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Noncategorical Syllogisms in the *Analytics*

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It is a commonplace now among logicians that the logic of categorical syllogisms, first developed by Aristotle, presupposes the now-familiar logic of unanalyzed propositions. Aristotle, however, clearly took the syllogistic to be "basic logic", presupposing *no* other logic. Since he was not unaware of many important principles now constitutive of the calculus of propositions, it can only be argued that either: (i) Aristotle was blind to the import of such principles for formal logic in general, or (ii) he believed such principles could be accounted for by the syllogistic. In spite of the numerous and illustrious supporters of (i), we shall attempt here a brief defense of (ii).

The question, of course, is not whether Aristotle himself substantiated (ii), but rather: can any syllogistic substantiate (ii)? In answering this question affirmatively we will first cite several arguments which are found in the *Analytics*, and which make use of well-known principles of the propositional calculus. We shall then make some historical remarks concerning the attempt to reduce the logic of unanalyzed propositions to the logic of analyzed propositions (the syllogistic). Finally, we hope to show how a recently developed syllogistic system offers a technique which can be used to successfully render the arguments cited from the *Analytics* as categorical syllogisms.

1 The first argument is from *Prior Analytics*, 34a6-7.

(1) . . . if when A is, B must be, then if A is possible, B must necessarily be possible.

The next two arguments are found together at *Prior Analytics*, 53b 12-15.

(2) If, when A is, B must be, then if B is not, A cannot be.
(3) Therefore, if A is true, B must be true.

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Here the premise "When *A* is, *B* must be" is understood. The fourth and fifth arguments are found together at *Prior Analytics*, 57b1-6.

(4) . . . when two things are so interrelated that when the first is the second must be, when the second is not, neither will the first be.
(5) . . . but when the second is, the first need not necessarily be.

The last two are from *Posterior Analytics*, 72b36-73a5.

(6) . . . if *A* is, *B* must be; if *B* is, *C* must be; therefore if *A* is, *C* must be.
(7) . . . if *A* is, *B* must be, and if *B* is, *A* must be, . . . if *A* is, *A* must be.

Given the propositional calculus, 1 and 3 are analyzed as modus ponens arguments (1 has a problematic premise and conclusion), 2 and 4 illustrate modus tollens, 6 and 7 are examples of "hypothetical syllogisms", or chain arguments, and 5 exemplifies the fallacy of affirming the consequent. None of these arguments is in the form of a categorical syllogism. All make use of variables taking propositions as values. So, if there is any way to give a syllogistic analysis of these inferences, it must, at least, succeed in eliminating the use of such variables.

2 It is known that there was a certain degree of rivalry between the ancient Peripatetic and Stoic schools of logic. They disagreed not only about the relative status of the logic of unanalyzed propositions with respect to the logic of analyzed propositions, but about the status of formal logic in general. While Aristotle took formal logic (demonstration) to be a *tool* for doing and teaching philosophy and science, the Stoics took logic to be a part of philosophy. The logic developed by them was a logic of unanalyzed propositions, closely related to the contemporary propositional calculus.

The teachings and doctrines of the two schools were often fused and confused over the next several centuries. And there was a tendency (as Aristotelianism grew stronger) to treat Stoic examples and principles in Aristotelian terms. Eventually syllogists were forced to confront the issue of just what the relationship should be between the so-called "hypotheticals" and the rest of syllogistic. By the seventeenth century the Cambridge philosopher John Wallis was arguing that hypothetical arguments could be construed as categorical syllogisms (see [10], p. 29 and [11], p. 306). But it was Leibniz who, later in that century, did the most to show just how such a reduction might be possible.

Leibniz realized that the logic of unanalyzed propositions could only be reduced to the logic of analyzed propositions if the hypothetical form of a proposition could be reduced to the categorical form. He wanted to do this because he hoped to develop a *universal* logic based on the syllogistic mode. It was a task he struggled at most of his life. And while he never fully succeeded, he did manage to make important progress. In 1679 Leibniz wrote that "the categorical proposition is the basis of the rest, and modal, hypothetical, disjunctive and all other propositions presuppose it" ([12, p. 17). He realized that "certain changes" would be required to make the hypothetical form categorical. But he was still not clear about just what those changes would be. In 1686 Leibniz wrote *General Inquiries about the Analysis of*

Concepts and of Truths. In it he attempted to bring together a wide variety of logical concepts and devices into a coherent whole, and claimed to have "made excellent progress". Much of the paper is devoted to finding a satisfactory mode of symbolization for categoricals. Leibniz saw clearly how important a uniform syntax for both categoricals and noncategoricals was for the development of a uniform, universal logic.

> If, as I hope, I can conceive all propositions as terms, and hypotheticals as categoricals, and if I can treat all propositions universally, this promises a wonderful ease in my symbolism and analysis of concepts, and will be a discovery of the greatest importance. ([12], p. 66)

The key to solving the problem of hypothetical reduction, as Leibniz saw, is the ability to conceive of entire propositions as themselves terms. In *General Inquiries* he used variables which can be interpreted either as terms or propositions. But, though he had no doubt that "any proposition can be conceived as a term" ([12], p. 71, see also p. 86), he was far from certain about just how this could be done.[1]

Before going on to show how Leibniz's problem can be solved, it is interesting to note that Leibniz believed that, given a means of reduction, a uniform reading could be given for categoricals, hypotheticals, and even entire inferences. He took all categoricals to have the general form: *A contains B*, where *A* is the subject and *B* the predicate. He said that a conditional can be viewed as having the same form since it merely says that the consequent is contained in the antecedent. In other words, a sentence of the form '*A* contains *B*' can be read as a categorical with *A* and *B* as its subject and predicate terms respectively, or as a conditional with *A* the antecedent and *B* the consequent.

> Une proposition catégorique est vrai quand le prédicat est contenu dans le sujet; une proposition hypothétique est vrai quand le conséquent est contenu dans l'antécédent. ([1], p. 483)

At the end of *General Inquiries* he suggested that the relation of logical entailment could likewise be reduced to one of containment since "that a proposition follows from a proposition is simply that a consequent is contained in an antecedent, as a term in a term. By this method we reduce inferences to propositions, and propositions to terms" ([12], p. 87). There is little doubt how strongly Leibniz believed that a logic of unanalyzed propositions could be reduced completely to a logic of analyzed propositions.

> We have, then, discovered many secrets of great importance for the analysis of all our thoughts and for the discovery and proof of truths. We have discovered how . . . absolute and hypothetical truths have one and the same laws and are contained in the same general theorems, so that all syllogisms become categorical . . . ([12], p. 78)

3 Inferences, such as those found in the *Analytics*, which are not *prima facie* categorical, can be reduced to the categorical form, as Leibniz saw, only by first reducing hypothetical propositions to categoricals. This last, as again Leibniz saw, requires the interpretation of entire propositions as terms. But how is this possible? Can such an interpretation, a procedure for turning propositions into terms, be devised?

F. Sommers has done just that. Since 1967 Sommers has produced a series of papers ([13]-[20]) in which he has moved ever closer to the fulfillment of Leibniz's dream of a uniform, universal logical calculus based on the categorical form. In the process Sommers has shown how all noncategoricals (including relational sentences, identity statements, and truth-functions) can be uniformly treated as categoricals[2]. Recall that the problem of reducing truth-functions (hypotheticals) to categoricals hinged on the problem of treating entire sentences as terms. In "On concepts of truth in natural languages" [15] Sommers, while primarily concerned with defending a version of the correspondence theory of truth, solves that problem. Tarski [21] had suggested the notion of an entire sentence having, like a term, a denotation. Sentences could be said to *designate states of affairs*. Indeed, long before this Frege had talked of the denotation, or reference, of a sentence as the circumstances of its truth or falsity. Sommers distinguishes between what a sentence is *about* and what it designates, or *specifies*. Suppose 'S' is a sentence (say 'Nixon is a liar'). Then '$[S]$' will be read as 'State of affairs in which S' (e.g., 'State of affairs in which Nixon is a liar'). An expression like '$[S]$' is a term (a "sentential term"), and the process of forming '$[S]$' from 'S' is called "nominalization". The nominalization of a sentence always results in a term, a sentential term. The states of affairs in which Nixon is a liar are what is specified by the sentence 'Nixon is a liar'. In general, a sentence 'S' specifies a state of affairs, $[S]$. Keep in mind that '$[S]$' is a term which denotes $[S]$, while 'S' is a sentence which specifies $[S]$. Sommers says that a sentence is about what its terms denote. Thus 'Nixon is a liar' is about Nixon and liars. Moreover, sentential terms, being terms, can be used in other sentences. For example, 'The state of affairs in which Nixon is free is deplorable' contains as one of its terms a sentential term. Let S be the sentence 'Nixon is free' and $S.1$ be the sentence 'The state of affairs in which Nixon is free is deplorable'. $S.1$ says '$[S]$ is deplorable'. One of the things $S.1$ is about is $[S]$. But what $S.1$ specifies is $[S.1]$, i.e., $[[S]$ is deplorable$]$.

The notion of sentence nominalization, and the resulting distinction between what a sentence is about and what it specifies, proves to be an impressively powerful tool for logical analysis. Sommers shows how it can be used, contra Tarski, to block paradoxes of self-reference in a natural language (e.g., the Liar), while permitting all innocuous self-reference (as in 'This sentence has five words'). But, more importantly for the development of syllogistic along Leibniz's lines, Sommers shows how the nominalization process can be used to convert truth-functions into categoricals.

A sentence like 'If Nixon is free then justice has failed' is clearly not a categorical. It is a truth-function of two other sentences (which could themselves be viewed as categoricals). But it can be conceived of as a categorical, given the notion of a sentential term. According to Sommers, 'If Nixon is free then justice has failed' is about two kinds of states of affairs: those in which Nixon is free and those in which justice has failed. What it says about these is that all the states of affairs of the first kind are states of affairs of the second kind. We could rewrite our sentence (using 'S' for 'Nixon is free' and 'P' for 'justice has failed') as: 'All [S] are [P]'. Now this sentence is a genuine categorical. It has a subject term, '[S]', and a predicate term, '[P]'. Again, sentential terms are terms, not sentences; they denote what are specified by sentences. Since any truth-function can be transformed into a categorical by the nominalization of its component sentences, Leibniz's quest for a means of making hypotheticals categorical is achieved in a simple and straightforward manner. Here are some simple examples of such transformations.

Truth-functional forms	Categorical forms
If p then q	All [p] are [q]
p and q	Some [p] are [q]
If p then if q then r	All [p] are [all [q] are [r]]
p	Some [p] exists (or holds)
Not p	No [p] exists.

Given Sommers's nominalization procedure it is a simple matter to show our seven arguments from the *Analytics* can be construed as categorical syllogisms.

(1′) All [A] are [B]
Some [A] is possible
Therefore, some [B] is possible

(2′) All [A] are [B]
No [B] exists
Therefore, no [A] exists

(3′) All [A] are [B]
Some [A] exists
Therefore, some [B] exists

(4′) All [first] are [second]
No [second] exists
Therefore, no [first] exists

(5′) All [first] are [second]
Some [second] exists
Therefore, some [first] exists

(6′) All [A] are [B]
All [B] are [C]
Therefore, all [A] are [C]

(7′) All [A] are [B]
All [B] are [A]
Therefore, all [A] are [A].

With the noted exception of 5′, these are all valid categorical syllogisms. 1′ and 3′ are in third figure (viz. Disamis) and the others are in first figure (2′ and 4′ are Celarent, 6′ and 7′ are Barbara).

A final note: it requires no great ingenuity to see Aristotle's use of phrases such as "when A is" as shorthand for something like "on all occasions where

A holds", which in turn is but a slight variation on Sommers's "All states of affairs in which A".

NOTES

1. An excellent account of Leibniz's attempt here is offered by Castañeda [1].
2. The papers in Sommers's series are listed in the references. It is unfortunate that thusfar Sommers's logical work has not received the attention it deserves. I have offered comments on this work in my papers [2]-[9].

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