Notre Dame Journal of Formal Logic
Volume XII, Number 1, January 1971

DOES OCKHAM ACCEPT MATERIAL IMPLICATION?

M. MULLICK

In recent years a great deal of research has been carried out on Ockham's logic, the most important of which is Boehner's. In this paper I shall discuss a problem raised by Boehner's account of Ockham's theory of consequences. Boehner's main contention here is that Ockham not only knew of, but wholly accepted material implication. I will argue that though the first half of Boehner's thesis is correct, and that Ockham (like many other medieval logicians) was aware of truth-functional relations between propositions, his attitude toward them was one of rejection rather than acceptance; secondly, that all the rules of consequences that Boehner refers to as evidence of Ockham's acceptance of material implication may be given more plausible alternative interpretations.

First, a word about the term 'consequence'. Boethius, it appears, is the originator of the term, and possibly also the source of confusion that surrounds it. For whereas the Stoics made a sharp distinction between conditional propositions and arguments, in the notion of consequences this distinction is once again obliterated. In the writings of the Schoolmen we find definitions such as the following of Buridan:

> A consequence is a hypothetical proposition since it is formed from several propositions by means of the connective 'if' or by this word 'therefore' or an equivalent.

The only significant difference that Buridan notes between the two, which he otherwise regards as equivalent, is that

> the word 'if' indicates that the proposition immediately following it is the antecedent and the other the consequent, whereas this word 'therefore' indicates the contrary.[1]

In fact it is the word 'therefore' or *'igitur'* that is most characteristic of consequences. And since it is clear from the available texts that medieval logicians, following in the tradition of Aristotle, were concerned primarily

1. Buridan, Jean, *Consequentiae*, I, Chap. 3.

Received December 9, 1969

with the conditions of valid arguments and their inter-reduction, we may agree with Moody when he says that

> the medieval theory of consequence is not a logical calculus, not a system of formulas expressed with variables and logical constants. It is rather a system of rules governing logically valid inferences to or from sentences of conditional form.[2]

It needs to be emphasized that some distinction *is* made between a conditional proposition and a consequence by Ockham, since when he classifies different types of propositions he mentions conditionals but not consequences; later when he discusses arguments he does so in terms of consequences, not conditionals. A consequence is then equivalent to, but not identical with, a true conditional–not any true conditional indeed, but one whose antecedent 'infers' the consequent. Thus Ockham:

> *Sed quia conditionalis aequivalet uni consequentia, ita quod tunc conditionalis est vera, quando antecedens infert consequens, et non aliter, ideo differatur usque ad tractatum de consequentiis.*[3]

According to Kneale,[4] Ockham distinguishes consequences from syllogisms also, describing the former as enthymemes (i.e. elliptic arguments), but this is not in fact borne out by the text to which Kneale refers–*Summa Logicae* pars III, 3, 1. Both complete and incomplete arguments are included as consequences by Ockham. This is made clear in his discussion on the divisions of consequences.

Boehner adduces all the evidence at his command in his attempt to prove that for Ockham, such conditionals as are equivalent to consequences are truth-functions. But it is far from clear that Boehner succeeds in this attempt.

Material relations, i.e., those based on truth values may be described as the weakest that hold between propositions. To assert then the conditions under which these relations hold is to leave the question of stronger logical relations, an open one. One may not, for instance, argue that since the statement that a consequence is false if the antecedent is true but not the consequent, holds for material implication, then that is the only relation that is being asserted in it. But Boehner quotes the following as evidence that Ockham is concerned with material implication:

> . . . in no true conditional proposition is it possible that the antecedent be true and the consequent be false. For every conditional is true in which the antecedent cannot be true without the consequent (being true).[5]

---

2. Moody, E. A., *Truth and Consequence in Mediaeval Logic* (Amsterdam, 1953), pp. 77-78.
3. Ockham, *Summa Logicae*, edited by P. Boehner (St. Bonaventure, New York, 1951), Part II, c. 30.
4. Kneale, W., and M. Kneale, *Development of Logic* (Oxford, 1962), p. 289.
5. Boehner, P., "Does Ockham know of material implication?" in *Collected Articles on Ockham*, edited by E. M. Buytaert (St. Bonaventure, New York 1958), p. 329.

This he says "is in perfect harmony with material implication," though he notes somewhat reluctantly that "it holds in strict implication as well." But surely the dictum, *"ex vero nunquam sequitur falsum; et ideo quando-cumque antecedens est verum et consequens est falsum, consequentia non valet,"*[6] is a universal truth not merely a law of material or for that matter, strict implication. A second passage says, "but if one part only is true, it must be the consequent." "This," says Boehner, "in our opinion expresses very nicely the basic definition of material implication as offered by modern logicians."[7] But surely this asserts no more than the first passage, for if from the true the false may not follow, and yet only one part of a true conditional or consequence is true, then it must be the consequent. From this statement of Ockham's however Boehner assumes that the former would accept both *CpCNpq* and *CpCqp* as valid consequences. But this of course just does not follow, since Ockham does not say that a consequence holds *whenever* the consequent is true or the antecedent false. In the strictest of strict systems, in systems of entailment, for that matter in any concept of logical consequence, it is an essential assumption that though there can be no 'following' relation between a true premise and a false conclusion, it is possible for the conclusion to be true while all or some of the premises are false. To hold this, however, it is not necessarily to hold with protagonists of material implication that if the consequent is true then it follows from *any* false proposition, much less that it follows from *any* proposition at all, true or false as *CpCqp* suggests. As a trump card for the cause of material implication, Boehner offers the following passage, which he says is "perhaps, the clearest presentation of material implication which we can expect."[8] It goes thus:

> And therefore as a *consequentia* can be valid *potest esse bona* though neither one of the propositions be true, and although the antecedent is false and the consequent is true, but is never valid, if the antecedent is true and the consequent is false; so likewise a hypothetical conditional proposition can be true, though neither one of the categorical propositions of which it is composed is true, and though the first is true and the second is true, but not if the first is true and the second is false.[9]

But surely there is a much clearer presentation of material implication, which if Ockham had wished he could easily have given instead of the above. For it involves the phrase *'potest esse bona,'* more specifically the use of the word *'potest.'* If Ockham indeed had a purely truth-functional definition of consequence (and incidentally this is an excellent example of the *distinction* Ockham makes between the *validity* of consequences and the *truth* of conditional propositions), he could easily have left out *'potest,'* and said that under the following conditions *'sicut consequentia est bona.'* The point is that he did not.

---

6. Ockham, *Op. cit.*, Part III, 3, c. 27.
7. Boehner, P., *Op. cit.*, p. 329.
8. Boehner, P., *Op. cit.*, p. 330. (Translation of ms. f. 242 vb.)
9. Boehner, P., *Op. cit.*, p. 330.

A parallel (mis)interpretation is offered by Boehner of Ockham's rule *"Quod ex falsis potest sequi verum."*[10] Here too he chooses to ignore the use of the modal term; more accurately, he rejects it. He says,

> The apparent modal formation of this rule should not mislead us, for the modality only serves to emphasise the fact that from a false proposition both a true and a false proposition follow.[11]

He then goes on to symbolise this rule as *CNpCpq* and *CqCpq*. That this is unwarranted is proved by the sequel in Ockham. For what the latter claims to be showing from this is, as he continues

> *et ideo ista consequentia non valet: Antecedens est falsum, igitur consequens est falsum. Sed ista consequentia est bona: Consequens est falsum, igitur et antecedens . . . .*[12]

And this of course is a law not peculiar to material implication, viz., the law of contraposition *CCpqCNqNp*.

Not satisfied with all the paradoxes he has produced, Boehner goes on to claim that "we have still another clear indication for Ockham's knowledge and use of material implication," and quotes the following rule:

> *Et ideo tales consequentiae sunt bonae: Oppositum consequentis stat cum antecedente, igitur consequentia non valet. Oppositum consequentis non stat cum antecedente, igitur, consequentia est bona. Sed sciendum, quod consequentia poterit esse bona ut nunc, quam vis ut nunc oppositum consequentis posset stare cum antecedente; sed si oppositum consequentis stet vel posset stare cum antecedente, non poterit esse consequentia simplex.*[13]

Boehner translates the above thus:

> therefore the following *consequentiae* are valid: the opposite of the consequent is true together with the antecedent, therefore the *consequentia* does not hold: the opposite of the consequent is not true together with the antecedent, therefore the *consequentia* may be valid as a factual one, although factually the opposite of the consequent might be true together with the antecedent. But if the opposite of the consequent should be true, or could be true, together with the antecedent, it cannot be an absolute *consequentia.*[14]

From the Latin original we see firstly that Ockham was perfectly capable of using the simple verb *"est bona"* when he wished to indicate a valid rule. Secondly the rule itself that Ockham states here and elsewhere might be interpreted as the very rejection of the idea of material implication as a principle of consequence. For it is not insignificant that in stating this meta-rule Ockham has to draw special attention to the one ex-

10. Ockham, *Op cit.*, Part III, 3, c. 36.
11. Boehner, P., *Medieval Logic* (Manchester, 1952), p. 61.
12. Ockham, *Op. Cit.*
13. *Ibid.*
14. Boehner, P., *Collected Articles*, p. 350.

ception – the case of the *'ut nunc'* consequence – in which he allows that the opposite of the consequent might be true with the antecedent, but, which he hastens to add, can never be an absolute (logical ?) consequence. Nor could it be a formal one.

Not this however, but the preceding rule excites Boehner's "greatest interest." "For here," he says,

> We finally meet with the crucial *consequentia* for which we were looking. For when the contradictory opposite of the consequent does not stand together, or is not true, together with the antecedent, that is if the copulative proposition formed by the antecedent and the denial of the consequent is false, then the *consequentia* is valid.[15]

Hence according to Boehner we have the thesis *CNKpNqCpq*. Now this is surely the least plausible formalisation of the rule described, since it leaves the exception that Ockham makes in the *'ut nunc'* case quite inexplicable. Secondly, other manuscripts give a much stronger version of the same rule. S2 Rule VIII, for instance, says:

> *Si oppositum consequentis non stat cum antecedente, consequentia erat bona. Et in omni consequentia bona oppositum consequentis repugnat antecedenti.*

and S3 Rule g,

> *Quando oppositum consequentis non stat cum antecedente, sed si repugnat, est bona.*[16]

It is amusing to read Boehner's remark regarding S2, Rule VIII which he says "presents first the rule in the milder form of 'not being true together' (*stare cum*), but then we could say spoils this clear presentation by adding that in every valid *consequentia* the opposite of the consequent is repugnant with the antecedent."

This would suggest that Ockham accepted nothing weaker than strict implication. But in fact it is not even clear that Ockham is saying that *whenever* the opposite of the consequent of any conditional is incompatible (i.e., not true together) with the antecedent then that proposition represents a true consequence. Rather it would appear, in the light of all the other classifications he offers, that he is stating a rule for testing a putative consequence which does satisfy other criteria of validity, i.e., it is either formally or materially valid, valid either by extrinsic or intrinsic means, and at least an absolute consequence. (As we have seen, Ockham explicitly draws attention to the consequence valid *'ut nunc'* as the only case to which the rule cannot apply).

The point of the foregoing discussion has not been to show that Ockham was not aware of material implication. His discussion of the *'ut nunc'* consequence is parallel in the most salient respects to contemporary dis-

---

15. Boehner, P., *Collected Articles*, p. 350.
16. Ockham, quoted by Boehner, *Op. cit.*, p. 347-348.

cussion of material implication, with one important difference: whereas the latter treats of propositions as absolute timeless entities, the Schoolmen's consequence *'ut nunc'* is much closer to a propositional function with time variables, true for at least 'one time' if true at all. The fact remains that the '⊃' of formal implication expresses essentially the same relation as in a material implication, and we may safely conclude that Ockham was fully aware of truth-functions. What is of as great significance is that he was aware of the essential difference between such factual consequences that are really consequences by courtesy, and every other kind. For instance, he notes that whereas for simple or absolute consequences the following two rules hold: (1) the contingent does not follow from the necessary, and (2) the impossible does not follow from the possible; neither of these hold for the *'ut nunc'* consequence. Thus Ockham:

> *Tamen consequentia ut nunc bene poterit sequi; sicut sequitur: Omne ens est, igitur omnis homo est, et tamen antecedens est necessarium et consequens contingens. Similiter bene sequitur: Omne coloratum est homo, igitur omnis assinus est homo, et tamen antecedens est possibile et consequens impossibile: et consequentia est bona solum ut nunc.*[17]

It is notable that even for these examples Ockham does not choose completely unconnected propositions as antecedents and consequents, which it is fair to suppose he would have done had he wanted to illustrate a purely truth-functional relation. In fact he does just this in the next rule quoted by Boehner, which proves beyond doubt that he did in fact accept the paradoxes of strict implication, though again he made haste to add,

> *Sed tales consequentiae non sunt formales nec sunt multum usitande.*

A point of some interest emerges from the above mentioned rule which also states that:

> *Quod ex impossibili sequitur quodlibet, et quod necessarium sequitur ad quodlibet.*[18]

For this rule Ockham says, validates not formally but materially. This then also throws light on a controversial passage in Ockham which has been variously interpreted. In connection with the classification of consequences into formal and material kinds, Ockham says that not only are those consequences that hold by means of a logical rule or extrinsic medium, formally valid, but also those that are directly validated by an intrinsic means, and only indirectly by an extrinsic means. (He wished no doubt to include as formally valid all enthymemes that could be converted into valid syllogisms by the addition of a true or necessary proposition, linking a term of the antecedent with one of the consequent). However Ockham lays down a restriction to this rule in the case of consequences that are validated indirectly,

---

17. Ockham, *Op. cit.*, Part III, 3, c. 36.
18. Ockham, S1 (10) as quoted by Boehner, *Op. cit.*, p. 349.

> *Per medium extrinsicum (non) respiciens conditiones . . . generales propositionum, scilicet veritatem, falsitatem, necessitatem, impossibilitatem.*[19]

It is clear then that Ockham listed even strict implication paradoxes under 'material consequences' valid only *'per medium intrinsicum'* and not holding by virtue of a formal rule.

Finally, it is necessary to study yet one more rule attributed by Boehner to Ockham which if correct would indeed go a long way towards establishing that Ockham accepted material implication, since the rule in question is valid in no other system. Thus Boehner claims that:

> Discussing the *'fallacia consequentia'* Ockham states that whenever there is a case in which the consequent does not follow from the antecedent, the antecedent will follow from the consequent. Hence every fallacy of the consequent can be transformed into a valid consequence by interchanging the antecedent and the consequent.[20]

If Boehner's interpretation is to be accepted, it would appear that Ockham was willing to subscribe to the material implication thesis *CNCpqCqp*. This would indeed be an important find in Ockham's logic. Closer study reveals, however, that Boehner's interpretation needs revision. The context in which this so-called law occurs, is Ockham's commentary on Aristotle's *Sophistici Elenchi*. Here Ockham observes:

> *Notandum est hic, quod semper quando est una consequentia, si sit fallacia consequentis, non tenet consequentia, sed e converso bene sequitur.*[21]

But the example which precedes this generalisation is in translation as follows:

> And in the same way if it is argued from one conditional composed of two negative members to one opposite conditional composed of the affirmative opposites to the former negatives, there will be a similar defect. And by such an argument arises the following fallacy of the consequent: 'If no animal runs, no man runs, therefore if an animal runs a man runs'; where the second conditional is equivalent to a single consequence in which there is a fallacy of the consequent from the position of the consequent.

Tracing this fallacy to its original source in Aristotle we find that Ockham here does no more than expand on Chapter 28 of the *Sophistici Elenchi* where Aristotle describes (to be sure, somewhat cryptically) this sort of fallacy. But there is no suggestion in Aristotle (nor could there be in Ockham), that if *Cpq* does not hold then *Cqp* does. He is clearly saying on the other hand that if *CNpNq* is a valid consequence then not *Cpq*, but *Cqp* (being the contrapositive of the original) is valid. Similarly if *Cpq* is valid, then not *CNpNq*, but its converse, namely *CNqNp*, is also valid.

---

19. The text is confused at this point. This is Kneale's reconstruction, *Op. cit.*, p. 290.
20. Boehner, P., *Medieval Logic*, p. 60.
21. Quoted by Boehner, *Medieval Logic*, p. 123.

We conclude that Boehner's account of Ockham's use and acceptance of material implication is unsatisfactory. For though Ockham realised that there were relations between propositions that hold *'ut nunc'* and others that hold on some material grounds, these were neither formal nor necessary, nor were they much used in actual argument.

Indian Institute of Technology
Kanpur, India