SOME NOTES ON ALMOST HERMITIAN MANIFOLDS

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Introduction.

Several years ago Ishihara [1], [2] introduced the concept of the half-symmetric φ-connection and semi-symmetric φ-connection in an almost complex manifold.

In the present paper, we study about a half-symmetric metric φ-connection and a semi-symmetric metric φ-connection in an almost Hermitian manifold M^{2n} and obtain some interesting results. In § 1, we get a necessary and sufficient condition that an almost Hermitian manifold may admit a half-symmetric metric φ-connection or a semi-symmetric metric φ-connection. In § 2, we determine the semi-symmetric metric φ-connection that an Hermitian manifold may admit.

§ 1. Half-symmetric metric φ-connection and semi-symmetric metric φ-connection on an almost Hermitian manifold.

Let M^{2n} be an almost Hermitian manifold having an almost Hermitian structure $(g_{ij}(x), \varphi^{\cdot i}_{\cdot j}(x))$. That is,

$$\text{(1. 1)} \qquad \begin{cases} \text{(i)} & \varphi^{\cdot i}_{\cdot a}\varphi^{\cdot a}_{\cdot j} = -\delta^i_j, \\ \text{(ii)} & g_{ab}\varphi^{\cdot a}_{\cdot i}\varphi^{\cdot b}_{\cdot j} = g_{ij}. \end{cases}$$

As is well known, the Nijenhuis tensor $N^{\cdot i}_{\cdot jk}(x)$ of an almost complex structure $\varphi^{\cdot i}_{\cdot j}(x)$ is given by

$$\text{(1. 2)} \qquad N^{\cdot i}_{\cdot jk} = (\varphi^{\cdot i}_{\cdot a, j} - \varphi^{\cdot i}_{\cdot j, a})\varphi^{\cdot a}_{\cdot k} - (\varphi^{\cdot i}_{\cdot a, k} - \varphi^{\cdot i}_{\cdot k, a})\varphi^{\cdot a}_{\cdot j}.$$

In the case of $N^{\cdot i}_{\cdot jk}(x) = 0$, the almost complex structure is integrable and consequently the M^{2n} becomes an Hermitian manifold.

In an almost Hermitian manifold M^{2n}, we consider a metric φ-connection $E^{i}_{jk}(x)$:

$$\text{(1. 3)} \qquad g_{jk/h} = g_{jk, h} - g_{ak}E^{a}_{jh} - g_{ja}E^{a}_{kh} = 0,$$

$$\text{(1. 4)} \qquad \varphi^{\cdot i}_{\cdot j/k} = \varphi^{\cdot i}_{\cdot j, k} + \varphi^{\cdot a}_{\cdot j}E^{i}_{ak} - \varphi^{\cdot i}_{\cdot a}E^{a}_{jk} = 0.$$

As is easily seen from (1. 3), such a $E^{i}_{jk}(x)$ is given by

$$\text{(1. 5)} \qquad E^{i}_{jk} = \left\{ {i \atop jk} \right\} + S^{\cdot i}_{\cdot jk} + S_{jk}^{\cdot\cdot i} + S_{kj}^{\cdot\cdot i},$$

where, $\left\{ {i \atop jk} \right\}$ is the Christoffel symbol formed by $g_{ij}(x)$ and $S^{\cdot i}_{\cdot jk}$ is the torsion tensor of E^{i}_{jk}:

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$$S^{i}_{\cdot jk}=\frac{1}{2}(E^{i}_{jk}-E^{i}_{kj}), \qquad S_{jk}^{\cdot\cdot i}=g_{ja}S^{a}_{\cdot kb}g^{bi}. \tag{1.6}$$

On the other hand, $\varphi_{ij}=g_{ia}\varphi^{a}_{\cdot j}$ is, as is seen from (1. 1), skew-symmetric with respect to i and j. Consequently,

$$\varphi_{ijk}=\varphi_{ij,k}+\varphi_{jk,i}+\varphi_{ki,j} \tag{1.7}$$

is skew-symmetric in all the indices. Taking notice of (1. 3), we have easily from (1. 4)

$$\varphi_{ijk}=-2(S_{ajk}\varphi^{a}_{\cdot i}+S_{aki}\varphi^{a}_{\cdot j}+S_{aij}\varphi^{a}_{\cdot k}). \tag{1.8}$$

And we see that $N^{i}_{\cdot jk}=0$ is equivalent to

$$S_{ijk}=S_{ibc}\varphi^{b}_{\cdot j}\varphi^{c}_{\cdot k}+S_{ajc}\varphi^{a}_{\cdot i}\varphi^{c}_{\cdot k}+S_{abk}\varphi^{a}_{\cdot i}\varphi^{b}_{\cdot j}. \tag{1.9}$$

N.B. In [5], Yano and Mogi conclude that $N^{i}_{\cdot jk}=0$ is equivalent to

$$S_{ijk}=S_{ibc}\varphi^{b}_{\cdot j}\varphi^{c}_{\cdot k}+S_{jca}\varphi^{c}_{\cdot k}\varphi^{a}_{\cdot i}+S_{kab}\varphi^{a}_{\cdot i}\varphi^{b}_{\cdot j}, \tag{1.10}$$

but in general this conclusion does not hold. The necessary and sufficient condition that (1. 9), (1. 10) may be equivalent is that S_{ijk} is skew-symmetric in all the indices. In the case that this condition is satisfied, comparing (1. 8) with (1. 10), we have[1)]

$$S_{ijk}=\frac{1}{2}\varphi_{abc}\varphi^{a}_{\cdot i}\varphi^{b}_{\cdot j}\varphi^{c}_{\cdot k}. \tag{1.9$'$}$$

As is well known, we define the following operators Φ_{α} ($\alpha=1, 2, 3, 4$):

$$\Phi_{1}T^{h}{}_{i}=O^{hb}_{ai}T^{a}{}_{b}=\frac{1}{2}(\delta^{h}_{a}\delta^{b}_{i}-\varphi^{h}_{\cdot a}\varphi^{b}_{\cdot i})T^{a}{}_{b},$$

$$\Phi_{2}T^{h}{}_{i}={}^{*}O^{hb}_{ai}T^{a}{}_{b}=\frac{1}{2}(\delta^{h}_{a}\delta^{b}_{i}+\varphi^{h}_{\cdot a}\varphi^{b}_{\cdot i})T^{a}{}_{b},$$

$$\Phi_{3}T_{ji}=O^{ba}_{ji}T_{ba}=\frac{1}{2}(\delta^{b}_{j}\delta^{a}_{i}-\varphi^{b}_{\cdot j}\varphi^{a}_{\cdot i})T_{ba},$$

$$\Phi_{4}T_{ji}={}^{*}O^{ba}_{ji}T_{ba}=\frac{1}{2}(\delta^{b}_{j}\delta^{a}_{i}+\varphi^{b}_{\cdot j}\varphi^{a}_{\cdot i})T_{ba}.$$

Now, we assume that the metric φ-connection E^{i}_{jh} is half-symmetric in the sense of [1], [2]. That is,

$$\Phi_{1}\Phi_{3}S^{h}_{\cdot jk}=O^{hb}_{aj}O^{cd}_{bk}S^{a}_{\cdot cd}=0. \tag{1.11}$$

As is easily seen, this can be rewritten as follows:

1) Compare Theorem 3. 2 in [5].

$$S_{ijk}-S_{ibc}\varphi^{b}_{\cdot j}\varphi^{c}_{\cdot k}+S_{ajc}\varphi^{c}_{\cdot k}\varphi^{a}_{\cdot i}+S_{abk}\varphi^{a}_{\cdot i}\varphi^{b}_{\cdot j}=0. \tag{1.12}$$

Hence from (1. 9) and (1. 12), in an Hermitian manifold, as a necessary and sufficient condition that a metric φ-connection E^{i}_{jk} may be half-symmetric, we obtain

$$S_{ijk}=S_{ibc}\varphi^{b}_{\cdot j}\varphi^{c}_{\cdot k}. \tag{1.13}$$

This can be rewritten as follows:

$$S_{ijc}\varphi^{c}_{\cdot k}=S_{ikb}\varphi^{b}_{\cdot j}. \tag{1.13'}$$

Since this shows that $S_{ijc}\varphi^{c}_{\cdot k}$ is symmetric with respect to j and k, we have the following

PROPOSITION 1. 1. *In an Hermitian manifold M^{2n}, a necessary and sufficient condition that a metric φ-connection $E^{i}_{jk}(x)$ may be half-symmetric is that $S_{ijc}\varphi^{c}_{\cdot k}$ is symmetric with respect to j and k.*

Next we assume that in an almost Hermitian manifold, a metric φ-connection $E^{i}_{jk}(x)$ is semi-symmetric in the sense of [1], [2].[2] That is

$$S^{h}_{\cdot jk}=\frac{2}{n}\Phi_{4}(S_{[j}\delta^{h}_{k]})=\frac{2}{n}{}^{*}O^{bc}_{jk}(S_{[b}\delta^{h}_{c]}),\ S_{j}=S^{a}_{\cdot ja}. \tag{1.14}$$

As is easily seen, this can be rewritten as

$$S_{ijk}=\frac{1}{2n}(S_{j}g_{ik}-S_{k}g_{ij}+\varphi_{ik}\varphi^{b}_{\cdot j}S_{b}-\varphi_{ij}\varphi^{c}_{\cdot k}S_{c}). \tag{1.15}$$

In this case we see that (1. 9) holds identically. That is, the following Proposition is obtained:

PROPOSITION 1. 2. *When an almost Hermitian manifold M^{2n} admits a semi-symmetric metric φ-connection, the M^{2n} is an Hermitian manifold.*

N.B. As is directly seen from (1. 15)

$$S_{ijc}\varphi^{c}_{\cdot k}=\frac{1}{2n}(\varphi_{ik}S_{j}+\varphi_{ij}S_{k}-g_{ij}\varphi^{c}_{\cdot k}S_{c}-g_{ik}\varphi^{b}_{\cdot j}S_{b})$$

holds. Since this is obviously symmetric with respect to j and k, from Proposition 1. 1 we see that the semi-symmetric metric φ-connection is half-symmetric. This is known from the general theory in [1].

Now we suppose that an Hermitian manifold M^{2n} admits a semi-symmetric metric φ-connection. Substituting (1. 15) into (1. 8), we have

$$\varphi_{ijk}=-\frac{2}{n}(\varphi_{jk}S_{i}+\varphi_{ki}S_{j}+\varphi_{ij}S_{k}). \tag{1.16}$$

Contracting on both sides of this with φ^{jk}, we obtain

2) The definition of semi-symmetric connection in [1], [2] is different from the definition in [5].

$$(1.17)\qquad \varphi_{ibc}\varphi^{bc}=-\frac{4(n-1)}{n}S_i,$$

and for $n\neq 1$, eliminating S_i from (1. 16) and (1. 17), we get

$$(1.18)\qquad \varphi_{ijk}=\frac{1}{2(n-1)}(\varphi_{jk}\varphi_{ibc}+\varphi_{ki}\varphi_{jbc}+\varphi_{ij}\varphi_{kbc})\varphi^{bc}.$$

Summing up the result, we see that in order that an Hermitian manifold M^{2n} $(n\neq 1)$ may admit a semi-symmetric metric φ-connection, it is necessary that (1. 18) holds.

Conversely, we assume that an Hermitian manifold M^{2n} $(n\neq 1)$ satisfies (1. 18). We consider to determine S_{ijk} from (1. 8) and (1. 18). Now if we contract on both sides of (1. 8) with φ^{jk} and put as

$$S_i=-\frac{n}{4(n-1)}\varphi_{ibc}\varphi^{bc},$$

then

$$(1.19)\qquad 2(n-1)S_i=n(S_{abc}\varphi^{a}_{\cdot\, i}\varphi^{bc}-2S_{abi}g^{ab})$$

is obtained. And we have, from (1. 8) and (1. 18),

$$(1.20)\qquad \begin{aligned} &S_{ajk}\varphi^{a}_{\cdot\, i}+S_{aki}\varphi^{a}_{\cdot\, j}+S_{aij}\varphi^{a}_{\cdot\, k}\\ &=\frac{1}{n}(\varphi_{jk}S_i+\varphi_{ki}S_j+\varphi_{ij}S_k).\end{aligned}$$

From (1. 19) and (1. 20), we will seek the solution S_{ijk} satisfying

$$g^{ab}S_{aib}=S_i.$$

Since we have (1. 19) by contracting on both sides of (1. 20) with φ^{jk}, we may consider (1. 20) only. Then, if we put as

$$(1.21)\qquad S_{ijk}=\frac{1}{2n}(g_{ik}S_j-g_{ij}S_k+\varphi_{ik}\varphi^{b}_{\cdot\, j}S_b-\varphi_{ij}\varphi^{c}_{\cdot\, k}S_c)+s_{ijk},$$

from $S_{aib}g^{ab}=S_i$, we get

$$(1.22)\qquad s_{ajb}g^{ab}=0.$$

Next substituting (1. 21) into (1. 20),

$$(1.23)\qquad s_{ajk}\varphi^{a}_{\cdot\, i}+s_{aki}\varphi^{a}_{\cdot\, j}+s_{aij}\varphi^{a}_{\cdot\, k}=0$$

is obtained. Hence if we put

$$(1.24)\qquad u_{ijk}=s_{ajk}\varphi^{a}_{\cdot\, i}$$

for simplicity, this u_{ijk} should satisfy the following conditions:

$$(1.25)\qquad \begin{cases}(\mathrm{i})\ \ u_{ijk}+u_{jki}+u_{kij}=0,\\ (\mathrm{ii})\ \ u_{ajc}\varphi^{ac}=0,\\ (\mathrm{iii})\ \ u_{ijk}=-u_{ikj}.\end{cases}$$

(i) is obtained from (1. 23), (ii) is equivalent to (1. 22) and (iii) is seen from the fact that S_{ijk} is skew-symmetric with respect to j and k. And the solution of (1. 20) is given by (1. 21) and

$$(1.26)\qquad s_{ijk}=-u_{ajk}\varphi^{a}_{\cdot i}.$$

We will seek the u_{ijk} satisfying the system of equations (1. 25). However $u_{ijk}=0$ obviously satisfies (1. 25), and $s_{ijk}=0$ is obtained from (1. 26). In this case as is seen from (1. 21), this gives the semi-symmetric metric φ-connection. Consequently, when (1. 18) holds, we know that an Hermitian manifold M^{2n} admits a semi-symmetric metric φ-connection.

Summarizing the above, we have the following

THEOREM 1. 3. *A necessary and sufficient condition that an Hermitian manifold M^{2n} $(n\neq 1)$ may admit a semi-symmetric metric φ-connection is that*

$$(1.18)\qquad \varphi_{ijk}=\frac{1}{2(n-1)}(\varphi_{jk}\varphi_{ibc}+\varphi_{ki}\varphi_{jbc}+\varphi_{ij}\varphi_{kbc})\varphi^{bc}$$

holds.

In an Hermitian manifold M^{2n}, when the torsion tensor S_{ijk} of the metric φ-connection E^{i}_{jk} is skew-symmetric in all the indices, then (1. 9)′ holds. And moreover, if this connection is half-symmetric, (1. 13) holds. Hence in an Hermitian manifold M^{2n}, when the torsion tensor S_{ijk} of a metric φ-connection E^{i}_{jk} is skew-symmetric in all the indices, in order that the E^{i}_{jk} may be half-symmetric, from (1. 9)′ and (1. 13) it is necessary that

$$(1.27)\qquad \varphi_{ijk}=\varphi_{ibc}\varphi^{b}_{\cdot j}\varphi^{c}_{\cdot k}$$

holds.

Conversely, since (1. 13) is obtained from (1. 27) and (1. 9)′, this is also sufficient.

Summarizing the above we have the following

THEOREM 1. 4. *In an Hermitian manifold M^{2n}, when the torsion tensor S_{ijk} of the metric φ-connection E^{i}_{jk} is skew-symmetric in all the indices, a necessary and sufficient condition that the connection E^{i}_{jk} may be half-symmetric is that*

$$(1.27)\qquad \varphi_{ijk}=\varphi_{ibc}\varphi^{b}_{\cdot j}\varphi^{c}_{\cdot k}$$

holds.

N.B. (1. 27) is equivalent to

$$(1.27)'\qquad \varphi_{ijc}\varphi^{c}_{\cdot k}=\varphi_{ikb}\varphi^{b}_{\cdot j}$$

which shows that $\varphi_{ijc}\varphi^{c}_{\cdot k}$ is symmetric with respect to j and k. Consequently, this shows that $\varphi_{ijc}\varphi^{c}_{\cdot k}$ is symmetric with respect to i and k.

§2. Determination of semi-symmetric metric φ-connection on an Hermitian manifold.

We suppose that an Hermitian manifold M^{2n} $(g_{ij}(x), \varphi^{i}_{\cdot j}(x))$ is conformal to a Kählerian manifold $\tilde{M}^{2n}$ $(\tilde{g}_{ij}(x), \tilde{\varphi}^{i}_{\cdot j}(x))$.

That is, we suppose that there exists a suitable function $\sigma(x)$ such that

$$\left\{\begin{array}{l} g_{ij}(x)=e^{\sigma(x)}\tilde{g}_{ij}(x), \\ \varphi^{i}_{\cdot j}(x)=\tilde{\varphi}^{i}_{\cdot j}(x) \end{array}\right. \tag{2. 1}$$

hold.

$$\varphi_{ij}(x)=g_{ia}(x)\varphi^{a}_{\cdot j}(x), \qquad \tilde{\varphi}_{ij}(x)=\tilde{g}_{ia}(x)\tilde{\varphi}^{a}_{\cdot j}(x)$$

are both skew-symmetric with respect to i and j and

$$\varphi_{ij}(x)=e^{\sigma(x)}\tilde{\varphi}_{ij}(x) \tag{2. 2}$$

holds. Since $\tilde{g}_{ij}(x)$ defines a Kählerian metric,

$$\left\{\begin{array}{l} N^{i}_{\cdot jk}=\varphi^{a}_{\cdot k}(\varphi^{i}_{\cdot a,j}-\varphi^{i}_{\cdot j,a})-\varphi^{a}_{\cdot j}(\varphi^{i}_{\cdot a,k}-\varphi^{i}_{\cdot k,a})=0, \\ \tilde{\varphi}_{ij,k}+\tilde{\varphi}_{jk,i}+\tilde{\varphi}_{ki,j}=0 \end{array}\right. \tag{2. 3}$$

hold and vice-versa. Now if we denote the covariant differentiation with respect to the Christoffel symbol $\left\{\widetilde{{h \atop ij}}\right\}$ of a Kählerian metric $\tilde{g}_{ij}(x)$ as $\tilde{\nabla}_{i}$, (2. 3) can be rewritten as follows:

$$\left\{\begin{array}{l} (\tilde{\nabla}_{j}\tilde{\varphi}_{ia}-\tilde{\nabla}_{a}\tilde{\varphi}_{ij})\tilde{\varphi}^{a}_{\cdot k}-(\tilde{\nabla}_{k}\tilde{\varphi}_{ia}-\tilde{\nabla}_{a}\tilde{\varphi}_{ik})\tilde{\varphi}^{a}_{\cdot j}=0, \\ \tilde{\nabla}_{i}\tilde{\varphi}_{jk}+\tilde{\nabla}_{j}\tilde{\varphi}_{ki}+\tilde{\nabla}_{k}\tilde{\varphi}_{ij}=0. \end{array}\right. \tag{2. 4}$$

This is, as is well known, equivalent to

$$\tilde{\nabla}_{i}\tilde{\varphi}_{jk}=0. \tag{2. 5}$$

Since we have from (2. 2)

$$\begin{aligned} \varphi_{ij,k}&=e^{\sigma}\tilde{\varphi}_{ij,k}+\sigma_{,k}\varphi_{ij}, \\ \varphi_{ijk}&=\varphi_{ij,k}+\varphi_{jk,i}+\varphi_{ki,j} \\ &=e^{\sigma}(\tilde{\varphi}_{ij,k}+\tilde{\varphi}_{jk,i}+\tilde{\varphi}_{ki,j}) \\ &\quad+(\sigma_{,i}\varphi_{jk}+\sigma_{,j}\varphi_{ki}+\sigma_{,k}\varphi_{ij}) \end{aligned}$$

is obtained. Since $\tilde{g}_{ij}$ is a Kählerian metric, taking notice of (2. 3), we get

$$\varphi_{ijk}=\sigma_{,i}\varphi_{jk}+\sigma_{,j}\varphi_{ki}+\sigma_{,k}\varphi_{ij}. \tag{2. 6}$$

Contracting on both sides of this with φ^{jk},

$$\varphi_{ibc}\varphi^{bc}=2(n-1)\sigma,_i \tag{2. 7}$$

is obtained, so substituting this into (2. 6), we have, ($n\neq 1$)

$$\varphi_{ijk}=\frac{1}{2(n-1)}(\varphi_{jk}\varphi_{ibc}+\varphi_{ki}\varphi_{jbc}+\varphi_{ij}\varphi_{kbc})\varphi^{bc}. \tag{2. 8}$$

Summing up, we see that in order that an Hermitian manifold M^{2n} ($n\neq 1$) (g_{ij}, $\varphi^{i}_{\cdot j}$) may be conformal to a Kählerian manifold $\tilde{M}^{2n}$, it is necessary that (2. 8) holds.

Conversely, for $n>2$ if (2. 8) holds, it is known that an Hermitian manifold M^{2n} is conformal to a Kählerian manifold $\tilde{M}^{2n}$. (See [3], [4].)

Summarizing the above, we have the following well known

THEOREM 2. 1. *A necessary and sufficient condition that a $2n$ ($n>2$) dimensional Hermitian manifold M^{2n} may be conformal to a Kählerian manifold $\tilde{M}^{2n}$ is that*

$$\varphi_{ijk}=\frac{1}{2(n-1)}(\varphi_{jk}\varphi_{ibc}+\varphi_{ki}\varphi_{jbc}+\varphi_{ij}\varphi_{kbc})\varphi^{bc} \tag{2. 8}$$

holds.

Now, since (1. 18) and (2. 8) are the same, from Proposition 1. 2, Theorem 1. 3 and Theorem 2. 1, we get the following

THEOREM 2. 2. *A necessary and sufficient condition that a $2n$ ($n>2$) dimensional almost Hermitian manifold M^{2n} may admit a semi-symmetric metric φ-connection is that the M^{2n} is conformal to a Kählerian manifold.*

If an almost Hermitian manifold M^{2n} ($n>2$) admits a semi-symmetric metric φ-connection, from (1. 15)

$$S^{i}_{\cdot jk}=\frac{1}{2n}(\delta^{i}_{k}S_j-\delta^{i}_{j}S_k+\varphi^{i}_{\cdot k}\varphi^{b}_{\cdot j}S_b-\varphi^{i}_{\cdot j}\varphi^{c}_{\cdot k}S_c)$$

holds. And since

$$S_i=-\frac{n}{2}\sigma,_i$$

holds from (1. 17) and (2. 7), when we substitute this into the above expression, we have

$$S^{i}_{\cdot jk}=\frac{1}{4}(\delta^{i}_{j}\sigma,_k-\delta^{i}_{k}\sigma,_j+\varphi^{i}_{\cdot j}\varphi^{c}_{\cdot k}\sigma,_c-\varphi^{i}_{\cdot k}\varphi^{b}_{\cdot j}\sigma,_b).$$

Consequently, from this

$$S_{jk}^{\cdot\cdot i}=\frac{1}{4}(g_{jk}g^{ia}\sigma,_a-\delta^{i}_{j}\sigma,_k-\varphi_{jk}\varphi^{ib}\sigma,_b+\varphi^{i}_{\cdot j}\varphi^{b}_{\cdot k}\sigma,_b)$$

holds. Hence substituting these results into (1. 5),

$$E_j{}^i{}_k=\left\{ {i \atop jk} \right\}+\frac{1}{2}(-\delta^{i}_{k}\sigma,_j+\varphi^{i}_{\cdot j}\varphi^{c}_{\cdot k}\sigma,_c+g_{jk}g^{ia}\sigma,_a) \tag{2. 9}$$

is obtained. On the other hand, as is well known from (2. 1), since

$$\left\{ {i \atop jk} \right\} = \left\{ \widetilde{i \atop jk} \right\} + \frac{1}{2}(\delta^i_j\sigma,_k + \delta^i_k\sigma,_j - g_{jk}g^{ia}\sigma,_a)$$

holds, substituting this into (2. 9), we have

$$E^i_{jk} = \left\{ \widetilde{i \atop jk} \right\} + \frac{1}{2}(\delta^i_j\sigma,_k + \varphi^i_{\cdot j}\varphi^c_{\cdot k}\sigma,_c),$$

namely

$$E^i_{jk} = \left\{ \widetilde{i \atop jk} \right\} + {}^*O^{ia}_{jk}\sigma,_a. \tag{2. 10}$$

This is the semi-symmetric metric φ-connection.

Summarizing the above, we have the following

THEOREM 2. 3. *The semi-symmetric metric φ-connection $E^i_{jk}(x)$ that a $2n$ $(n>2)$ dimensional Hermitian manifold M^{2n} $(g_{ij}(x) = e^{\sigma(x)}\tilde{g}_{ij}(x), \varphi^i_{\cdot j}(x))$ conformal to a Kählerian manifold $\tilde{M}^{2n}$ $(\tilde{g}_{ij}(x), \varphi^i_{\cdot j}(x))$ admits is given by*

$$E^i_{jk} = \left\{ \widetilde{i \atop jk} \right\} + {}^*O^{ia}_{jk}\sigma,_a. \tag{2. 10}$$

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