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ON PROPOSITIONAL FORM

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> Die allgemeine Form des Satzes ist: Es verhält sich so und so.
>
> Wittgenstein

For Aristotle the proper domain of logic was not just sentences in general, but propositions. Propositions are sentences "that have truth or falsity in them".

Aristotle believed that all propositions could be viewed as having, logically, one of the four basic categorical forms. In other words, all propositions are logically subject-predicate in form. If he thought anything about other forms of propositions (viz., those more complex forms built up from combinations of categoricals) we have little evidence of it from his extant writings. Contemporary mathematical logicians recognize both simple subject-predicate propositions and propositional "truth functions". A theory of the logical form of simple subject-predicate propositions can be gleaned from mathematical as well as Aristotelian logic. I think the Aristotelian view is closer to the truth.

This essay has four parts.* In the first part we examine Aristotle's theory of the logical form of a categorical. We then look at the notion of the form of a simple subject-predicate proposition as found in the contemporary logician's calculus of (one-place) predicates. In Section **3** a

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*I owe many of the ideas presented in this essay to a variety of sources. One source, however, is all-pervasive, namely the most recent work of Fred Sommers [8]-[12]. Correspondence from Professor Sommers and his unpublished manuscript [13] have also been sources of motivation and insight behind this present essay.

Aside from [1] my thinking about Aristotle has been prodded by Bocheński [2], McCall [6], Corcoran [3], and Łukasiewicz [5]. To the latter (p. 6) I owe the suggestion that apodeictic and problematic propositions might be viewed as strongly and weakly assertoric.

I should also mention Strawson [14] and [15], Quine [7], and Geach [4], to whom I owe the notion of the connotation of singular terms.

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general picture of the logical form of simple categoricals is offered which is similar to the classical view.

1 *Aristotle on propositional form* Throughout the *Prior Analytics* Aristotle employed the notations *AB*, *BC*, *PΣ*, etc., to indicate propositions predicating the first term of the second (*in some way or other*). We might call these "proto-propositions". Of course, in actual discourse no proposition is a proto-proposition *simpliciter*. Every predication is some kind of predication. Thus, for example, every predication is either an affirmation or a denial. In *Prior Analytics* 25a1-5 Aristotle said that in addition to proto-propositions being modified by quality they are also modified by quantity and modality. Moreover, it is clear from what he said in *On Interpretation* 21b26-33 that these modifications are applied to a proto-proposition in a particular order. A predication is first affirmative or negative. Second, it is universal, particular, or indefinite. Finally, every predication is assertoric, apodeictic, or problematic.

But there is more. In *Prior Analytics* Aristotle was developing a logic fit for science. Since for him science had to deal in universal principles there was no need for him to consider propositions whose subjects are individual. Our interest here is in what Aristotle thought of propositions in general, not just those he thought constitutive of science. In *On Interpretation* he often gave examples of propositions having individuals as subjects (e.g., 'Socrates is ill'). Furthermore, at 17b5-12 we are told that a proposition like 'Man is white' has a universal subject but is not "universal in character". These kinds of propositions he called *indefinite*. All of the examples of indefinite propositions which Aristotle gave are like this. They have universal subjects but are not so characterized (i.e., they omit the initiating 'all' or 'every').

As is clear from the second and third chapters of *On Interpretation*, Aristotle considered a simple proposition to consist of a noun (*ονομα*) and a verb (ῥῆμα). Nouns are either general names (e.g., 'man') or proper names (e.g., 'Philo'). Verbs are adjectives or nouns having a time reference (usually implicit) and indicating "that something is said of something". It seems now that Aristotle's talk of the quantitative modification of a proposition was confused. We have universal, particular, indefinite, and individual propositions. But all of these are clearly not on par with one another. I think the confusion here is due to Aristotle's failure to adequately separate how he characterized propositions and how he characterized subjects of propositions. Let us say that subjects (nouns) can be either universal (general names) or individual (proper names, definite descriptions, etc.). When Aristotle talked of what is universal, particular, or indefinite he meant not subjects but propositions. In *On Interpretation* 17b12-15 he said that the universal quantifier when applied to the subject does not make *it* universal—it makes the "whole proposition" universal. In other words, the ambiguity of "universal" is resolved by saying that a subject is universal by virtue of its being a general name. Propositions are universal (as a whole) by virtue of their subjects being universally quantified. Quantification characterizes propositions, not subjects. We could say, then, that an indefinite proposition is one which has no *explicit* quantifier, universal, or particular.

Universal (in the sense of being a general name) and individual are not kinds of predications. While every proposition has either a universal subject or an individual subject, this fact is unimportant for characterizing the form of the whole proposition. What this means is that such propositions as 'Men are white' and 'Socrates is white' could, contra Russell, be treated on all logical fours with each other. The fact that Aristotle's logic dealt only with the former kinds of propositions indicates no logical derivation or inferiority on the part of propositions with individual subjects, but rather a peculiarly Aristotelian view of what the use of logic should be. From *Prior Analytics* 43a25-44 it is clear that for science only terms which can serve both as subjects and as predicates are of importance. In Aristotle's overall theory of propositional form there was no need to exclude from logic propositions with individual subjects. If, then, we take individual subjects on par with universal subjects we must say that every proposition, regardless of the nature of its subject, is either universal, particular, or indefinite. Moreover, if we take indefinite propositions as simply implicitly universal or particular, then we can say that every proposition, including those with individual subjects, is either universal or particular. The reason, which I am sure Aristotle saw, that in ordinary discourse we leave propositions with individual subjects indefinite (i.e., we dispense with any overt quantification) is that the universal quantification of such a proposition and the particular quantification of it are logically equivalent.

For Aristotle, any term could be negated. Examples such as 'not-man' and 'not-ill' are numerous throughout the *Organon.* When Aristotle divided propositions into those which are affirmative and those which are negative he was again talking about the "whole proposition". In *On Interpretation* 19b20-30, for example, we are told that 'Man is just' and 'Man is not-just' are *both* affirmative; and also that 'Man is not just' and 'Man is not not-just' are negative. Thus, it is important to avoid the easy confusion of negation as a propositional characterization with negation as a term characterization. In other words, the first and second 'not's of 'Man is not not-just' are not equivalent; they do not cancel each other. The second is a sign of term negation. It applies only to the predicate term 'just'. The first is the sign of predicate denial. It indicates that the predicate ('not-just') is denied of the subject. Every predicate is either affirmed or denied of its subject. Aristotle said, "we mean by negation a statement denying one thing of another" (*On Interpretation* 17a25-27).

We can say, now, that for Aristotle every proposition affirmed or denied a (negated or unnegated) predicate term (verb: adjective or noun) of a (negated or unnegated) subject term (noun: general or individual name), which was either implicitly or explicitly quantified universally or particularly. A proto-proposition, say AB, (where A is the predicate term and B

is the subject term, either or both of which may or may not be negated) was first modified by quality. The predicate term was either affirmed or denied of the subject. Let us indicate the possible results of quality modification like this:

(not-) B is (not) (not-) A .

Only parenthetical phrases can be omitted. Next a qualified proposition would be quantified. Let us indicate those possible results like this

(all/some) (not-) B is (not) (not-) A .

Aristotle said that quantified propositions are modalized. Now, like quality and quantity, modality is meant to characterize the whole proposition. However, if we look carefully at what has been said thusfar we see that propositional quantity was affected by attaching a quantifier to the subject term alone. Also, propositional quality was the result of attaching the sign of affirmation ('is') or the sign of denial ('is not') to the predicate term. Indeed, we can say that for Aristotle a proposition just consisted of a subject and a predicate. The subject consisted of a subject term (noun) and a (possibly implicit) quantifier. The predicate consisted of a predicate term (verb) and a sign of predication quality (affirmation or denial). A proto-proposition, e.g., *AB*, then, was just a subject term and a predicate term—not a subject and predicate. Modifications of quality and quantity applied to the proposition as a whole, not by adding any third item to the proto-proposition, but by modifying the *terms* of the proposition. The same holds true for modality.

In *Prior Analytics* 25a1-5 actuality, necessity, and possibility are said to be ways in which an attribute applies to some subject. The passage in *On Interpretation* 21b26-33 seems to indicate that just as 'is' and 'is not' are "added to the underlying things" (i.e., the terms constituting the proto-proposition), 'is possible', etc., are likewise so added. (What underlies a modalized proposition is a quantified one.) Thus, modality, it would seem, modifies a proposition by characterizing the way in which the predicate is predicated of the subject. In other words, a predicate is predicated of a subject (i.e., affirmed or denied of it) assertorically, apodiectically, or problematically. Like quality and quantity, modality modifies a proposition without adding any third item to the proto-proposition, but rather by modifying one of the underlying terms. In this case it is the predicate term, which happens to have already been qualified. We can indicate the possible results of modalizing a quantified proposition ~~like~~ this:

(all/some) (not-) B is (not) (necessarily/possibly) (not-) A .

Aristotle's logic is a logic of terms rather than unanalyzed propositions. For him logic was a tool for discerning validity of arguments. Arguments are analyzed into propositions, which are in turn analyzed into terms. The validity of an argument is intimately tied to the terms of its propositions. Thus, it would have been essential for Aristotle to have had a fairly well-worked out theory of propositional form—a theory of how terms constitute propositions. Thusfar I have tried to indicate that theory. Several points concerning the Aristotelian view deserve special emphasis, however, I will merely list them here, for each is worthy of its own independent study.

1. Every proposition is categorical (i.e., has exactly one subject and one predicate).

2. Any term, subject or predicate, can be negated.

3. Any term can be either a subject term or a predicate term.

4. A term may be either universal (general) or individual.

5. While quality, quantity and modality characterize whole propositions, they do so not by adding a new item to the subject-predicate pair, but by modifying the subject or the predicate.

2 *Predication and the predicate calculus* In a sense the contemporary mathematical logician has two logics—two calculi. The propositional or sentential calculus is a logic of unanalyzed propositions. It is usually claimed that this is the basic logic, the logic underlying any other logic. The contemporary logic of analyzed propositions is the predicate calculus. It is the theory which corresponds to Aristotle's logic of terms—his theory of categorical propositions. As its name implies, predicate calculus puts much logical weight on the predicate of a proposition.

Let us begin by examining a proposition like 'All singers are performers'. In the view now taught in the schools this is paraphrased as 'Each thing is such that if it is a singer then it is a performer'. Now, 'each thing is such that' is taken to be a quantifier; 'it is a singer' and 'it is a performer' are taken as two propositions connected by the propositional connective 'if . . . then'. The whole thing is symbolized as

$$(x)(Sx \supset Px)$$

which is meant to reveal the logical form of the original proposition. But how does that 'if . . . then' get in there? And all those x's? The contemporary logician agrees. Indeed, it is not a simple categorical. The original proposition, 'All singers are performers', is supposedly deceptive in its simplicity. The whole point, says he, is to reveal its hidden logical complexity. When this is done we see that the proposition is really a quantified conditional. It is in the antecedent and consequence of this conditional that we see how the contemporary logician analyzes categoricals. We interpret these as 'it is a singer' and 'it is a performer'. We can see now why this logic is called "*predicate* calculus". Our original natural language proposition had a subject term, 'singers', and a predicate term, 'performers'. After formalization we are left with two predicate terms, 'singer' and 'performer'. The contemporary logician takes all the terms of such propositions to be logical predicates, with the exception of certain logical operators such as negation and quantification. What happens to subjects? As we say, '$(x)(Sx \supset Px)$' is no longer the form of simple categorical. 'Sx' and 'Px' are. Here the subject is indicated by 'x', which is called an "individual variable". It is a variable name for individuals. No subject is general. All propositions which are categorical are singular. This is an important fact, and true for all categoricals according to the theory: the logical subject of any such proposition is just the pronoun 'it'.

In the classical view every simple assertoric categorical has a (possibly implicit) sign of quantity and a sign of quality. In the contemporary view no simple assertoric categorical has a sign of quantity. To add a quantifier is to raise the proposition's level of logical complexity. In contemporary terms 'Px' is atomic, but '$(x)Px$' and '$(x)(Sx \supset Px)$' are molecular. The predicate does, however, admit something like a sign of quality. A predicate can be denied of a subject—but only by negating the entire proposition. In the classical view 'x is P' and 'x is not P' differ only in quality. The first affirms P of x while the second denies P of x. Affirming and denying are the two ways of predicating a predicate of a subject assertorically. They are operations between subjects and predicates at the same logical level. Not so today. In the contemporary view, while an affirmation is logically simple, its corresponding denial is logically complex. This is because such a logic requires that a predicate be denied only be negating the entire corresponding affirmative proposition. Given 'Px', the denial is, supposedly, just its negation, '$\sim Px$'. But while 'Px' is atomic, '$\sim Px$' is molecular. In other words, in today's view, all simple categoricals are affirmations.

And what about the place of modal modifiers in the contemporary theory of propositional form? It seems that no uniform acceptable position has found sufficient support to become standard. For some logicians, modality is not part of a proposition, but merely a metalinguistic characterization of the proposition as a whole. For others, modality is a direct operation on propositions. Such operations form new propositions of a logically greater complexity. For still others, modality operates on predicates to form new predicates.

We can summarize now the theory of predication for simple assertoric categoricals found in contemporary predicate calculus as follows.

1. The logical subject is always singular.

2. The logical predicate is always universal.

3. The quality is always affirmative.

4. Every term is positive.

5. Predicate denial is equivalent to propositional negation.

3 *The form of a proposition* A proposition has exactly two parts: a *subject* and a *predicate*. Each are modified *terms*. A term is any noun, noun phrase, pronoun, verb, verb phrase, or adjective. Thus 'man', 'the man', 'tall girls', 'he', 'they', 'run', 'believed', 'wrote slowly', 'red', and 'rare' are examples of terms. A subject is a modified subject term. It is modified by a mark of quantity. Words like 'all', 'every', 'each', and their synonyms are *universal* qualifiers. Words like 'a', 'some', 'at least one', and their synonyms are *particular* quantifiers.

Every term has a denotation. Whatever a term applies to is denoted by that term. Thus 'men' denotes all men, 'red' denotes all red things, and 'Jones' denotes Jones. A pronoun simply denotes whatever is denoted by the noun it replaces. Thus in 'Jones said he was angry' the pronoun 'he' denotes Jones since it replaces 'Jones'. Terms which denote several things are *general* terms. Terms which denote exactly one thing are *individual* terms. So, while 'men', 'man', 'they', and 'red' are general, 'Jones', 'the man', and 'he' are individual.

Every term has a *connotation*. Its connotation is its definition or meaning. While 'red' denotes apples, fire engines, cardinals, etc., it connotes something like: *primary colour of the visual light spectrum which . . . .* While 'bachelor' denotes a large number of men, it connotes something like: *unmarried male adult person*. Of any individual term we can ask the question: What is Jones? What is it? What is Ursa Major? The answer will always involve a general term, thus: a man, a number, a constellation. The connotation of an individual term is simply the connotation of the general term which answers the What is it? question for that case.

Subjects do not have denotations or connotations. Subjects *refer*. For example, 'all men' refers to all men, 'some men' refers to some men (remember that in each case the term 'men' *denotes* all men). Since 'some Jones' and 'all Jones' both refer to the same thing, the denotation of 'Jones' (viz., Jones), subjects whose terms are individual usually omit the modifying quantifier. Logically, some quantifier must be understood to accompany such subject terms. But it does not matter which one. It is important, then, to keep in mind the distinction between referring (a role played by subjects) and denoting (a role played by terms). The classical doctrine of distribution was, no doubt, based upon a recognition of the fact that some quantified phrases refer to just what their terms denote while others do not. Now in addition to universally quantified subjects, particularly quantified subjects, and individual subjects there are often subjects which consist, explicitly at least, of just general terms. 'Stars are falling' and 'Men are animals' are examples of propositions with such *indefinite* subjects. But *every* subject must logically have a quantifier. Indefinite subjects, like individual subjects, have implicit quantifiers. However, indefinite subjects, unlike individual subjects, do not admit of either quantifier arbitrarily. The quantifier is omitted from indefinite subjects not because it does not matter but because it is understood. 'Stars are falling' is logically equivalent to 'Some stars are falling', and 'Men are animals' is logically equivalent to 'All men are animals'.

One way of characterizing a proposition is according to the type of subject it contains, so propositions are prima facie universal, particular, individual, or indefinite. But, logically, propositions are either universal or particular.

A predicate is a modified predicate term. It is modified by a mark of *predication.* (Sometimes predicates are also modified in a second way, which will be discussed later.) Predicates are either *affirmed* or *denied.* Words like 'is', 'are', 'was', 'does', etc., and their synonyms are marks of affirmation. Words like 'is not', 'isn't', 'are not', 'don't', etc., and their synonyms are marks of denial. 'Jones is angry', 'Men are mortal', 'We were along', and 'Mary doesn't sew' are propositions whose predicates are affirmed. 'Civilization isn't dead', 'Whales are not fish', 'Some student isn't in the classroom', and 'Words don't mean anything' are propositions whose predicates are denied. A proposition whose predicate is affirmed is an *affirmation.* A proposition whose predicate is denied is a *denial*. Some propositions omit any explicit mark of predication. Examples are: 'Jones runs' and 'All students desire good marks'. Such propositions are implicitly affirmative. Thus 'Jones runs' is logically equivalent to 'Jones does run' or 'Jones is running', and 'All students desire good marks' is logically equivalent to 'All students do desire good marks' or 'All students are persons who desire good marks'. *Logically,* every proposition is either an affirmation or a denial.

By letting 'S' be any subject term, 'P' be any predicate term, 'all' be any universal quantifier, 'some' be any particular quantifier, 'are' be any mark of affirmation, and 'aren't' be any mark of denial we can represent the *logical form* of any proposition, thusfar, as

$$\text{all/some } S \text{ are/aren't } P \quad .$$

But there is still more about this basic form.

If I tell you that my car is nonred you know that it is blue or green or black or white or pink or some other colour. If I tell you that the number of planets is uneven you know that it is odd. If I tell you that my friend is unmarried you know that my friend is either a bachelor or a spinster or a child. Term pairs like 'red'/'blue', 'red'/'green', 'odd'/'even', or 'married'/'bachelor' are said to be *contraries*. Some terms, like 'red', have any number of contraries. Others, like 'odd' have just one. If I want to specify all the contraries of a term then it is often handy to have a device for doing so without listing all the contraries. This is so for terms like 'red' with very large numbers of contraries. I used just such a device at the beginning of this paragraph. Terms like 'nonred', 'uneven', and 'unmarried' serve to replace the disjunctions of all the contraries of those terms. Thus: 'nonred' is logically equivalent to 'blue or green or black or . . .', 'uneven' is logically equivalent to 'odd', 'unmarried' is logically equivalent to 'a bachelor or a spinster or a child'. Pairs such as 'red'/'nonred', 'even'/'uneven', and 'married'/'unmarried' are *logical contraries*. Every term has exactly one logical contrary. The logical contrary of any term is formed by adding to it such prefixes as 'non' or 'un' or their synonyms (such as 'dis' in 'likes'/'dislikes'). Such a prefix is called a *term negator*. Terms such as 'nonred' and 'unhappy' are *negative terms*. It is important to notice that, while in any pair of logical contraries one term is negated and the other unnegated, each is equally the logical contrary of the other and can be formed from the other by adding a term negator. So, 'red' and 'nonred' are each the logical contrary of the other; and the logical contrary of any term is logically equivalent to that term prefixed by a term negator. Therefore, 'red' and 'nonnonred' are both logical contraries of 'nonred' and are logically equivalent to one another. In other words, the logical contrary of any term can be achieved either by adding to or deleting from it a term negator.

If we let 'non' be any term negator then we can fill out our scheme for the logical form of any proposition as

all/some (non)*S* are/aren't (non)*P* ,

where parenthetical marks may be omitted.

Finally, predicates are predicated of subjects either assortorically or modally. *Modality* is a modification of a predicate by the introduction of 'possibly' and 'necessarily' and their synonyms. We can predicate of a subject a predicate of assertoric form: 'are *P*' or 'aren't *P*'. Or, we can predicate a predicate of modal form: 'are possibly *P*', 'are necessarily *P*', 'aren't possibly *P*', and 'aren't necessarily *P*'.

From a logical point of view we can always eliminate one of our two types of modal modifiers, given negative terms. Thus, '*S* aren't necessarily *P*' can be reparsed as '*S* are possibly non-*P*'. '*S* are necessarily *P*', then, is '*S* aren't possibly non-*P*'. If we let '?' be 'possibly' or any synonym of 'possibly' then we can give our final version of the logical form of any proposition as

all/some (non)*S* are/aren't (?) (non)*P* .

I will conclude with one final remark of comparison between the contemporary and classical views of propositional form. According to contemporary mathematical logic, every proposition is either asserted or negated (p or $\sim p$). No such contrast exists for either Aristotle or for us. The opposition between affirmation and denial is a contrast between two ways of predicating a predicate of a subject. When Aristotle says of a proposition that it is negative he merely means that its predicate is denied rather than affirmed of its subject. In modern logic negation is a new element added to the unnegated proposition. Thus, while 'p' is atomic, '$\sim p$' is molecular. For Aristotle and ourselves a negated proposition is neither more nor less logically complex than an unnegated one. In this view asserted propositions are not contrasted with negated propositions. Indeed, in such a theory all propositions are asserted, whether they are affirmations or denials. Asserted propositions cannot possibly be contrasted with negated propositions. What asserted propositions *are* contrasted with (quite naturally) are unasserted propositions. (It can easily be argued that apodeictic and problematic, as well as assertoric, propositions are asserted. We can think of apodeictic and problematic propositions as strongly and weakly assertoric respectively.) An asserted proposition, an assertion, was, for Aristotle, a sentence which has meaning *and* truth or falsity. He simply called them "propositions" (*On Interpretation* 17a1-8). Nonassertions are meaningful sentences which are not either true or false. A prayer is an example Aristotle gave of a nonasserted proposition. Other examples would be guesses, questions, requests, commands, oaths, and promises.

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