An Alternative Proof of an Extension Theorem of T. Ohsawa

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Introduction

In [DHOh; OhT; Oh2; Oh3], extension theorems for weighted square-integrable holomorphic functions that are defined on intersections of lower-dimensional affine subspaces with a pseudoconvex domain D were proved on the basis of L^2-estimates for the $\bar\partial$-operator. (See also [Oh2; De; Bou; Mv] for generalizations to holomorphic differential forms with values in certain vector bundles.) They have proved to be useful in many applications, among them the behavior of the Bergman kernel [DH2; McN; JP] and the construction of integral kernels for the $\bar\partial$-equation, [BonD].

It is therefore of interest to have proofs for such extension results that are as elementary as possible. For the theorem of Ohsawa and Takegoshi (see [OhT]), such new proofs have been given, for instance, in [Bs; McN; Siu] and also by T. Ohsawa himself (oral communication). Our goal here is to give also an elementary proof for the refined extension theorem of Ohsawa [Oh3] that allows so-called negligible weights in the extension. Our proof will be free of tools from Kähler geometry.

Let us first clarify some notations and state the theorem. Let $D \subset \mathbb{C}^n$ be an arbitrary pseudoconvex domain in $\mathbb{C}^n$. For a plurisubharmonic function ψ on D, we denote by $H^2(D, \psi)$ the Hilbert space of holomorphic functions in $L^2(D, \psi)$. We also fix an affine linear subspace $H \subset \mathbb{C}^n$ of codimension k for which $D' = D \cap H \neq \emptyset$. Then the extension theorem of [Oh3] can be stated in the following form.

0.1. THEOREM. *Assume that there exists on D a plurisubharmonic function V such that*

$$C_V := \sup_D (V + 2k \log \operatorname{dist}(\cdot, H)) < \infty.$$

Then there exists a continuous linear extension operator $E_\psi^V : H^2(D', \psi + V) \to H^2(D, \psi)$ whose operator norm is bounded by

$$\|E_\psi^V\|^2 \le C_n e^{C_V}.$$

The constant C_n depends only on the dimension and not on the choice of ψ and D.

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In Section 1 we will reduce the proof to a simpler situation. Section 2 contains the basic estimate that we use. The proof of Theorem 0.1 will be given in Section 3, and Section 4 contains our application to the Bergman kernel.

1. Reduction Steps

Let D, D', ψ, and V be as in Theorem 0.1. At first we show that it is enough to prove:

Any $f \in H^2(D', \psi+V)$ admits a holomorphic extension $\tilde{f} \in H^2(D, \psi)$ such that

$$\|\tilde{f}\|_{\psi}^2 \leq C_n e^{C_V} \|f\|_{D', \psi+V}^2. \tag{1}$$

Namely, suppose that we have shown this. Let $h^2(D, \psi)$ denote the (closed) subspace of all functions $g \in H^2(D, \psi)$ such that $g|D' = 0$, and denote by $\pi'\colon H^2(D, \psi) \to h^2(D, \psi)$ the orthogonal projection. Then, for a function $f \in H^2(D', \psi + V)$, we choose an extension $\tilde{f} \in H^2(D, \psi)$ and put

$$E_{\psi}^{V}(f) := \tilde{f} - \pi'(\tilde{f}).$$

It is easy to check that this definition is independent of the choice of the extension $\tilde{f}$ and hence is consistent. Also, it is elementary to show that E_{ψ}^{V} defines the desired extension operator.

For the proof of (1), the following further reductions are possible. Let $\Phi\colon D \to \mathbb{R}^+$ be a strongly plurisubharmonic exhaustion function and $T \subset \mathbb{R}^+$ an unbounded set such that $D_t := \{\Phi < t\}$ is strictly pseudoconvex with a smooth boundary for all $t \in T$. Let $D_t' = D_t \cap H$.

A routine argument based upon the Alaoglu–Bourbaki theorem on weak-$\star$-compactness of the unit ball in a normed space then justifies that it suffices to show:

For each $t \in T$ and $f \in H^2(D_t', \psi + V)$, there is an extension $\tilde{f}_t \in H^2(D_t, \psi)$ for f satisfying

$$\|\tilde{f}_t\|_{D_t, \psi}^2 \leq C_n e^{C_V} \|f\|_{D', \psi+V}^2 \tag{2}$$

with a constant C_n independent of ψ and t.

Finally it obviously suffices to prove (2) under the additional assumption that ψ and V are smooth. Namely, on D_t one can choose decreasing sequences $(\psi_s)_s$ and $(V_s)_s$ of smooth plurisubharmonic functions that converge to ψ and V, respectively. Then (2) applies with ψ_s and V_s instead of ψ and V. Hence the smoothness assumption on ψ and V can be removed by applying the Alaoglu–Bourbaki theorem once more.

2. A Basic Estimate for the $\bar{\partial}$ Operator

Let Ω denote a smooth bounded domain in $\mathbb{C}^n$ with a defining function r that is normalized in such a way that $|\nabla r| = 1$ on $\partial\Omega$. Let φ be a C^2-smooth function on

$\bar{\Omega}$. Let $q \in \{0, \dots, n-1\}$. The standard $\bar{\partial}$-operator on $L^2_{(0,q)}(\Omega, \varphi)$ has a closure, also denoted by $\bar{\partial}$. By $\bar{\partial}^*_\varphi \colon L^2_{(0,q+1)}(\Omega, \varphi) \to L^2_{(0,q)}(\Omega, \varphi)$ we denote the closure of the formal adjoint of $\bar{\partial}$. The space

$$\mathcal{F}_q(\Omega, \varphi) := C^1_{(0,q+1)}(\bar{\Omega}) \cap \operatorname{dom}(\bar{\partial}^*_\varphi)$$

consists of all $(0, q+1)$-forms $u \in C^1_{(0,q+1)}(\bar{\Omega})$ satisfying the Neumann condition

$$u \rfloor \partial r = 0 \quad \text{on } \partial\Omega. \tag{3}$$

This space is known to be dense in $L^2_{(0,q+1)}(\Omega, \varphi) \cap \operatorname{dom}(\bar{\partial}^*_\varphi)$ with respect to the graph norm $u \mapsto \|u\|_\varphi + \|\bar{\partial}u\|_\varphi + \|\bar{\partial}^*_\varphi u\|_\varphi$ (see [Hör, p. 100]).

For a function $f \in C^2(\bar{\Omega})$ and a $(0, 1)$-form $u = \sum_{j=1}^n u_j d\bar{z}_j$, we write for short

$$\mathcal{L}_f(z; u) := \sum_{j,k=1}^n \frac{\partial^2 f}{\partial z_j \partial \bar{z}_k}(z) u_j(z) \overline{u_k}(z).$$

2.1. Lemma (The a priori formula for $\bar{\partial}$). *Let Ω and φ be as before and let $\eta \in C^2(\bar{\Omega})$ be a positive function. Then for $u = u_1 d\bar{z}_1 + \cdots + u_n d\bar{z}_n \in \mathcal{F}_1(\Omega, \varphi)$ we have*

(a)
$$\begin{aligned} &\|\sqrt{\eta}\bar{\partial}u\|^2_\varphi + \|\sqrt{\eta}\bar{\partial}^*_\varphi u\|^2_\varphi \\ &\quad = \int_{D_t} (\eta\mathcal{L}_\varphi - \mathcal{L}_\eta)(z; u) e^{-\varphi}\, d\lambda(z) + 2\operatorname{Re}(u \rfloor \partial\eta, \bar{\partial}^*_\varphi u)_\varphi \\ &\quad\quad + \sum_{i,j=1}^n \int_\Omega \eta \left| \frac{\partial u_i}{\partial \bar{z}_j} \right|^2 e^{-\varphi}\, d\lambda + \int_{\partial\Omega} \eta\mathcal{L}_r(\zeta; u) e^{-\varphi}\, d\sigma(\zeta); \end{aligned} \tag{4}$$

(b) *in particular, if Ω is pseudoconvex then*

$$\begin{aligned} &\|\sqrt{\eta}\bar{\partial}u\|^2_\varphi + \|\sqrt{\eta}\bar{\partial}^*_\varphi u\|^2_\varphi \\ &\quad \geq \int_\Omega (\eta\mathcal{L}_\varphi - \mathcal{L}_\eta)(z; u) e^{-\varphi}\, d\lambda(z) + 2\operatorname{Re}(u \rfloor \partial\eta, \bar{\partial}^*_\varphi u)_\varphi. \end{aligned} \tag{5}$$

Here $d\lambda$ denotes the Lebesgue measure and $d\sigma$ the area measure on $\partial\Omega$.

Proof. The first formula is stated in [BoS]. For the reader's convenience we include a proof here. It uses the same technique (based upon integration by parts) as applied by Hörmander [Hör]. Similar computations have been carried out in [McN; Bs; Siu]. First we recall the integral formula of Gauss:

$$\int_\Omega \frac{\partial f}{\partial z_j} \bar{g}\, d\lambda = -\int_\Omega f \frac{\partial \bar{g}}{\partial z_j}\, d\lambda + \int_{\partial\Omega} \frac{\partial r}{\partial z_j} f \bar{g}\, d\sigma \tag{6}$$

for functions $f, g \in C^1(\bar{\Omega})$. For $1 \leq k \leq n$, let δ_k denote the operator

$$\delta_k h = e^\varphi \frac{\partial(e^{-\varphi} h)}{\partial z_k} \quad \text{for } h \in C^1(\bar{\Omega}).$$

With this notation, for $u = u_1 d\bar{z}_1 + \cdots + u_n d\bar{z}_n \in \mathcal{F}_1(\Omega, \varphi)$ we can write

$$\bar{\partial}_\varphi^* u = -\sum_{j=1}^{n} \delta_j u_j. \tag{7}$$

We start by computing

$$\|\sqrt{\eta}\bar{\partial}_\varphi^* u\|_\varphi^2 = \sum_{j,k=1}^{n} \int_\Omega \eta \delta_j u_j \overline{\delta_k u_k} e^{-\varphi}\, d\lambda,$$

using integration by parts. From formula (6) for $f = e^{-\varphi} u_j$ and $g = \eta \delta_k (u_k)$, we obtain

$$\begin{aligned}\int_\Omega \eta \delta_j (u_j) \overline{\delta_k (u_k)} e^{-\varphi}\, d\lambda &= \int_\Omega \eta \frac{\partial (e^{-\varphi} u_j)}{\partial z_j} \overline{\delta_k (u_k)}\, d\lambda \\ &= -\int_\Omega e^{-\varphi} u_j \frac{\partial (\eta \overline{\delta_k (u_k)})}{\partial z_j}\, d\lambda + \int_{\partial\Omega} \frac{\partial r}{\partial z_j} u_j \overline{\delta_k u_k} \eta e^{-\varphi}\, d\sigma \\ &= -\int_\Omega e^{-\varphi} u_j \frac{\partial \eta}{\partial z_j} \overline{\delta_k (u_k)}\, d\lambda - \int_\Omega e^{-\varphi} u_j \eta \frac{\partial (\overline{\delta_k (u_k)})}{\partial z_j}\, d\lambda \\ &\quad + \mathcal{I}_1(j,k),\end{aligned} \tag{8}$$

where

$$\mathcal{I}_1(j,k) := \int_{\partial\Omega} \frac{\partial r}{\partial z_j} u_j \overline{\delta_k u_k} \eta e^{-\varphi}\, d\sigma.$$

Now one has a commutator relation for the δ_k, namely,

$$\left[\delta_k, \frac{\partial}{\partial \bar{z}_j}\right] = \frac{\partial^2 \varphi}{\partial z_k\, \partial \bar{z}_j}.$$

Substituting this into the second member on the right-hand side of (8), we have

$$\begin{aligned}-\int_\Omega e^{-\varphi} u_j \eta \frac{\partial (\overline{\delta_k (u_k)})}{\partial z_j}\, d\lambda &= -\int_\Omega e^{-\varphi} u_j \eta \overline{\frac{\partial (\delta_k (u_k))}{\partial \bar{z}_j}}\, d\lambda \\ &= \int_\Omega e^{-\varphi} u_j \eta \overline{\left(\frac{\partial^2 \varphi}{\partial z_k\, \partial \bar{z}_j} - \delta_k \frac{\partial}{\partial \bar{z}_j}\right) u_k}\, d\lambda \\ &= \int_\Omega e^{-\varphi} \eta \frac{\partial^2 \varphi}{\partial z_j\, \partial \bar{z}_k} u_j \overline{u_k}\, d\lambda - \overline{\int_\Omega e^{-\varphi} \eta \overline{u_j} \delta_k \frac{\partial u_k}{\partial \bar{z}_j}\, d\lambda}.\end{aligned}$$

The second integral on the right side is again transformed by Gauss' formula:

$$\begin{aligned}-\int_\Omega e^{-\varphi} \eta \overline{u_j} \delta_k \left(\frac{\partial u_k}{\partial \bar{z}_j}\right) d\lambda &= -\int_\Omega \eta \overline{u_j} \frac{\partial}{\partial z_k} \left(e^{-\varphi} \frac{\partial}{\partial \bar{z}_j} u_k\right) d\lambda \\ &= \int_\Omega e^{-\varphi} \eta \frac{\partial \overline{u_j}}{\partial z_k} \frac{\partial u_k}{\partial \bar{z}_j}\, d\lambda \\ &\quad + \int_\Omega \frac{\partial \eta}{\partial z_k} \overline{u_j} \frac{\partial u_k}{\partial \bar{z}_j} e^{-\varphi}\, d\lambda - \mathcal{I}_2'(j,k),\end{aligned} \tag{9}$$

where

$$\mathcal{I}_2'(j,k) = \int_{\partial\Omega} \eta \frac{\partial r}{\partial z_k} \overline{u_j} \frac{\partial u_k}{\partial \bar{z}_j} e^{-\varphi}\, d\sigma.$$

The complex conjugate of this is

$$-\overline{\int_\Omega e^{-\varphi}\eta\overline{u_j}\delta_k \frac{\partial u_k}{\partial \bar{z}_j}\, d\lambda} = \int_\Omega e^{-\varphi}\eta \frac{\partial u_j}{\partial \bar{z}_k} \overline{\frac{\partial u_k}{\partial \bar{z}_j}}\, d\lambda + \int_\Omega \frac{\partial \eta}{\partial \bar{z}_k} u_j \overline{\frac{\partial u_k}{\partial \bar{z}_j}} e^{-\varphi}\, d\lambda - \mathcal{I}_2(j,k),$$

where

$$\mathcal{I}_2(j,k) := \int_{\partial\Omega} \eta \frac{\partial r}{\partial \bar{z}_k} u_j \frac{\partial \overline{u_k}}{\partial z_j} e^{-\varphi}\, d\sigma.$$

If we substitute this into the computations carried out so far, we obtain

$$\begin{aligned}\int_\Omega \eta\delta_j(u_j)\overline{\delta_k(u_k)}e^{-\varphi}\, d\lambda = &-\int_\Omega u_j \frac{\partial\eta}{\partial z_j}\overline{\delta_k u_k} e^{-\varphi}\, d\lambda + \int_\Omega \frac{\partial \eta}{\partial \bar{z}_k} u_j \overline{\frac{\partial u_k}{\partial \bar{z}_j}} e^{-\varphi}\, d\lambda \\ &+ \int_\Omega \eta \frac{\partial^2\varphi}{\partial z_j\, \partial \bar{z}_k} u_j \overline{u_k} e^{-\varphi}\, d\lambda + \int_\Omega \eta \frac{\partial u_j}{\partial \bar{z}_k} \overline{\frac{\partial u_k}{\partial \bar{z}_j}} e^{-\varphi}\, d\lambda \\ &+ \mathcal{I}_1(j,k) - \mathcal{I}_2(j,k).\end{aligned}$$

Next we want to sum this over all indices $j, k \in \{1, \ldots, n\}$. The Neumann condition (3) at the level of $(0,1)$-forms reduces to

$$\sum_{j=1}^n u_j \frac{\partial r}{\partial z_j} = 0 \quad \text{on } \partial\Omega; \tag{10}$$

in particular, $\sum_{j=1}^n \mathcal{I}_1(j,k) = 0$ for all $k = 1, \ldots, n$. Hence we obtain

$$\begin{aligned}\|\sqrt{\eta}\bar\partial_\varphi^* u\|_\varphi^2 = &-\int_\Omega \sum_{j,k} u_j \frac{\partial\eta}{\partial z_j} \overline{\delta_k u_k} e^{-\varphi}\, d\lambda + \int_\Omega \sum_{j,k} \frac{\partial\eta}{\partial \bar{z}_k} u_j \overline{\frac{\partial u_k}{\partial \bar{z}_j}} e^{-\varphi}\, d\lambda \\ &+ \int_\Omega \eta \sum_{j,k} \frac{\partial^2\varphi}{\partial z_j\, \partial\bar{z}_k} u_j \overline{u_k} e^{-\varphi}\, d\lambda + \int_\Omega \sum_{j,k} \eta \frac{\partial u_j}{\partial \bar{z}_k} \overline{\frac{\partial u_k}{\partial \bar{z}_j}} e^{-\varphi}\, d\lambda \\ &- \sum_{j,k} \mathcal{I}_2(j,k).\end{aligned} \tag{11}$$

We now observe (see [Hör, p. 102]) that

$$\|\sqrt{\eta}\bar\partial u\|_\varphi^2 = \int_\Omega \eta \left(\sum_{j,k=1}^n \left| \frac{\partial u_k}{\partial \bar{z}_j} \right|^2 - \sum_{j,k} \frac{\partial u_j}{\partial \bar{z}_k} \overline{\frac{\partial u_k}{\partial \bar{z}_j}} \right) e^{-\varphi}\, d\lambda.$$

This is substituted into (11) to yield

$$\begin{aligned}\|\sqrt{\eta}\bar\partial u\|_\varphi^2 + \|\sqrt{\eta}\bar\partial_\varphi^* u\|_\varphi^2 = &\int_\Omega \eta \sum_{j,k=1}^n \left| \frac{\partial u_k}{\partial \bar{z}_j} \right|^2 e^{-\varphi}\, d\lambda + T_2 + T_3 \\ &+ \int_\Omega \eta \sum_{j,k=1}^n \frac{\partial^2\varphi}{\partial z_j \partial \bar{z}_k} u_j \overline{u_k} e^{-\varphi}\, d\lambda - \sum_{j,k} \mathcal{I}_2(j,k),\end{aligned} \tag{12}$$

where

$$T_2 := -\int_\Omega \sum_{j,k} u_j \frac{\partial\eta}{\partial z_j} \overline{\delta_k u_k} e^{-\varphi}\, d\lambda \quad \text{and} \quad T_3 := \int_\Omega \sum_{j,k} \frac{\partial\eta}{\partial \bar z_k} u_j \overline{\frac{\partial u_k}{\partial \bar z_j}} e^{-\varphi}\, d\lambda.$$

Finally, the proof of (a) will be complete if we show that the sum of the second and third terms on the right (T_2 and T_3) is equal to

$$-\int_\Omega \sum_{j,k=1}^n \frac{\partial^2\eta}{\partial z_j\, \partial \bar z_k} u_j \overline{u_k} e^{-\varphi}\, d\lambda + 2\operatorname{Re}(u \lrcorner \partial\eta, \bar\partial_\varphi^* u)_\varphi,$$

and that the last term (which we will denote T_5) is equal to

$$\sum_{j,k=1}^n \int_{\partial\Omega} \eta \frac{\partial^2 r}{\partial z_j\, \partial \bar z_k} u_j \overline{u_k} e^{-\varphi}\, d\sigma.$$

Let again $j, k \in \{1, \ldots, n\}$ be fixed. We transform the single terms that appear in T_2. In the first step we write

$$\begin{aligned} -\int_\Omega u_j \frac{\partial\eta}{\partial z_j} \overline{\delta_k u_k} e^{-\varphi}\, d\lambda \\ = -2\operatorname{Re} \int_\Omega u_j \frac{\partial\eta}{\partial z_j} \overline{\delta_k u_k} e^{-\varphi}\, d\lambda + \int_\Omega \overline{u_j} \frac{\partial\eta}{\partial \bar z_j} \delta_k u_k e^{-\varphi}\, d\lambda. \end{aligned} \tag{13}$$

The second term on the right side can be computed by the Gauss formula as follows:

$$\begin{aligned} \int_\Omega \overline{u_j} \frac{\partial\eta}{\partial \bar z_j} \delta_k u_k e^{-\varphi}\, d\lambda &= \int_\Omega \overline{u_j} \frac{\partial\eta}{\partial \bar z_j} \frac{\partial (u_k e^{-\varphi})}{\partial z_k}\, d\lambda \\ &= -\int_\Omega \frac{\partial^2\eta}{\partial \bar z_j\, \partial z_k} \overline{u_j} u_k e^{-\varphi}\, d\lambda - \int_\Omega \frac{\partial \overline{u_j}}{\partial z_k} \frac{\partial\eta}{\partial \bar z_j} u_k e^{-\varphi}\, d\lambda \\ &\quad + \int_{\partial\Omega} \frac{\partial r}{\partial z_k} \frac{\partial\eta}{\partial \bar z_j} \overline{u_j} u_k e^{-\varphi}\, d\sigma. \end{aligned} \tag{14}$$

We now sum over all $j, k = 1, \ldots, n$. Again, the sum of the boundary integrals vanishes because of (10). Likewise, we see that

$$\sum_{j,k=1}^n \int_\Omega \frac{\partial \overline{u_j}}{\partial z_k} \frac{\partial\eta}{\partial \bar z_j} u_k e^{-\varphi}\, d\lambda = T_3.$$

Summation of (13) over all j, k thus yields (by means of (7))

$$\begin{aligned} T_2 &= -\sum_{j,k=1}^n \int_\Omega u_j \frac{\partial\eta}{\partial z_j} \overline{\delta_k u_k} e^{-\varphi}\, d\lambda \\ &= -2\sum_{j,k=1}^n \operatorname{Re} \int_\Omega u_j \frac{\partial\eta}{\partial z_j} \overline{\delta_k u_k} e^{-\varphi}\, d\lambda - \int_\Omega \sum_{j,k} \frac{\partial^2\eta}{\partial \bar z_j\, \partial z_k} \overline{u_j} u_k e^{-\varphi}\, d\lambda - T_3 \\ &= 2\operatorname{Re}(u \lrcorner \partial\eta, \bar\partial_\varphi^* u)_\varphi - \int_\Omega \sum_{j,k} \frac{\partial^2\eta}{\partial \bar z_j\, \partial z_k} \overline{u_j} u_k e^{-\varphi}\, d\lambda - T_3. \end{aligned}$$

In order to transform the term T_5, we need only recall the argument of [Hör, p. 103]. It was shown there that (3) implies

$$\sum_{k=1}^{n}\left(\frac{\partial\overline{u_k}}{\partial z_j}\frac{\partial r}{\partial\bar z_k}+\overline{u_k}\frac{\partial^2 r}{\partial z_j\,\partial\bar z_k}\right)=0\quad\text{on }\partial\Omega$$

for all fixed j. We multiply by $u_j\eta e^{-\varphi}$ and then sum over all j. Finally, integration over $\partial\Omega$ gives us

$$T_5=-\sum_{j,k}\mathcal{I}_2(j,k)=\sum_{j,k=1}^{n}\int_{\partial\Omega}\eta\frac{\partial^2 r}{\partial z_j\,\partial\bar z_k}u_j\overline{u_k}e^{-\varphi}\,d\sigma.$$

The proof of the a priori formula for $\bar\partial$ is complete.

For the proof of (b) we need only observe that, for pseudoconvex Ω, the boundary integral on the right-hand side of (4) is nonnegative if $u\in\mathcal{F}_1(\Omega,\varphi)$. Namely, from (10) we see that, for any $\zeta\in\partial\Omega$, the vector $(u_1(\zeta),\dots,u_n(\zeta))$ belongs to the holomorphic tangent space $T_\zeta^{(1,0)}\partial\Omega$. Hence $\mathcal{L}_r(\zeta;u(\zeta))\geq 0$. □

3. Proof of the Theorem

We now give the proof of (2) under the assumption that ψ and V are smooth and the codimension of H is $k=1$. (The general case is settled by iterating the result from codimension 1). After a suitable choice of coordinates, we may assume that

$$H=\{\,z=(z',z_n)\mid z_n=0\,\}.$$

Then $V(z)+2\log|z_n|\leq C_V$ on D_t. There exists a number $\varepsilon>0$ such that $(z',0)\in D'$ whenever $z\in D_t$ and $|z_n|<\varepsilon$. Let $\chi\in C^\infty(\mathbb{R})$ be a function with $\chi(x)=1$ on $(-\infty,1/4]$ and $\chi(x)=0$ on $[3/4,\infty)$. For an arbitrary function $f\in H^2(D',\psi+V)$, we define the following $(0,1)$-form:

$$\alpha_f:=\bar\partial\left[\chi\left(\frac{|z_n|^2}{\varepsilon^2}\right)f(z',0)\right]=\chi'\left(\frac{|z_n|^2}{\varepsilon^2}\right)f(z',0)\frac{z_n}{\varepsilon^2}\,d\overline{z_n}.\tag{15}$$

This form is $\bar\partial$-closed and smooth on D_t.

Let

$$\mathcal{J}_\varepsilon(f):=\int_{D'}|f(z',0)|^2\int_{\{1/2<|z_n|<1\}}\exp[-(\psi+V)(z',\varepsilon z_n)]\,d\lambda(z_n)\,d\lambda(z').\tag{16}$$

We look for smooth weight functions φ and $\eta>0$ such that $(\eta+\eta^3)e^{-\psi}\leq e^{C_V-\varphi}$ and such that the quadratic form $\eta\mathcal{L}_\varphi-\mathcal{L}_\eta$ tames these data and yields a basic estimate of the form

$$|(u,\alpha_f)_\varphi|^2\leq C'\mathcal{J}_\varepsilon(f)\|\sqrt{\eta+\eta^3}\,\bar\partial_\varphi^* u\|_\varphi^2\quad\text{for all}\quad(0,1)\text{-forms }u\in\operatorname{dom}(\bar\partial_\varphi^*).\tag{17}$$

For an arbitrary number $0 < \tau < \varepsilon^2$ we put

$$\varphi(z) = \varepsilon|z|^2 + \log(|z_n|^2 + \tau) + \psi(z) + V(z) \tag{18}$$

and

$$w = V + \log(|z_n|^2 + \varepsilon^2) - C_V - 4.$$

Then we have $w \le -3$ on D_t if ε is chosen small enough. Moreover, w is plurisubharmonic. The function

$$\eta = 2(-w + \log(-w))$$

is plurisuperharmonic, and $-4w \ge \eta > 8$ everywhere. By explicit computation we obtain

$$-\mathcal{L}_\eta = 2\left(1 - \frac{1}{w}\right)\mathcal{L}_w + 2\frac{\partial w \otimes \overline{\partial w}}{w^2} \ge 2\mathcal{L}_w + 4\frac{\partial \eta \otimes \overline{\partial \eta}}{\eta^3}. \tag{19}$$

We will prove that (17) is satisfied for these functions η and φ.

It will suffice to check (17) for all (0, 1)-forms $u \in \mathcal{F}_t := \mathcal{F}_1(D_t, \varphi)$, since this space is dense in $\operatorname{dom}(\bar\partial_\varphi^*)$ with respect to the graph norm

$$u \mapsto \|u\|_\varphi + \|\bar\partial u\|_\varphi + \|\bar\partial_\varphi^* u\|_\varphi.$$

(Note that, because of the smoothness assumption on V, the function $\gamma := \sqrt{\eta + \eta^3}$ is bounded on each D_t.) We may furthermore restrict ourselves to forms in the null space $N_{(0,1)}(\bar\partial)$ of $\bar\partial$ (for forms $u \perp N_{(0,1)}(\bar\partial)$, the estimate (17) is trivial).

Let $u \in \mathcal{F}_1(D_t, \varphi) \cap N_{(0,1)}(\bar\partial)$. We can apply (5). Using (19), we can split the mixed term $(u \lrcorner \partial\eta, \bar\partial_\varphi^* u)_\varphi$ that appeared in (5). By the Cauchy–Schwarz inequality, we obtain:

$$\begin{aligned} -2|(u \lrcorner \partial\eta, \bar\partial_\varphi^* u)_\varphi| &= -2|(u \lrcorner \eta^{-3/2}\partial\eta, \eta^{3/2}\bar\partial_\varphi^* u)_\varphi| \\ &\ge -\|u \lrcorner \eta^{-3/2}\partial\eta\|_\varphi^2 - \|\eta^{3/2}\bar\partial_\varphi^* u\|_\varphi^2 \\ &\ge \frac{1}{4}\int_{D_t} \mathcal{L}_\eta(z; u)e^{-\varphi}\, d\lambda - \|\eta^{3/2}\bar\partial_\varphi^* u\|_\varphi^2. \end{aligned} \tag{20}$$

Let Q denote the quadratic form defined by the coefficients of $\eta\mathcal{L}_\varphi - \frac{3}{4}\mathcal{L}_\eta$. Substituting (20) into (5), we have

$$\|\sqrt{\eta}\bar\partial u\|_\varphi^2 + \|\sqrt{\eta + \eta^3}\bar\partial_\varphi^* u\|_\varphi^2 \ge \int_{D_t} Q(z; u)e^{-\varphi}\, d\lambda(z); \tag{21}$$

hence (with $\gamma = \sqrt{\eta + \eta^3}$),

$$\int_{D_t} Q(z; u)e^{-\varphi}\, d\lambda \le \|\gamma\bar\partial u\|_\varphi^2 + \|\gamma\bar\partial_\varphi^* u\|_\varphi^2. \tag{22}$$

The form Q is even positive definite, since

$$Q \ge \varepsilon\eta\mathcal{L}_{|z|^2} + \frac{3}{2}\mathcal{L}_w \ge 8\varepsilon\mathcal{L}_{|z|^2} + \frac{\varepsilon^2}{(\varepsilon^2 + |z_n|^2)^2}\mathcal{L}_{|z_n|^2}.$$

Thus, $Q^{-1}(z; u)$ is also meaningful. In combination with the Cauchy–Schwarz inequality, we obtain

$$|(u,\alpha_f)_\varphi|^2 \le \left(\int_{D_t} Q^{-1}(z;\alpha_f)e^{-\varphi}\,d\lambda\right)\left(\int_{D_t} Q(z;u)e^{-\varphi}\,d\lambda\right)$$
$$\le 2\left(\int_{D_t} Q^{-1}(z;\alpha_f)e^{-\varphi}\,d\lambda\right)(\|\gamma\bar\partial u\|_\varphi^2 + \|\gamma\bar\partial_\varphi^* u\|_\varphi^2)$$

for all $u \in \mathcal{F}_t$. On $\operatorname{supp}(\alpha_f)$ we even have

$$Q \ge 8\varepsilon\mathcal{L}_{|z|^2} + \frac{1}{4\varepsilon^2}\,dz_n\,\overline{dz_n}.$$

Substituting (15), we derive that

$$\int_{D_t} Q^{-1}(z;\alpha_f)e^{-\varphi}\,d\lambda$$
$$\le C' \int_{D_t\cap\{\varepsilon/2<|z_n|\le\varepsilon\}} |z_n|^2 \frac{4}{\varepsilon^2}|f(z',0)|^2 \frac{e^{-\psi-V}}{|z_n|^2+\tau}\,d\lambda$$
$$\le C''\varepsilon^{-2}\int_{D'} |f(z',0)|^2 \int_{\{\varepsilon/2<|z_n|<\varepsilon\}} e^{-(\psi+V)(z',z_n)}\,d\lambda(z_n)\,d\lambda(z') = C''\mathcal{J}_\varepsilon(f),$$

where $\mathcal{J}_\varepsilon(f)$ is defined as in (16). This yields

$$|(u,\alpha_f)_\varphi|^2 \le C^*\mathcal{J}_\varepsilon(f)\|\gamma\bar\partial_\varphi^* u\|_\varphi^2 \tag{23}$$

for all forms $u \in \mathcal{F}_t \cap N_{(0,1)}(\bar\partial)$ and hence for any $u \in \operatorname{dom}(\bar\partial_\varphi^*)$. By the Hahn–Banach theorem combined with the Riesz theorem, we obtain a solution $u_\varepsilon \in L^2(D_t,\varphi)$ of the equation $\bar\partial(\gamma u_\varepsilon) = \alpha_f$ such that

$$\|u_\varepsilon\|_\varphi^2 \le C''\mathcal{J}_\varepsilon(f).$$

The function u_ε is even smooth, and

$$\tilde f_{\varepsilon,\tau} := \chi\left(\frac{|z_n|^2}{\varepsilon^2}\right) f(z',0) - \gamma u_\varepsilon$$

is holomorphic on D_t.

We estimate the norm of this function as follows:

$$\|\tilde f_{\varepsilon,\tau}\|_\psi^2 \le 2\int_{D_t\cap\{|z_n|<\varepsilon\}} |f(z',0)|^2 e^{-\psi}\,d\lambda + 2\|\gamma u_\varepsilon\|_\psi^2; \tag{24}$$

we have $(\varepsilon^2+|z_n|^2)^{-1}e^{-V}e^{C_V} \ge 1/2$ on D_t if ε is small enough, so

$$\frac{1}{2}e^{-C_V}\int_{D_t\cap\{|z_n|<\varepsilon\}} |f(z',0)|^2 e^{-\psi}\,d\lambda$$
$$\le \int_{D_t\cap\{|z_n|<\varepsilon\}} |f(z',0)|^2 \frac{1}{\varepsilon^2+|z_n|^2} e^{-\psi-V}\,d\lambda$$
$$= \int_{D_t\cap\{|z_n|<1\}} |f(z',0)|^2 \frac{1}{1+|z_n|^2} e^{-(\psi+V)(z',\varepsilon z_n)}\,d\lambda$$
$$\le \mathcal{J}_\varepsilon^*(f) := \int_{\{|z_n|<1\}} |f(z',0)|^2 e^{-(\psi+V)(z',\varepsilon z_n)}\,d^{2n}z.$$

Furthermore, since for $\tau < \varepsilon^2$ we can estimate

$$V(z) + \log(|z_n|^2 + \tau) \leq w(z) + C_V + 4 \leq -\frac{\eta(z)}{4} + 4 + C_V,$$

the second term in (24) is dominated by some constant times $e^{C_V}\mathcal{J}_\varepsilon(f)$, because

$$\begin{aligned}|\gamma u_\varepsilon|^2 e^{-\psi} &= (\eta + \eta^3)e^{\varepsilon|z|^2 + V + \log(|z_n|^2+\tau)}|u_\varepsilon|^2 e^{-\varphi} \\ &\leq C' C_t^\varepsilon e^{C_V}(\eta + \eta^3)e^{-\eta/4}|u_\varepsilon|^2 e^{-\varphi} \leq C'' C_t^\varepsilon e^{C_V}|u_\varepsilon|^2 e^{-\varphi},\end{aligned}$$

where $C_t = \exp(\max_{D_t}|z|^2)$ and $C', C'' > 0$ are unimportant constants.

We next show that, for any point $(z', 0) \in D_t \cap H$, there exists a constant $C = C'(\varepsilon, z', t)$ such that, for all $0 < \tau < \varepsilon^2$,

$$|\tilde{f}_{\varepsilon,\tau}(z',0) - f(z',0)|^2 = |\gamma u_\varepsilon(z',0)|^2 \leq C(z',\varepsilon,t)\frac{1}{\log(1+\varepsilon^2/4\tau)}\|u_\varepsilon\|_\varphi^2. \quad (25)$$

Here we have used the fact that γu_ε is holomorphic on $\{|z_n| < \varepsilon/2\}$. It satisfies on D_t the upper estimate

$$\frac{|\gamma u_\varepsilon|^2}{|z_n|^2 + \tau} = \gamma^2 e^{\varepsilon|z|^2 + \psi + V}|u_\varepsilon|^2 e^{-\varphi} \leq C(t,\varepsilon)|u_\varepsilon|^2 e^{-\varphi}, \quad (26)$$

with a constant $C(t,\varepsilon)$ that does not depend on τ. Let $(z', 0) \in D_t \cap H$ be arbitrary. Then, after shrinking ε if necessary, we find a radius $\rho(z')$ such that the polydisc $P(z') = \Delta_{n-1}(z', \rho(z')) \times \Delta(0, \varepsilon/2)$ is contained in D_t. Let

$$\gamma u_\varepsilon(w) = \sum_{\beta \in \mathbb{N}_0^n} A_\beta(z')(w - (z',0))^\beta$$

denote the Taylor expansion of γu_ε about $(z', 0)$ on $P(z')$. Then, using the orthogonality of the monomials $(w - (z',0))^\beta$ in conjunction with (26), we have

$$|\gamma u_\varepsilon(z',0)|^2 \int_{P(z')} \frac{1}{|w_n|^2 + \tau}\, d\lambda(w) \leq \int_{P(z')} \frac{|\gamma u_\varepsilon(w)|^2}{|w_n|^2 + \tau}\, d\lambda(w) \leq C(t,\varepsilon)\|u_\varepsilon\|_\varphi^2.$$

From

$$\int_{P(z')} \frac{1}{|w_n|^2 + \tau}\, d\lambda(w) = C_n^* \rho(z')^{2n-2} \log\left(1 + \frac{\varepsilon^2}{4\tau}\right),$$

with an unimportant constant C_n^* we obtain (25). We saw already that $\|\tilde{f}_{\varepsilon,\tau}\|_\psi^2 \leq C' C_t^\varepsilon e^{C_V}(\mathcal{J}_\varepsilon(f) + \mathcal{J}_\varepsilon^*(f))$ for all $\tau < \varepsilon^2$, with a constant C_t that does not depend on anything but t. After selecting a weak-$\star$-convergent subsequence $(\tilde{f}_{\varepsilon,\tau_j})_j$ from the $\tilde{f}_{\varepsilon,\tau}$, we obtain by means of (25) an extension $\tilde{f}_\varepsilon \in H^2(D_t, \psi)$ of f. Finally we let ε tend to zero. Then $\mathcal{J}_\varepsilon(f) \to 3\pi/4\|f\|_{D',\psi+V}^2$ and $\mathcal{J}_\varepsilon^*(f) \to \pi\|f\|_{D',\psi+V}^2$. After once more choosing a weak-$\star$-convergent subsequence from the $\tilde{f}_\varepsilon$, we will gain the desired extension $\tilde{f}_t \in H^2(D_t, \psi)$, satisfying (2). □

By checking all the steps in the foregoing proof, we see that we indeed obtain our next result, which generalizes the case of codimension 1.

3.2. THEOREM. *Let D be a pseudoconvex domain in $\mathbb{C}^n$ and h a holomorphic function such that $Z_h = \{h = 0\}$ becomes a 1-codimensional complex submanifold of D. Assume that a plurisubharmonic function V exists on D for which*

$$C_{V,h} = \sup_D (V + 2\log|h|) < \infty.$$

Then there exists a constant $C_n > 0$ such that, for all plurisubharmonic functions ψ on D, one can find a bounded linear extension operator $E_\psi^{V,h} \colon H^2(D \cap Z_h, \psi + V) \to H^2(D, \psi)$ whose operator norm can be estimated by $\|E_\psi^{V,h}\|^2 \le C_n e^{C_{V,h}}$, with a constant C_n that does not depend on anything but the dimension.

4. An Application to the Bergman Kernel

We want to give an application of the Ohsawa extension theorem to the Bergman kernel of a class of pseudoconvex domains with a C^2-smooth boundary—an application covering all domains that are regular in the sense of [DF].

Let $\Omega \subset\subset \mathbb{C}^n$ be a pseudoconvex domain. For a plurisubharmonic function u on Ω, we denote by $K_{\Omega,u}$ the Bergman kernel for the Hilbert space $H^2(\Omega, u)$ of all holomorphic functions f such that $\int_\Omega |f|^2 e^{-u}\, d\lambda < \infty$. As usual we put $K_\Omega = K_{\Omega,0}$. Furthermore, we define $\mathcal{P}(\bar{\Omega})$ as the family of all functions that are continuous on $\bar{\Omega}$ and plurisubharmonic on Ω. Let δ_Ω denote the boundary distance function on Ω.

If now H is an affine linear subspace of codimension k that meets Ω, and if u is a plurisubharmonic function on Ω satisfying

$$u(z) + 2k \log \operatorname{dist}(z, D \cap H) \le 0$$

on Ω, then from Theorem 0.1 we obtain, for all $w \in \Omega \cap H$,

$$K_\Omega(w) \ge C_n K_{\Omega\cap H,u}(w). \tag{27}$$

Our result on the Bergman kernel is as follows.

4.1. THEOREM. *Assume that $\partial\Omega \in C^2$ and that each point $\zeta \in \partial\Omega$ is a peak point for $\mathcal{P}(\bar{\Omega})$. Let $z^0 \in \partial\Omega$ be a point such that the Levi form of $\partial\Omega$ has $p < n-1$ positive eigenvalues at z^0. Then, for the Bergman kernel of Ω, we have*

$$\lim_{\Omega \ni w \to z^0} \delta_\Omega(w)^{p+2} K_\Omega(w) = \infty. \tag{28}$$

REMARKS. (a) In (28), the approach of $w \in \Omega$ toward z^0 is not required to be nontangential.

(b) By work of Sibony [Si], it is known that the hypothesis concerning the plurisubharmonic peak functions is satisfied when Ω is regular in the sense of [DF].

(c) Stronger quantitative estimates have been obtained for the large class of domains of finite type. See, for example, [DHOh] or [Cat] and the references given in those papers.

(d) An extremely large Levi degeneracy set E of the boundary is not an obstacle for (28) to hold. One should note that Sibony [Si] found pseudoconvex regular domains in $\mathbb{C}^2$ such that E has a positive Hausdorff measure of dimension 3.

Before giving a proof of the theorem, we summarize some facts about plurisubharmonic peak functions. In [Si, Thm. 2.1], the following is proved.

4.2. LEMMA. *Assume that $G \subset\subset \mathbb{C}^n$ is pseudoconvex, with a C^1-smooth boundary, such that each point $\zeta \in \partial G$ is a peak point for $\mathcal{P}(\bar{G})$. If $u \in C^0(\partial G)$, then the function*

$$\tilde{u}(z) := \sup\{\, v(z) \mid v \in \mathcal{P}(\bar{G}),\ v \le u \text{ on } \partial G \,\}$$

is also an element of $\mathcal{P}(\bar{G})$; moreover, $\tilde{u} \mid \partial G = u$.

Let now G be as in the lemma, and let $d_q(z) := |z - q|^2$ for $q \in \partial G$ and $z \in \mathbb{C}^n$. Then we have our next lemma.

4.3. LEMMA. *The functions $\psi_q := \widetilde{-d_q}$, where $q \in \partial G$, have the following properties*:

(a) $\psi_q \le -d_q$ *on* $\bar{G}$;
(b) $|\psi_p - \psi_q| \le 2\,\mathrm{diam}(G) \cdot |p - q|$ *for* $p, q \in \partial G$.

Proof. (a) is a consequence of the maximum principle. Let us prove (b). For $\zeta \in \partial G$, we have

$$\begin{aligned}\psi_q(\zeta) + 2\,\mathrm{Re}\langle \zeta - p, p - q\rangle &= -|\zeta - q|^2 + 2\,\mathrm{Re}\langle \zeta - p, p - q\rangle \\ &= -|\zeta - p|^2 - |p - q|^2 \le -d_p(\zeta).\end{aligned}$$

Hence $z \mapsto \psi_q(z) + 2\,\mathrm{Re}\langle z - p, p - q\rangle$ is a candidate for the supremum that defines ψ_p, and therefore

$$\psi_q(z) \le \psi_p(z) + 2|\mathrm{Re}\langle z - p, p - q\rangle| \le \psi_p(z) + 2\,\mathrm{diam}(G) \cdot |p - q|.$$

Since the roles of p and q can be interchanged, the claim now follows. □

Proof of Theorem 4.1. Choose open neighborhoods $W \subset\subset V$ for z^0, a smoothly bounded pseudoconvex domain $D \subset\subset \mathbb{C}^n$, and a linear subspace E of $\mathbb{C}^n$ of dimension $p + 1$ such that the following hold.

(i) $D \cap V \subset \Omega \cap V$ and $\partial D \cap V = \partial\Omega \cap V$.
(ii) For each $\zeta \in \partial\Omega \cap V$, the intersection $D'_\zeta := (\zeta + E) \cap D$ is strongly pseudoconvex and has a C^2-smooth boundary.
(iii) There is a number $\lambda > 0$ such that the eigenvalues of the Levi form of $\partial D'_\zeta$ are bounded from below by λ whenever $\zeta \in \partial\Omega \cap V$.
(iv) For any point $w \in \Omega \cap W$, there exists $w^* \in \partial\Omega \cap V$ with $w \in D'_{w^*}$ and $|w - w^*| \approx \delta_\Omega(w)$.

By the localization lemma for the Bergman kernel (see e.g. [Oh1]) we have, with a constant C that depends only on V and W,

$$K_\Omega \ge C K_{\Omega\cap V} = C K_{D\cap V} \ge C K_D$$

on $\Omega \cap W$. Let $w \in \Omega \cap W$ be an arbitrary point, and let w^* be a boundary point according to (iv). Then, using Lemma 4.3(a) we can apply formula (27) to the function

$$u := -(n-p-1)\log(-\psi_{w^*})$$

and obtain (with a new constant C')

$$K_D(w) \geq C' K_{D'_{w^*},u}(w).$$

But in [DHM] it is shown that (with a constant that depends only on λ):

$$K_{D'_{w^*},u}(w) \geq C_\lambda |w-w^*|^{-p-2} e^{u(w)} \geq C'' \delta_\Omega(w)^{-p-2} |\psi_{w^*}(w)|^{-(n-p-1)}.$$

Lemma 4.3(b) implies that, for all $w \in \Omega \cap W$,

$$|\psi_{w^*}(w)| \leq |\psi_{z^0}(w)| + |w^* - z^0| \leq \hat{C}\big(|\psi_{z^0}(w)| + \delta_\Omega(w) + |w - z^0|\big)$$

and, consequently,

$$\delta_\Omega(w)^{p+2} K_\Omega(w) \geq \hat{C} \frac{1}{(|\psi_{z^0}(w)| + \delta_\Omega(w) + |w-z^0|)^{n-p-1}}.$$

From this the theorem follows. □

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