Akabc$ (i.e., a statement of the condition under which $Akabc$ is admissible in the system) which, while they are not theses of the system, can be asserted as true material implications outside the system.

(i) $Aac \supset \sim Okabc$	C1, definition of O
(ii) $\sim Okabc \,\&\, Iab \supset Akabc$	admission of $Akabc$ in the system
(iii) $\sim Okabc \supset (Iab \supset Akabc)$	(ii), exportation
(iv) $Aac \supset (Iab \supset Akabc)$	(i), (iii), **H.S., M.P.**
(v) $Iab \,\&Aac \supset Akabc$	(iv), exportation
(vi) $Akabc \supset Ikabc$	subaltern inference-S6
(vii) $Ikabc \supset Iakbc$	C18.11-partial conversion
(viii) $Iab \,\&Aac \supset Iakbc$	(v), (vii), **H.S., M.P.**

4.42 Disjunctive Rules In certain cases complex propositions can be derived from a disjunction of simple propositions. The deduction of these rules makes use of de Morgan's Laws in the propositional calculus.

C63 $Aac \vee Abc \supset Akabc$

(i) $Okabc \supset Oac \,\&\, Obc$	C7 + C7 a/b
(ii) $\sim(Oac \,\&\, Obc) \supset \sim Okabc$	transposition
(iii) $Aac \vee Abc \supset Akabc$	de Morgan's Laws, definition of O

C64 $Eac \vee Ebc \supset Ekabc$

(i) $Aanc \vee Abnc \supset Akabnc$	C63, c/nc
(ii) $Eac \vee Ebc \supset Aanc \vee Abnc$	obversion
(iii) $Akabnc \supset Ekabc$	obversion
(iv) $Eac \vee Ebc \supset Ekabc$	**H.S., M.P.**

C65 $Eab \vee Eac \supset Eakbc$ C64, a/b, b/c, c/a, conversion, **H.S., M.P.**

C66 $Oab \vee Oac \equiv Oakbc$ C61.1 transposition, de Morgan's Laws, definition of O

4.43 Mediate Omission of Terms The syllogistic parallel to "disjunctive syllogism" of the propositional calculus is the argument form $Aavbc \,\&\, Eab \supset Aac$. The conjunctive analogue of this is the form $Akabc \,\&Aab \supset Aac$.

C67 $Akabc \,\&\, Aab \supset Aac$

This can be deduced from previous rules and indirect reduction.

(i) $Akabc \supset Ekabnc$	obversion
(ii) $Ekabnc \supset \sim Ikancb$	C17, c/nc, definition of E and I
(iii) $Akabc \supset \sim Ikancb$	(i), (ii), **H.S., M.P.**
(iv) $\sim Ikancb \,\&\, Oac \supset Ekancb$	admission of $kanc$ in the system
(v) $\sim Ikancb \supset (Oac \supset Ekancb)$	exportation (iv)
(vi) $Akabc \supset (Oac \supset Ekancb)$	(iii), (v), **H.S., M.P.**
(vii) $Akabc \,\&\, Oac \supset Ekancb$	(vi) exportation
(viii) $Ekancb \supset Okancb$	subaltern inference

(ix) $Okancb \supset Oab$	C7, b/nc, c/b—omission of conjuncts
(x) $Ekancb \supset Oab$	(viii), (ix), **H.S., M.P.**
(xi) $Akabc$ & $Oac \supset Oab$	(vii), (x), **H.S., M.P.**
(xii) $Akabc$ & $Aab \supset Aac$	(xi), indirect reduction, definition of O

An alternative, simpler proof is possible if we make use of the propositional form $Aakab$.

(i) $Akabc$ & $Aakab \supset Aab$	Barbara
(ii) $Aab \supset Aakab$	from C9, addition of a superfluous conjunct
(iii) $Akabc$ & $Aab \supset Aac$	second principle of direct reduction

C68 $Akabc$ & $Oac \supset Oab$	proved in the proof of C67
C69 Oac & $Aab \supset Okabc$	C67, indirect reduction, definition of O
C70 Aab & $Oakbc \supset Oac$	C61, indirect reduction, definition of O

There are no corresponding rules for E and I propositions, as these lead to contradictions in the system. For example, the parallel to C69 would be Iac & $Eab \supset Ikabc$, where the formation of the term kab is incompatible with the premise Eab.

C71 $Akabc$ & $Akanbc \supset Aac$

(i) $Akabc \supset Ekabnc$	obversion
(ii) $Ekabnc \supset \sim Ikabnc$	definition of E and I
(iii) $\sim Ikabnc \supset \sim Ikancb$	partial conversion
(iv) $\sim Ikancb$ & $Oac \supset Ekancb$	admission of $kanc$ in the system
(v) $Ekancb \supset Akancnb$	obversion
(vi) $Akancnb \supset Ikanbnc$	subaltern inference and partial conversion
(vii) $Ikanbnc \supset Okanbc$	obversion
(viii) $Akabc$ & $Oac \supset Okanbc$	(i) to (vi), **H.S.,** direct reduction, **M.P.**
(ix) $Akabc$ & $Akanbc \supset Aac$	(vii), indirect reduction, definition of O

C71 is the syllogistic analogue of the simple constructive dilemma. There is no analogue of the simple destructive dilemma as the appropriate form, $Akabc$ & $Akabnc \supset Eab$, can occur only if kab is an empty term.

5. *Axiom-sets for Simple and Complex Terms*

5.1 *Combined Axiom-sets* Axioms and theorems have now been set out for the 71 main theses special to complex terms. If we now combine the axioms used for the deduction of complex terms with the axiom-sets for simple terms mentioned earlier, we obtain the following two main axiom-sets.

System **A** (n, k)

S5	$Iab \supset Iba$	*I* conversion
S6	$Aab \supset Iab$	*A* subalternation
S7	$Ecb \;\&\; Aac \supset Eab$	Celarent
C1	$Aab \supset Akabc$	addition of conjuncts
C2	$Aakbc \supset Aac$	omission of conjuncts
C9	$Akabc \supset Akabkac$	addition of superfluous conjuncts
C10	$Ekabkac \supset Ekabc$	omission of superfluous conjuncts
C61	$Aab \;\&\; Aac \supset Aakbc$	conjunction of predicates

System **CS** (n, k)

S1	Aaa	
S2	Iaa	
S4	$Ecb \;\&\; Iac \supset Iab$	Ferio
C1	$Aab \supset Akabc$	
C2	$Aakbc \supset Aac$	
C9	$Akabc \supset Akabkac$	
C10	$Ekabkac \supset Ekabc$	
C61	$Aab \;\&\; Aac \supset Aakbc$	

5.2 *Deduction of Syllogisms* As with the system of simple terms, variations are possible in the axioms used to derive rules for complex terms. More important, however, are variations which allow a reduction in the number of axioms. In this regard, the introduction of complex terms has the interesting consequence that whichever syllogistic mood has been taken as an axiom that mood is deducible from rules about complex terms. A key case is the use of C67 ($Akabc \;\&\; Aab \supset Aac$) and C1 (addition of conjuncts) to deduce Barbara (S3).

(i) $Akacb \;\&\; Aac \supset Aab$	C67, b/c, c/b
(ii) $Acb \supset Akacb$	C1, a/c, b/c, c/b, commutation of terms
(iii) $Acb \;\&\; Aac \supset Aab$	direct reduction, **M.P.**

Use of the same method together with obversion enables us to deduce Celarent as a theorem and to dispense with it as an axiom in System **A** (n,k).

(i) $A\mathrm{k}ac\mathrm{n}b \,\&\, Aac \supset Aa\mathrm{n}b$	C67, b/c, $c/\mathrm{n}b$
(ii) $Ac\mathrm{n}b \supset A\mathrm{k}ac\mathrm{n}b$	C1, a/c, b/a, $c/\mathrm{n}b$, commutation of terms
(iii) $Ecb \supset Ac\mathrm{n}b$	obversion
(iv) $Ecb \,\&\, Aac \supset Aa\mathrm{n}b$	(i), (ii), (iii), direct reduction, **M.P.**
(v) $Aa\mathrm{n}b \supset Eab$	obversion
(vi) $Ecb \,\&\, Aac \supset Eab$	(iv), (v), **H.S., M.P.**

At the same time, the axiom Ferio in System **CS**(n,k) can be deduced from other complex rules and obversion.

(i) $Ica \,\&\, Ac\mathrm{n}b \supset Ic\mathrm{k}a\mathrm{n}b$	C62, a/c, b/a, $c/\mathrm{n}b$—conjunction of predicates
(ii) $Ic\mathrm{k}a\mathrm{n}b \supset I\mathrm{k}ac\mathrm{n}b$	C18.11, $b/\mathrm{n}b$—partial conversion
(iii) $I\mathrm{k}ac\mathrm{n}b \supset Ia\mathrm{n}b$	C5, b/c, $c/\mathrm{n}b$—omission of conjuncts
(iv) $Ica \,\&\, Ac\mathrm{n}b \supset Ia\mathrm{n}b$	(i), (ii), (iii), **H.S., M.P.**
(v) $Iac \,\&\, Ecb \supset Ia\mathrm{n}b$	conversion of Iac, obversion of Ecb, direct reduction, **M.P.**
(vi) $Ecb \,\&\, Iac \supset Oab$	obversion of $Ia\mathrm{n}b$, transposition of premises, **H.S., M.P.**

In this connection, it might be thought that two alternative types of system are possible, one in which syllogistic moods are deduced from complex rules and the other in which complex rules are deduced from moods. But in fact, while some complex rules can be derived from moods, such key rules as C1, C2 and C62 must either be taken as axioms or else derived from other complex rules, so that the second of the alternative types of system does not arise.

Moreover, given the deduction of one mood such as Celarent or Ferio, we can deduce all the other moods as theorems. But it is also possible to use complex rules in the deduction of any syllogistic mood. Thus, by proofs similar to those of Barbara and Celarent given above, any of the moods which have a universal conclusion can be deduced, and by proofs similar to that of Ferio, any of the moods with a particular conclusion can also be deduced. So, although only one mood *needs* to be deduced by whatever method, these two types of proof provide a wholesale method of deducing all 24 moods.

5.3 *The System of Contingent Propositions* If we confine ourselves to admissible contingent A, E, I, O propositions and deduce Celarent in the way set out above, we arrive at an axiom-set which consists of the other axioms of System **A**(n,k). Let us use the heading "**S & C**" in referring to systems of this kind.

System **S & C, 1** (Contingent)

Axioms: S5, S6, C1, C2, C9, C10, C61.

The formation rules for a contingent system which have been given exclude the forms *Aakab* and *Oakab*, although these are, in fact, contingent forms. Their exclusion is no doubt desirable if we seek to have a system which is both contingent and in the spirit of traditional subject-predicate logic, for while *Aakab* and *Oakab* are contingent, the other two forms *Eakab* and *Iakab* are not contingent, so that the admission of the former pair would have the unusual consequence for the square of opposition that a contrary and a subcontrary of the relevant forms would always be lacking.

If, however, *Aakab* and *Oakab* are admitted, axiom C61 can then be deduced as a theorem.

(i)	$Aab \supset Akaab$	C1, b/a, c/b
(ii)	$Akaab \supset Akaakab$	C9, b/a, c/b
(iii)	$Akaakab \supset Aakab$	omission of a redundant ingredient
(iv)	$Aab \supset Aakab$	(i), (ii), (iii), **H.S., M.P.**
(v)	$Aac \supset Akabc$	C1
(vi)	$Akabc \supset Akabkbc$	C9, a/b, b/a, commutation of terms
(vii)	$Aac \supset Akabkbc$	(v), (vi), **H.S., M.P.**
(viii)	$Akabkbc \,\&\, Aakab \supset Aakbc$	Barbara, b/kbc, c/kab
(ix)	$Aab \,\&\, Aac \supset Aakbc$	(iv), (vii), (viii), first and second principles of direct reduction, **M.P.**

This reduces the axioms for a contingent system to six:

System **S & C, 2** (Contingent)

Axioms: S5, S6, C1, C2, C9, C10.

5.4 *The System of Contingent and Non-contingent Propositions*

With the admission of non-contingent A, E, I, O propositions, new deductions become possible. Two of the resulting possible axiom-sets will now be explained.

Once non-contingent propositions are admitted, there can be no queries about the forms $Aa\mathrm{k}ab$ and $Oa\mathrm{k}ab$, so that we can, if we wish, dispense with C61 as an axiom. At the same time, S4 (Ferio) of System **CS** (n,k) can be deduced as shown earlier. But we cannot, on pain of circularity, straightforwardly list the axioms which remain of that system as a new set of axioms[15]. One convenient solution is to introduce S6 and C4 as axioms and to turn some of the previous axioms into theorems, as follows.

S2	Iaa	(i) $Aaa \supset Iaa$	S6, b/a
		(ii) Iaa	(i), S1, **M.P.**
C2	$Aa\mathrm{k}bc \supset Aac$	(i) $A\mathrm{k}bcc \,\&\, Aa\mathrm{k}bc \supset Aac$	Barbara, b/c, $c/\mathrm{k}bc$[16]
		(ii) $Acc \supset A\mathrm{k}bcc$	C1, a/c, commutation of terms
		(iii) $Acc \,\&\, Aa\mathrm{k}bc \supset Aac$	(i), (ii), direct reduction, **M.P.**
		(iv) $Acc \supset (Aa\mathrm{k}bc \supset Aac)$	(iii), exportation
		(v) Acc	S1, a/c
		(vi) $Aa\mathrm{k}bc \supset Aac$	(iv), (v), **M.P.**

S5 can be deduced in a new, non-circular way, providing we first establish a new complex rule, C72.

C72	$Iab \supset I\mathrm{k}abb$	(i) $I\mathrm{k}ab\mathrm{k}ab \supset \sim E\mathrm{k}ab\mathrm{k}ab$	definition of E and I
		(ii) $\sim E\mathrm{k}ab\mathrm{k}ab \supset \sim E\mathrm{k}abb$	C4, $a/\mathrm{k}ab$, b/a, c/b, transposition
		(iii) $I\mathrm{k}ab\mathrm{k}ab \supset \sim E\mathrm{k}abb$	(i), (ii), **H.S., M.P.**
		(iv) $\sim E\mathrm{k}abb \,\&\, Iab \supset I\mathrm{k}abb$	admission of $I\mathrm{k}abb$ in the system
		(v) $I\mathrm{k}ab\mathrm{k}ab \,\&\, Iab \supset I\mathrm{k}abb$	(iii), (iv), direct reduction, **M.P.**
		(vi) $I\mathrm{k}ab\mathrm{k}ab \supset (Iab \supset I\mathrm{k}abb)$	(v), exportation
		(vii) $Iab \supset I\mathrm{k}abb$	(vi), S2, $a/\mathrm{k}ab$, **M.P.**
S5	$Iab \supset Iba$	(i) $Iab \supset I\mathrm{k}abb$	C72
		(ii) $I\mathrm{k}abb \supset I\mathrm{k}ab\mathrm{k}ab$	C11, c/b
		(iii) $I\mathrm{k}ab\mathrm{k}ab \supset Ib\mathrm{k}ab$	C5, a/b, b/a, $c/\mathrm{k}ab$, commutation of terms
		(iv) $Ib\mathrm{k}ab \supset Iba$	C6, a/b, c/a, commutation of terms
		(v) $Iab \supset Iba$	(i)-(iv), **H.S., M.P.**

15. Circularity arises because the standard deductions of S5 and S6 employ Datisi, which can be deduced from, or in the same way as, Ferio, by employing C62. But this is a theorem which depends on S5 and S6 for its own deduction.

16. Barbara is deduced from C67 (as shown above) and C67 can be deduced in a non-circular way from C4, S6 and C1.

At the same time, given *Aaa*, C10 can be made into a theorem. To do so, we need a new complex rule C73, $Iab \supset Iakab$, which can be either proved in a similar way to, or deduced from, C72.

C10 $Ekabkac \supset Ekabc$	(i) $Ikabc \supset Ikabkkabc$	C73, a/kab, b/c[17]
	(ii) $Ikabkkabc \supset Ikabkac$	from C6—omission of an ingredient of a conjunct
	(iii) $Ikabc \supset Ikabkac$	(i), (ii), **H.S.**, **M.P.**
	(iv) $Ekabkac \supset Ekabc$	(iii), transposition, definition of *E* and *I*

Bringing together these changes, we have a new axiom-set:

System **S & C, 3** (Contingent and Non-contingent)

S1 Aaa
S6 $Aab \supset Iab$
C1 $Aac \supset Akabc$
C4 $Eac \supset Eakbc$
C9 $Akabc \supset Akabkac$

As a final example, an alternative axiom-set is formed if we retain C61 ($Aab \,\&\, Aac \supset Aakbc$) as an axiom—a procedure which has the intuitively satisfactory result that the main mediate complex rule appears as an axiom. In this case the one adjustment needed is the deduction of C9 as a theorem.

C9 $Akabc \supset Akabkac$	(i) $Aaa \,\&\, Aab \supset Aakab$	C61, b/a, c/b
	(ii) $Aaa \supset (Aab \supset Aakab)$	(i), exportation
	(iii) $Aab \supset Aakab$	(ii), S1, **M.P.**
	(iv) $Akabc \supset Akabkkabc$	(iii), a/kab, b/c
	(v) $Akabkkabc \supset Akabkac$	from C2, omission of an ingredient of a conjunct
	(vi) $Akabc \supset Akabkac$	(iv), (v), **H.S.**, **M.P.**

As with System **S & C, 3,** this gives us an exiguous set of five axioms from which we can derive all the rules for the complete system of simple and complex terms.

System **S & C, 4** (Contingent and Non-contingent)

Axioms: S1, S6, C1, C4, C61.

Macquarie University
Sydney, Australia

17. Here kk followed by three ingredients is introduced as a term-form of the system.