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THE DISTRIBUTION OF TERMS

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In traditional logic the validity of a categorical syllogism was determined by application of various familiar rules which employ the medieval doctrine of the distribution of terms. This doctrine has not fared well in recent times. In [2], for example, P. T. Geach argues that the doctrine is incoherent (p. 4). This conclusion is reinforced by an examination of the current textbook formulations of the doctrine: roughly, that a general term is distributed if it is used to stand for all the individuals to which the term applies.[1] Why then has the doctrine survived? Geach answers: ". . . it looks intelligible if you are not too curious; and it supplies easy mechanical rules for judging the validity of inferences" (p. 21).

In order for the traditional rules of syllogistic to provide a useful mechanical test for evaluating categorical inferences there is no need for the doctrine of distribution to be explained at all. It would suffice merely to stipulate that a term is distributed in a categorical statement if it is the subject of a universal statement or the predicate of a negative one. We think, however, that the traditional doctrine can be explicated in a manner which is reasonably straightforward and intuitively clear. It will be instructive, however, to begin by looking at a recent attempt to reformulate the doctrine.

Stephen Barker in [1] claims that the traditional definitions of the doctrine of distribution are unsatisfactory and offers a new interpretation (pp. 40-42). Let T be a term which occurs as subject or predicate in a categorical sentence s. Let s' be the sentence that results from replacing the term T in s with the compound term T' & T. According to Barker "T is said to be distributed in s if and only if, for every term T', s logically implies s'" (p. 42). Barker does not explicitly explain what a compound term T' & T is, but it seems that what he intends is the concatenation of an adjective and count noun. Consider an example provided by Barker: Let T be the term 'prohibitionists' and s be the sentence 'Some seamen are not prohibitionists'. Now if T' is the term 'rich', s' would be the sentence

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'Some seamen are not rich prohibitionists'.[2] "To say that T is distributed in s is to say that every sentence of the form 'Some seamen are not . . . prohibitionists' is logically implied by s" (p. 42).

To see that Barker's interpretation as it stands is unsatisfactory, let T' be the expression 'sham'. Now the falsehood of s' ('Some seamen are not sham prohibitionists') is compatible with the truth of s ('Some seamen are not prohibitionists'), and hence the latter does not logically imply the former. The adjective 'sham' (like the adjectives 'counterfeit', 'spurious', 'toy', and 'fake' amongst others) belongs to a class of adjectives which in a certain manner negate the terms following them. Since the proposed criterion is intended as a formal one (that is, the relation between s and s' is supposed to be formal), there does not seem to be any good reason for excluding such adjectives. Nor would the exclusion of this class of adjectives help matters much. 'White' is not a 'negator' adjective, but white ants are not ants. Similarly, an alleged criminal may or may not be a criminal, and while a broken arm is an arm, a broken vase is very likely not a vase.

Barker also provides an informal explanation of the doctrine of distribution: "A term S occurring as the subject of a categorical sentence is said to be distributed in that sentence if and only if the sentence, in virtue of its form, says something about *every kind of* S" (p. 40). He goes on to make a similar remark about a term which occurs as the predicate of a categorical sentence. As this is stated, however, it does not make clear sense. The subject term of 'All men are mortals' is distributed but there is no clear sense in which that sentence says something about every kind of 'man': there seems to be a confusion between the term 'man', and what the term stands for.

Despite the unsoundness of Barker's formulation, it can be revised in a way which is both sound and avoids talk of compound terms. Let t be the set of objects to which the term T applies, and let t' be the set of objects to which the term T' applies.[3] Further suppose that t' is a subset of t. Now we may say that T is distributed in s if and only if, for all sentences s', s logically implies s' (where s' is the result of replacing T in s with T'). One of the undesirable results of this formulation, however, is that it introduces fairly recent set theoretic notions to explain a point about the much older logic of terms. In particular, in order to understand what is asserted when it is claimed that a term is distributed we must understand what it means to say that a term designates a set and what it means to say that one term designates a subset of the set designated by another term. We think it would be preferable if the doctrine could be characterized without any such appeal.

The notion of distribution is quite intuitive and susceptible to a reasonably straightforward explanation. It is not unnatural to think that a common noun, such as 'man' or 'animal', "stands for" every particular of which the term is true. On the other hand, it is not always true that a statement, in which the term occurs, says or implies something about *each* particular for which the term stands.[4] Whether the statement does say or imply something about each particular depends on whether the term is distributed in that statement: if the term is distributed, it does, and if the term is not distributed, it does not. For example, the statement 'All men are animals' implies that it is true of each particular man that he is some animal or other, because 'men' is distributed in that statement. But the statement does not imply that it is true of each particular animal that it is some man or other, because 'animals' is undistributed. In other words, 'All men are animals' implies 'Socrates is some animal or other', 'The teacher of Aristotle is some animal or other' and so on, for every sentence of the form 'x is some animal or other' where 'x' stands in the place of a singular term which designates a particular man. On the other hand, the statement does not imply 'Buridan's ass is some man or other', even though 'Buridan's ass' designates a particular animal.[5] The notion of distribution in a negative statement will be slightly different, but it can be treated in an analogous manner. Using this basis, we shall make the doctrine more precise.

By a *categorical statement* we mean a statement which has one of the following forms:

(A) All S are P.
(E) No S are P.
(I) Some S are P.
(O) Some S are not P.

where 'S' and 'P' are replaced by plural count noun phrases.[6] The notion of distribution may now be characterized as follows: Let T^1 and T^2 be the terms of a categorical statement C, and let α be a singular term designating some individual such that $\ulcorner\alpha$ is a $T^1\urcorner$ is true.

The term T^1 is distributed in C if and only if either:

(1) $\ulcorner C$ and α is a $T^1\urcorner$ logically implies $\ulcorner$For some x which is a T^2, α is the same as $x\urcorner$,

or

(2) $\ulcorner C$ and α is a $T^1\urcorner$ logically implies $\ulcorner$For some x which is a T^2, α is not the same as $x\urcorner$.

It will be observed that the desired results are obtained—namely, that the subject terms of universal statements and the predicate terms of negative statements are distributed. We shall consider the application of this formulation to the A-form and the O-form. 'All men are mortals and Socrates is a man' logically implies 'For some mortal, Socrates is the same as that mortal'. Hence the subject term is distributed. On the other hand, the predicate is not since 'All men are mortals and Buridan's ass is a mortal' fails to imply either that 'For some man, Buridan's ass is the same as that man' or 'For some man, Buridan's ass is not the same as that man'. Now consider the O-form, which is usually thought to be the most puzzling. The statement 'Some men are not bachelors and Smith is a man' fails to imply either 'For some bachelor, Smith is the same as that bachelor' or 'For some bachelor, Smith is not the same as that bachelor'. Hence the subject term is not distributed. But 'Some men are not bachelors and Jones is a bachelor' does imply 'For some man, Jones is not the same as that man'. (In particular, Jones is not the same as any man who is not a bachelor.) Hence the predicate term is distributed.

It is not our intention to argue that the doctrine of distribution is indispensible for an adequate semantical or logical theory. We think that we have shown that the doctrine can be given a coherent explanation and has at least *prima facie* plausibility. At least this much should be expected for a doctrine that is standardly used (for better or worse) in the perennial introduction to logic.

NOTES

1. E.g., "A term is said to be distributed when reference is made to *all* the individuals denoted by it . . ." on p. 95 of J. N. Keynes' *Studies and Exercises in Formal Logic*, Macmillan, London (1906). A more recent version provided by I. M. Copi on p. 140 of *Introduction to Logic*, Macmillan, New York (1961) reads: "A proposition *distributes* a term if it refers to all members of the class designated by the term." For a thorough criticism of formulations of this sort see Chapter One of [2].

2. Barker's admission of adjectives to the class of terms in his doctrine of distribution is inconsistent with his definition of 'term': "Let us insist that in a sentence in strictly categorical form . . . the terms must be plural substantive general terms. Thus, the sentence 'All gold is valuable' will not be regarded strictly in categorical form, . . . because its predicate is an adjective rather than a substantive (that is, nounlike) expression" (p. 35).

3. In accordance with Barker's definition of 'term', terms are plural count nouns and not adjectives.

4. See Peter of Spain, *Summulae Logicales* 6.09.

5. A. N. Prior, on p. 110 of *Formal Logic*, 2nd ed., Clarendon Press, Oxford (1960), attributes something like this formulation to Peter of Spain. This is misleading in several respects. Peter, following William of Sherwood, applies the doctrine of distribution only to the subject terms of universal propositions; and, again following William, he uses it as part of his theory of meaning but not as a method for testing the validity of syllogisms. (See *Summulae Logicales*, 6.11, 6.13, 12.26 and 12.43.) The idea of using the doctrine of distribution to test syllogisms came later, perhaps first with the Pseudo-Scot, the author of *Super Libros Elenchorum*. (See W. Kneale and M. Kneale, *The Development of Logic*, Clarendon Press, Oxford (1962), pp. 272-273.)

6. We assume for the sake of our analysis that universal, as well as particular, statements have existential import; that is, each term is true of at least one particular.

REFERENCES

[1] Barker, Stephen F., *The Elements of Logic*, 2nd ed., McGraw Hill, New York (1974).

[2] Geach, Peter Thomas, *Reference and Generality*, Cornell University Press, Ithaca, New York (1962).

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