SOME CHARACTERIZATIONS OF STEIN MANIFOLD THROUGH THE NOTION OF LOCALLY REGULAR BOUNDARY POINTS

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Dedicated to Professor K. Kunugi on his sixtieth birthday

Introduction.

The main purpose of the present paper is to investigate the intersection of a Cousin-I domain and a domain of holomorphy. Oka [14] proved that a domain of holomorphy in C^n is a *Cousin*-I *domain*, that is, a domain in which any additive Cousin's distribution has a solution. On the other hand, a Cousin-I domain in C^2 is a domain of holomorphy from Cartan [5] and Behnke-Stein [2]. Therefore a domain in C^2 is a Cousin-I domain if and only if it is a domain of holomorphy. Cartan [6] proved that $E=\{(z_1, z_2, z_3); |z_1|<1, |z_2|<1, |z_3|<1\}-\{(0, 0, 0)\}$ is not a domain of holomorphy but a Cousin-I domain. For any domain of holomorphy D in C^3, $E\cap D$ is a Cousin-I open set. Making use of the results of Scheja [16] and Andreotti-Grauert [1] concerning the prolongation of cohomology classes, we can construct systematically other Cousin-I domains in C^n which are not domains of holomorphy for $n\geqq 3$. For $G=\{(z_1, z_2, z_3); |z_1|<1, |z_2|<1, |z_3|<1\}-\{(z_1, z_2, z_3); z_1=z_2=0, |z_3|\leqq 1/2\}$, there holds $H^1(G, \mathfrak{O})=0$ from Scheja [16] where $\mathfrak{O}$ is the sheaf of all germs of holomorphic functions. Therefore G is not a domain of holomorphy but a Cousin-I domain. But G has a different property from E. The intersection $G\cap Z=[\{(z_1, z_2); |z_1|<1/2, |z_2|<1/2\}-\{(0, 0)\}]\times\{z_3; |z_3|<1/2\}$ of G and a tridisc $Z=\{(z_1, z_2, z_3); |z_1|<1/2, |z_2|<1/2, |z_3|<1/2\}$ is not a Cousin-I domain as the first component of the above direct product is not a Cousin-I domain from the results of Cartan [5] and Behnke-Stein [2].

A domain in C^n, which is a direct product $K_1\times K_2\times\cdots\times K_n$ of relatively compact subdomains K_i of a complex plane, is called a *polycylinder* hereafter. An open set G in C^n is called *regular* if $G\cap(K_1\times K_2\times\cdots\times K_n)$ is a Cousin-I open set for any polycylinder $K_1\times K_2\times\cdots\times K_n$ in C^n. From the previous paper [12] of the author G is a Cousin-I open set. Cartan's example E is a regular domain in C^3 but the above example G is not a regular domain. We say that a domain G in C^n is *exhausted by regular domains* if there exists a sequence $\{G_p; p=1, 2, 3, \cdots\}$ of regular domains G_p such that $G_p\Subset G_{p+1}$ $(p=1, 2, 3, \cdots)$ and $G=\bigcup_{p=1}^{\infty}G_p$. From the previous paper [12] of the author G is a Cousin-I domain as it is a limit of mono-

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tonously increasing sequence of Cousin-I domains G_p. Moreover we shall prove that a domain in C^n is a domain of holomorphy if and only if it can be exhausted by regular domains. This is a characterization of a domain of holomorphy by means of Cousin-I problems. This means that a regular domain in C^n, which is not a domain of holomorphy, is isolated in the set of regular domains in some sense.

We shall define a continuous boundary point of an open set in C^n in such a way that a smooth boundary point of an open set in C^n in the usual sense is a continuous boundary point. An open set G in C^n is called *locally regular at a boundary point* z^0 of G if there exists an open neighbourhood U of z^0 such that $G\cap U$ is regular. An open set is called *locally regular* if it is locally regular at each of its boundary points. We shall prove that a domain is pseudoconvex at its continuous boundary point z^0 if and only if G is locally regular at z^0. Hence from the affirmative solution of the Levi problem due to Bremermann [4], Norguet [13] and Oka [15] a domain with a continuous boundary is a domain of holomorphy if and only if it is locally regular. This is a characterization of a domain of holomorphy with a continuous boundary by means of Cousin-I problems. Making use of Docquier-Grauert [8] we shall extend this fact to a domain in a Stein manifold.

§1. Domain exhausted by regular domains.

LEMMA 1. *Let G be a regular domain in C^n. Then $D=G\cap\{z=(z_1, z_2, \cdots, z_n); z_j\in K_j\ (j=s_1, s_2, \cdots, s_r)\}$ is a Cousin-*I *open set for any $1\leqq s_1<s_2<\cdots<s_r\leqq n$ and for any domains K_j in a complex plane $(j=s_1, s_2, \cdots, s_r)$. Especially G itself is a Cousin-*I *domain.*

Proof. We put $K_j^p=\{z_j; |z_j|<p\}$ for $j\notin\{s_1, s_2, \cdots, s_r\}$ and $K_j^p=K_j\cap\{z_j; |z_j|<p\}$ for $j\in\{s_1, s_2, \cdots, s_r\}$. Then $D_p=G\cap(K_1^p\times K_2^p\times\cdots\times K_n^p)$ is a Cousin-I open set for each p as G is a regular domain. Since D is the limit of a monotonously increasing sequence of Cousin-I open sets D_p, D is a Cousin-I open set from the previous paper [12] of the author. In the same way we can prove that G itself is a Cousin-I domain.

The proof of the following Lemmas 2 and 3 is similar to the method of Hitotumatu [10].

LEMMA 2. *Let G be a Cousin-*I *domain in C^n and H be an $(n-1)$-dimensional analytic plane in C^n. Then the inclusion map $G\cap H\rightarrow G$ induces naturally a homomorphism of $H^0(G, \mathfrak{Q})$ onto $H^0(G\cap H, \mathfrak{Q})$.*

Proof. Without loss of generality we may suppose that $H=\{(z, w)=(z_1, z_2, \cdots, z_{n-1}, w); w=0\}$. Let $u(z)$ be a holomorphic function in $G\cap H$. If $x^0=(z^0, 0)=(z_1^0, z_2^0, \cdots, z_{n-1}^0, 0)$ is a point of $G\cap H$, there exists a neighbourhood $U(x^0)=\{(z, w); |z_j-z_j^0|<\varepsilon, |w|<\varepsilon\ (j=1, 2, \cdots, n-1)\}$ of x^0 in G. If x^0 is a point of $G-G\cap H$, we put $U(x^0)=G-G\cap H$. If we put $m_{x^0}=u/w$ for $x^0\in G\cap H$ and $m_{x^0}=0$ for $x^0\in G-G\cap H$, then $\mathfrak{C}=\{(m_{x^0}, U(x^0)); x^0\in G\}$ forms an additive Cousin's distribution in G. Since

G is a Cousin-I domain, there exists a meromorphic function m in G which is a solution of $\mathfrak{C}$. We put $v=wm$. For $x^0 \in G\cap H$, $h=m-u/w$ is a holomorphic function in $U(x^0)$. Hence $v=wh+u$ is holomorphic in $U(x^0)$ and $v=u$ in $U(x^0)\cap H$. Hence v is holomorphic and coincides with u in $G\cap H$. Since v is holomorphic in $G-G\cap H$, v is a holomorphic function in G with $v=u$ in $G\cap H$. Hence the canonical homomorphism $H^0(G, \mathfrak{O})\rightarrow H^0(G\cap H, \mathfrak{O})$ is surjective.

LEMMA 3. *Let G be a domain in the space* C^n *of variables* $z=(z_1, z_2, \cdots, z_n)$. *Then G is a domain of holomorphy if and only if the intersection* $G\cap H$ *of G and an l-dimensional analytic plane* $H=\{z;\ z_j=c_j\ (j=s_1, s_2, \cdots, s_{n-l})\}$ *is a Cousin-*I *open set for any integers* $1\leqq l\leqq n$, $1\leqq s_1<s_2<\cdots<s_{n-l}\leqq n$ *and complex numbers* c_j $(j=s_1, s_2, \cdots, s_{n-l})$.

Proof. Since a domain of holomorphy is a Cousin-I domain from Oka [14] and the intersection of a domain of holomorphy and an analytic plane is an open set of holomorphy, it suffices to prove the sufficiency by induction with respect to n. For $n=1$ any domain is a domain of holomorphy from Weierstrass' theorem. For $n=2$ any domain is a domain of holomorphy if and only if it is a Cousin-I domain from Oka [14], Cartan [5] and Behnke-Stein [2]. Suppose that our assertion is valid for $n<k$ $(k\geqq 2)$. We consider the case $n=k$. Let $z^0=(z_1^0, z_2^0, \cdots, z_k^0)$ be any boundary point of G. Two cases (1) and (2) may occur. In the case (1) there exists j such that z^0 is a boundary point of $G\cap H$ for $H=\{z;\ z_j=z_j^0\}$. In the case (2) z^0 is not a boundary point of $G\cap H$ for $H=\{z;\ z_j=z_j^0\}$ for any j.

Case (1) Since $G\cap H$ is an open set of holomorphy in H from the assumption of our induction, there exists a holomorphic function u in $G\cap H$ which is unbounded at z^0. From Lemma 2 there exists a holomorphic function v in G with $v=u$ in $G\cap H$. v is a holomorphic function in G which is unbounded at z^0.

Case (2) We shall prove that there exists a sequence $\{z^p;\ p=1, 2, 3, \cdots\}$ of $z^p\in\partial G\cap U$ such that each z^p has the property as in the case (1) and $z^p\rightarrow z^0$ when $p\rightarrow\infty$. If this is not true, there exists $\varepsilon>0$ such that $G\cap U\cap\{z;\ z_j=\zeta_j\}=U\cap\{z;\ z_j=\zeta_j\}$ for $U=\{z;\ |z_j-z_j^0|<\varepsilon\ (j=1, 2, \cdots, k)\}$ and for any j and $\zeta\in G\cap U$. Let $z^1=(z_1^1, z_2^1, \cdots, z_k^1)$ be any point of $G\cap U$ and $z^2=(z_1^2, z_2^2, \cdots, z_k^2)$ be any point of U. By induction we can prove that $(z_1^2, z_2^2, \cdots, z_m^2, z_{m+1}^1, \cdots, z_k^1)\in G\cap U$ for $1\leqq m\leqq k$. Therefore we have $z^2\in G\cap U$. Hence it holds that $G\cap U=U$. This means that z^0 is an interior point of G. But this is a contradiction. Therefore there exists a sequence $\{f_p;\ p=1, 2, 3, \cdots\}$ of holomorphic functions f_p in G which is unbounded at z^p tending to z^0 when $p\rightarrow\infty$. From Bochner-Martin [3] there exists a holomorphic function which is unbounded at z^0.

Thus we have proved the existence of a holomorphic function in G which is unbounded at z^0. Since z^0 is any boundary point of G, there exists a holomorphic function f in G which is unbounded at each boundary point of G from Bochner-Martin [3]. Hence G is a domain of holomorphy of f.

Quite similarly we can prove that a domain G in the space C^n of variables $z=(z_1, z_2, \cdots, z_n)$ is a domain of holomorphy if and only if the canonical homomorphism of $H^0(G, \mathfrak{O})$ into $H^0(G\cap H, \mathfrak{O})$ is surjective for any analytic plane H as in Lemma 3. This is a characterization of a domain of holomorphy.

LEMMA 4. *If a domain G in C^n is exhausted by regular domains, then the intersection $G\cap H$ of G and an l-dimensional analytic plane $H=\{z=(z_1, z_2, \cdots, z_n); z_j=c_j\ (j=s_1, s_2, \cdots, s_{n-l})\}$ is a Cousin-*I *open set for any integers $1\leqq l\leqq n$, $1\leqq s_1<s_2<\cdots<s_{n-l}\leqq n$ and complex numbers $c_j\ (j=s_1, s_2, \cdots, s_{n-l})$.*

Proof. There exists a sequence $\{G_p; p=1, 2, 3, \cdots\}$ of regular domains G_p such that $G_p\Subset G_{p+1}\ (p=1, 2, 3, \cdots)$ and $G=\cup_{p=1}^{\infty}G_p$. We may suppose that $H=\{(z, w)=(z_1, z_2, \cdots, z_l, w_1, w_2, \cdots, w_{n-l}); w_j=0\ (j=1, 2, \cdots, n-l)\}$. There exists $\varepsilon_p>0$ such that $E_p=G_p\cap\{(z, w); |w_j|<\varepsilon_p; (j=1, 2, \cdots, n-l)\}\subset\{(z, w); |w_j|<\varepsilon_p, (z, 0)\in G\cap H\ (j=1, 2, \cdots, n-l)\}$ for any p. Since G_p is regular, E_p is a Cousin-I open set from Lemma 1. Let $\mathfrak{C}=\{(m_i, V_i); i\in I\}$ be an additive Cousin's distribution in $G\cap H$. If we put $V_i^p=G_p\cap\{(z, w); |w_j|<\varepsilon_p, (z, 0)\in V_i\ (j=1, 2, \cdots, n-l)\}$ and $M_i^p(z, w)=m_i(z)$ in V_i^p, then $\mathfrak{C}_p=\{(M_i^p, V_i^p); i\in I\}$ is an additive Cousin's distribution in E_p. Since E_p is a Cousin-I open set, $\mathfrak{C}_p$ has a solution $M^p(z, w)$ for any p. Since the set of all poles of $M^p(z, w)$ does not contain connected components of $G_p\cap H$ for any p, the restriction $m^p(z)$ of $M^p(z, w)$ to $G_p\cap H$ is a solution of the restriction $\{(m_i|G_p\cap H, V_i\cap G_p); i\in I\}$ of $\mathfrak{C}$ to $G_p\cap H$ for any p. Since the canonical homomorphism of $H^1(G\cap H, \mathfrak{O})$ into $\lim_{p\to\infty}H^1(G_p\cap H, \mathfrak{O})$ is injective (Lemma 6 in the previous paper [12] of the author), $\mathfrak{C}$ has a solution in $G\cap H$. Therefore $G\cap H$ is a Cousin-I open set.

PROPOSITION 1. *A domain G in C^n is a domain of holomorphy if and only if it is exhausted by regular domains.*

Proof. If G is a domain of holomorphy, G is exhausted by domains of holomorphy G_p. Since each G_p is a regular domain, G is exhausted by regular domains. Conversely, if G is exhausted by regular domains, G is a domain of holomorphy from Lemmas 3 and 4.

Proposition 1 gives a characterization of a domain of holomorphy by means of Cousin-I problem and means that regular domains which are not domains of holomorphy are isolated in some sense in the set of regular domains.

§2. Behaviour of a regular domain at a continuous boundary point.

A subset S of R^n is called *smooth* at $x^0\in S$ if there exists a continuously differentiable function f in a neighbourhood U of x^0 such that $S\cap U=\{x; f(x)=0, x\in U\}$ and $\sum_{j=1}^n(\partial f/\partial x_j)^2>0$ at x^0. If $\partial f/\partial x_j\neq 0$ at x^0, there exists a continuously differentiable function g in a neighbourhood $V\subset U$ of x^0 such that $S\cap V=\{x; x_j=g(x_1, x_2, \cdots, \hat{x}_j, \cdots, x_n), x\in V\}$. The notion of smoothness is invariant under continuously bidifferentiable mappings. A subset S of R^n is called *continuous* at $x^0\in S$ if there exists a continuous function g in a neighbourhood V of x^0 such that $S\cap V=\{x; x_j=g(x_1, x_2, \cdots, \hat{x}_j, \cdots, x_n), x\in V\}$ for some j. This definition may depend on the special choice of coordinates in R^n. A boundary point x^0 of an open set G in R^n is called *continuous* (or *smooth*) if ∂G is continuous (or smooth) at x^0.

An open set G in a complex manifold is called *pseudoconvex at* $x^0 \in \partial G$ if there exists an open neighbourhood V of x^0 such that $G \cap V$ is holomorphically convex. G is called *pseudoconvex* if G is pseudoconvex at each point of ∂G.

PROPOSITION 2. *A regular open set G in C^n is pseudoconvex at a continuous boundary point z^0 of G.*

Proof. Without loss of generality we may suppose that $\partial G \cap V = \{z=(z_1, z_2, \cdots, z_n);\ x_n = g(z_1, z_2, \cdots, z_{n-1}, y_n),\ z \in V\}$ for a continuous function g in a polycylindrical neighbourhood V of z^0 where $z_n = x_n + \sqrt{-1}\,y_n$. Then two cases (1) and (2) may occur for a sufficiently small V. In the case (1) there holds $G \cap V = \{z;\ x_n < g(z_1, z_2, \cdots, z_{n-1}, y_n),\ z \in V\}$ or $G \cap V = \{z;\ x_n > g(z_1, z_2, \cdots, z_{n-1}, y_n),\ z \in V\}$. In the case (2) there holds $G \cap V = \{z;\ x_n \neq g\ (z_1, z_2, \cdots. z_{n-1}, y_n),\ z \in V\}$.

Case (1) We have only to consider the case $G \cap V = \{z;\ x_n < g(z_1, z_2, \cdots, z_{n-1}, y_n),\ z \in V\}$. There exists a family $\{V_t;\ 0 \leqq t \leqq t_0\}$ of polycylinders V_t containing z^0 such that $V_{t_1} \Subset V_{t_2} \Subset V$ for $0 \leqq t_2 < t_1 \leqq t_0$, $V_0 = \cup_{0 < t \leqq t_0} V_t$ and $\{z;\ (z_1, z_2, \cdots, z_n - t) \in V_t\} \subset V$ for $0 \leqq t \leqq t_0$. We shall prove that $E_t = \{z;\ x_n < g(z_1, z_2, \cdots, z_{n-1}, y_n) - t,\ z \in V_t\}$ is a regular open set for $0 \leqq t \leqq t_0$. Let P be a polycylinder. We consider a biholomorphic mapping $(z_1, z_2, \cdots, z_n) \rightarrow (w_1, w_2, \cdots, w_n)$ defined by $w_j = z_j\ (j=1, 2, \cdots, n-1)$ and $w_n = z_n + t$. Then $E_t \cap P$ is mapped onto $\{w;\ u_n < g\ (w_2, w_2, \cdots, w_{n-1}, v_n),\ (w_1, w_2, \cdots, w_{n-1}, w_n - t) \in V_t \cap P\} = G \cap V \cap \{z;\ (z_1, z_2, \cdots, z_{n-1}, z_n - t) \in V_t \cap P\}$ which is a Cousin-I open set for $0 \leqq t \leqq t_0$ as the third element of the right-hand side of the above equation is a polycylinder. Hence E_t is a regular open set. Since $E = G \cap V_0$ is exhausted by regular open sets E_t, E is an open set of holomorphy from Proposition 1. Hence G is pseudoconvex at z^0.

Case (2) If we put $E_1 = \{z;\ x_n < g\ (z_1, z_2, \cdots, z_{n-1}. y_n).\ x \in V\}$ and $E_2 = \{z;\ x_n > g(z_1, z_2, \cdots, z_{n-1}, y_n),\ x \in V\}$, then E_1 and E_2 are regular open sets. Therefore from the case (1) E_1 and E_2 are pseudoconvex at z^0. Hence G is pseudoconvex at z^0.

§ 3. Global character of locally regular domains.

An open set G in a complex manifold M is called *strongly regular* if $G \cap D$ is a Cousin-I open set for any Stein manifold $D \subset M$. This is invariant under biholomorphic mappings of M. We say that a domain G in a complex manifold is *exhausted by strongly regular domains* if there exists a sequence of strongly regular domains G_p such that $G_p \Subset G_{p+1}\ (p=1, 2, 3, \cdots)$ and $G = \cup_{p=1}^{\infty} G_p$.

PROPOSITION 3. *A domain G in a Stein manifold is a Stein manifold if and only if G is exhausted by strongly regular domains.*

Proof. If G is a Stein manifold, it is obvious that G is exhausted by strongly regular domains. Conversely suppose that G is exhausted by strongly regular domains G_p. Let x^0 be any point of ∂G. There exists a biholomorphic mapping τ of a holomorphically convex neighbourhood U of x^0 into a complex Euclidean space. U is exhausted by holomorphically convex domains U_p. Since $\tau(G\cap U)$ is exhausted by strongly regular open sets $\tau(G_p\cap U_p)$, $\tau(G\cap U)$ is an open set of holomorphy from Proposition 1. G is a Stein manifold from Docquier-Grauert [8].

An open set G in a complex manifold is called *locally regular* (or *locally strongly regular*) *at a point* $x^0\in\partial G$ if there exists a biholomorphic mapping τ of a neighbourhood U of x^0 into a complex Euclidean space such that $\tau(G\cap U)$ is a regular (or strongly regular) open set. We say that G is *locally regular* (or *locally strongy regular*) if G is locally regular (or locally strongly regular) at each point of ∂G. We say that a boundary point x^0 of an open set G in a differentiable manifold is a *smooth boundary point* of G if there exists a continuously bidifferentiable mapping τ of a neighbourhood U of x^0 into a Euclidean space R^n such that $\tau(x^0)$ is a smooth boundary of $\tau(G\cap U)$. We say that G has a *smooth boundary* if each point of ∂G is a smooth boundary point of G.

PROPOSITION 4. *Let G be a domain with a smooth boundary in a Stein manifold. Then G is a Stein manifold if and only if G is locally regular.*

Proof. If G is a Stein manifold, it is obvious that G is locally regular. Conversely suppose that G is locally regular. Let x^0 be any point of ∂G. Since G is locally regular at x^0, there exists a biholomorphic mapping τ of a neighbourhood U of x^0 into a complex Euclidean space such that $\tau(G\cap U)$ is a regular open set. Since x^0 is a smooth boundary point, there exists a continuously bidifferentiable mapping τ' of a neighbourhood V of x^0 such that $\tau'(x^0)$ is a smooth boundary point of $\tau'(G\cap V)$. Let W be a polycylinder such that $\tau(x^0)\in W\subset\tau(U\cap V)$. Since the continuously bidifferentiable mapping $\tau\circ\tau'^{-1}$ maps $\tau'(\tau^{-1}(W))$ onto W, $\tau(x^0)$ is a smooth boundary point of a regular open set $\tau(G\cap U)\cap W$. From Proposition 2 $\tau(G\cap U)\cap W$ is pseudoconvex at $\tau(x^0)$. Therefore G is pseudoconvex at x^0. From Docquier-Grauert [8] G is a Stein manifold.

We say that a boundary point x^0 of an open set G in a complex manifold is a *continuous boundary point* of G if there exists a biholomorphic mapping τ of a neighbourhood U of x^0 into a complex Euclidean space such that $\tau(x^0)$ is a continuous boundary point of $\tau(G\cap U)$. Moreover, if $\tau(G\cap U)$ is a regular open set simultaneously, x^0 is called a *continuous and locally regular boundary point* of G. We say that G has a *continuous* (or *continuous and locally regular*) *boundary* if each boundary point of G is a continuous (or continuous and locally regular) boundary point of G. These definitions are not so good that a boundary point x^0 of an open set U in a complex Euclidean space C^n may not be a continuous boundary point of U even if x^0 is a continuous boundary point of U which is considered as a subset of a complex manifold C^n and that a boundary point which is continuous and which is locally regular, separately may not be continuous and locally regular.

PROPOSITION 5. *Let G be a domain with a continuous boundary in a Stein manifold. Then G is a Stein manifold if and only if G is locally strongly regular.*

Proof. If G is a Stein manifold, it is obvious that G is locally strongly regular. Conversely suppose that G is locally strongly regular. Let x^0 be any point of ∂G. Since ∂G is continuous at x^0, there exists a biholomorphic mapping τ of a neighbourhood U of x^0 into a complex Euclidean space such that $\tau(x^0)$ is a continuous boundary point of $\tau(G \cap U)$. Since G is locally strongly regular at x^0, there exists a biholomorphic mapping τ' of a neighbourhood V of x^0 into a complex Euclidean space such that $\tau'(G \cap V)$ is a strongly regular open set. Let W be a holomorphically convex neighbourhood of x^0 such that $W \subset U \cap V$. Then $\tau'(G \cap V \cap W)$ is a strongly regular open set. Since the biholomarphic mapping $\tau \circ \tau'^{-1}$ maps $\tau'(G \cap V \cap W)$ onto $\tau(G \cap V \cap W)$, $\tau(G \cap V \cap W)$ is a strongly regular open set. Therefore $\tau(G \cap V \cap W)$ is pseudoconvex at the continuous boundary point $\tau(x^0)$ from Proposition 2. Hence G is pseudoconvex at x^0. From Docquier-Grauert [8] G is a Stein manifold.

PROPOSITION 6. *A domain G with a continuous and locally regular boundary in a Stein manifold is a Stein manifold.*

Proof. Let x^0 be any point of ∂G. Since x^0 is a continuous and locally regular boundary point of G, there exists a biholomorphic mapping τ of a neighbourhood U of x^0 into a complex Euclidean space such that $\tau(x^0)$ is a continuous boundary point of a regular open set $\tau(G \cap U)$. From Proposition 2 $\tau(G \cap U)$ is pseudoconvex at $\tau(x^0)$. Hence G is pseudoconvex at x^0. From Docquier-Grauert [8] G is a Stein manifold.

§ 4. Example.

Let E be a relatively compact open subset with a smooth boundary in a Stein manifold M. Then from Andreotti-Grauert [1] and Fujimoto-Kasahara [9] the canonical homomorphism $H^0(M, \mathfrak{O}) \rightarrow H^0(M-\bar{E}, \mathfrak{O})$ is surjective. Therefore $M-\bar{E}$ is not holomorphically convex. Therefore from Proposition 4, $M-\bar{E}$ is not locally regular at some point of ∂E. Let x^0 be a point of ∂E at which $M-\bar{E}$ is not locally regular. For any neighbourhood U of x^0, there exists a holomorphically convex subdomain D of U such that $(M-\bar{E}) \cap D$ is not a Cousin–I open set. Making use of Andreotti-Grauert [1], we can take E such that $M-\bar{E}$ is a Cousin–I domain. This gives an example of a Cousin–I domain with a smooth boundary which is not locally regular.

PROPOSITION 7. *Let E be a relatively compact open subset of a Stein manifold M. Then there exists an arbitrarily small holomorphically convex subdomain D of M such that $(M-\bar{E}) \cap D$ is not a Cousin*–I *open set.*

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