Notre Dame Journal of Formal Logic
Volume XXI, Number 1, January 1980
NDJFAM

LEWIS' POSTULATE OF EXISTENCE DISARMED

ROBERT W. MURUNGI

C. I. Lewis [5], pp. 178-179, introduced his so-called existence postulate to delineate his system of strict implication from material implication. Cresswell [2], p. 291, adopted a postulate according to which in a Kripke [4] normal model structure (G, K, R), it must be that $K > \{G\}$. This formula, as Cresswell points out, is equivalent to the validity of Lewis' postulate. Now according to Kripke's modelling for classical sentential modal logic (see [3] and [4]), with respect to $K = \{G\}$, all true sentences would be necessary. Lewis' postulate is designed to block against such modal collapse. The problem with which we have to contend is this: Is this postulate a necessary truth to be assumed as an axiom of modal logic? Should the possibility that $K = \{G\}$ be excluded for fear of modal collapse? I wish to show that Lewis' postulate is a contingent, not a necessary, truth and, consequently, that modal collapse is no ground for excluding such possiblities as $K = \{G\}$.

Most modal logicians appear to think that Lewis' postulate is a necessary truth. Church alone in [1] was inclined to cast doubt on the majority position. The following considerations justify Church's position.

We take a *possible world* to be not, as in Kripke modelling, a truth-value assignment for a wff α, but a set of truth-value assignments for α. By a *valuation space* for a wff α let us understand a set of possible worlds W such that for any truth-value assignment Σ to the variables of α, there is a member $w_i \in W$ in which Σ is represented. Now if, as in classical sentential calculus, there are exactly 2^k possible truth-value assignments to the variables of a wff α which contains occurrences of exactly k distinct sentential variables, then there are exactly $2^{2^k} - 1$ non-empty possible worlds in W. Semantics for L are then stipulated classically, not across possible worlds but within them as follows:

(L) Let $w_i \in W$ be any set of truth-value assignments satisfying the usual conditions for sentential calculus *de inesse*. Then for any wff α, and any truth-value assignment $\Sigma \epsilon w_i$, the truth-value, V, of $L\alpha$ is defined by the clause $V(L\alpha, w_i) = t$, iff for every $\Sigma \epsilon w_i, V(\alpha, w_i) = t$. Otherwise, $V(\alpha, w_i) = f$.

Received June 17, 1977

Besides semantics for 'L', in order to evaluate Lewis' postulate adequately, we need semantics for wff with quantified sentential variables.

(Π) Let $w_i \in W$ be as in (L) above. Then the truth-value, V, of a universally quantified wff $\Pi p\alpha$ is defined as follows: $V(\Pi p\alpha, w_i) = t$ iff for each $\Sigma \in w_i$, $V(\alpha, w_i) = t$. Otherwise, $V(\Pi p\alpha, w_i) = f$.

A wff α is said to be *true* in w_i iff $V(\alpha, w_i) = t$ for each $\Sigma \in w_i$. A wff α is said to be *contingent* iff $V(\alpha, w_i) = t$ under some $\Sigma \in w_i$ and $V(\alpha, w_i) = f$ under some $\Sigma' \in w_i$, where, of course, $\Sigma \neq \Sigma'$. A wff α is *valid* iff for every $w_i \in W$, $V(\alpha, w_i) = t$. Validity is thus defined classically as truth in all possible worlds. Finally, a wff α is a *necessary truth* iff α is valid.

Now as Cresswell in [2], p. 291, has pointed out, Lewis' postulate is equivalent to

(1) $N\Pi pCpLp$.

To see that (1) is contingent, consider the $M'(T')$ thesis in Murungi [6], namely, *necessitas consequentis*,

(2) $CpLp$.

The valuation space for (2) has exactly three non-empty members: two possible worlds in which p and Lp have the same truth-value and in which (2) is true, and one possible world in which there is some $\Sigma \in w_3$ such that (2) is true and some $\Sigma' \in w_3$ in which (2) is false. Hence, by (Π), $V(\Pi pCpLp, w_1) = V(\Pi pCpLp, w_2) = t$ while $V(\Pi pCpLp, w_3) = f$. Hence $V((1), w_1) = V((1), w_2) = f$ while $V((1), w_3) = t$. It follows that (1) is contingent. Therefore, Lewis' postulate is not a necessary truth to be assumed as an axiom of modal logic. The possibility that $W = \{w_3\}$ cannot be excluded on grounds of modal collapse, since in this case the Axiom of Necessity $CLpp$ is true while (2) is contingent. Classical modal logic is in need, not of Lewis' postulate, but of a pregnant notion of possible worlds.

REFERENCES

[1] Church, A., "A formulation of the logic of sense and denotation," in P. Henle, H. M. Kallen, and S. K. Langer, eds., *Structure, Method and Meaning: Essays in honor of H. M. Sheffer*, Liberal Arts Press, New York, 1951, p. 22.

[2] Hughes, G. E., and M. J. Cresswell, *An Introduction to Modal Logic*, Methuen & Co., Ltd., London, 1968.

[3] Kripke, S. A., "A completeness theorem in modal logic," *Journal of Symbolic Logic*, Vol. 24 (1959), pp. 1-14.

[4] Kripke, S. A., "Semantical analysis of modal logic I: normal modal propositional calculi," *Zeitschrift fur Mathematische Logik und Grundlagen der Mathematik*, Vol. 9 (1963), pp. 67-96.

[5] Lewis, C. I., and C. H. Langford, *Symbolic Logic*, Dover Publications, Inc., New York, 1959.

[6] Murungi, R. W., "On a nonthesis of classical modal logic," *Notre Dame Journal of Formal Logic*, Vol. XV (1974), pp. 494-496.

Southern Illinois University
Edwardsville, Illinois

and

Kenyatta University College
Nairobi, Kenya