J. Math. Kyoto Univ. (JMKYAZ)
24-1 (1984) 197-203

On a differentiable map commuting with an elliptic pseudo-differential operator

Dedicated to Professor Hisaaki Yoshizawa on his 60-th birthday

By

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(Received and communicated by Prof. H. Yoshizawa, November 11, 1982)

Introduction.

In [1] it is proved that a differentiable map commuting with a Laplacian must be a Riemannian submersion. As a generalization of this it is noted in [2] that a differentiable map commuting with an elliptic differential operator is a submersion. In this note we generalize them to the case that the operators are elliptic pseudo-differential operators.

Let X and Y be smooth manifolds (connected, without boundary) $\pi: X\rightarrow Y$ a smooth map. Also let $P: C_0^\infty(X)\rightarrow C^\infty(X)$ and $Q: C_0^\infty(Y)\rightarrow C^\infty(Y)$ be elliptic pseudo-differential operators of order $m_1>0$ and $m_2>0$, type(ρ, δ), respectively (for the difinition of pseudo-differential operators see § 1). Denote by $\pi^*: C^\infty(Y)\rightarrow C^\infty(X)$ the map $\pi^*(f)(x)=f(\pi(x))$, $f\in C^\infty(Y)$. We show

Theorem 1. *Assume that one of the following conditions* (i) *or* (ii) *is satisfied:*
(i) *X and Y are compact,*
(ii) *P and Q are properly supported.*
Assume that for all $f\in C_0^\infty(Y)$, $P\circ\pi^*(f)=\pi^*\circ Q(f)$. Then the orders of the operators P and Q coincide with each other and the map π is a submersion.

Denote by $\pi^*(TY)$ the induced bundle of the tangent bundle TY of the manifold Y by the map $\pi: X\rightarrow Y$. As a set $\pi^*(TY)$ is the fiber product of $TY\rightarrow Y$ and $X\xrightarrow{\pi} Y$. Let $d\pi: TX\rightarrow\pi^*(TY)$ be the bundle map (the differential of π) on X and $d\pi^*: \pi^*(T^*Y)\rightarrow T^*X$ the dual bundle map of $d\pi$, where T^*X is the cotangent bundle of X. By the definition of the induced bundle we have the map $p: \pi^*(T^*Y)\rightarrow T^*Y$, that is, the projection $p: T^*Y\times X\rightarrow T^*Y$ restricted to $\pi^*(T^*Y)$.

Corollary 1. *Let X, Y, P and Q be the same as in Theorem* 1. *If P has the homogeneous principal symbol $\sigma_m(P)$, then Q has also the homogeneous principal symbol $\sigma_m(Q)$ (m=order of P=order of Q) and*

$$\sigma_m(Q)\circ p=\sigma_m(P)\circ(d\pi)^* \quad on \quad \pi^*(T^*Y)\backslash X,$$

i.e., on the complement of the zero-section of $\pi^*(T^*Y)$.

Note that for submersions the bundle map $(d\pi)^*: \pi^*(T^*Y)\rightarrow T^*X$ is injective.

Corollary 2. *Let $\pi: X\rightarrow Y$ be a submersion and P and Q pseudo-differential operators of order $m>0$, type (ρ, δ) on X and Y respectively. Assume that Q is elliptic and $P\circ\pi^*=\pi^*\circ Q$, then*

$$Ch(P)\cap(d\pi)^*(\pi^*(T^*Y))=\phi,$$

where $Ch(P)$ is the characteristic set of P.

Corollary 3. *Let π, P and Q be as above, but we assume that P is elliptic. Then Q must be elliptic.*

The proof of these corollaries is included in the proof of Theorem 1.

In §1 we give a review on pseudo-differential operators. Also for more details for pseudo-differential operators see [4] and [5]. In §2 we show Theorem 1 and finally in §3 we give an application of Theorem 1.

§1. Review on pseudo-differential operators.

Let U be an open set in $\boldsymbol{R}^n$ and denote by $S^m_{\rho,\delta}(U\times\boldsymbol{R}^N)(N\geqq 1)$ the set of functions $a(x, \theta)\in C^\infty(X\times\boldsymbol{R}^N)$ such that for every multi-indices α, β the derivative $D^\beta_x D^\alpha_\theta(a(x, \theta))$ satisfies

$$\sup_{(x,\theta)\in K\times\boldsymbol{R}^N}\frac{|D^\beta_x D^\alpha_\theta a(x, \theta)|}{(1+|\theta|)^{m-\rho|\alpha|+\delta|\beta|}}<+\infty,$$

where K is an arbitrary compact set in U, and ρ and δ satisfies the inequalities $0\leqq 1-\rho\leqq\delta<\rho\leqq 1$, and m is a real number. Let $L^m_{\rho,\delta}(U)$ be the set of operators of the form:

$$C^\infty_0(U)\ni u\longmapsto\iint_{U\times\boldsymbol{R}^n}e^{i<x-y,\theta>}a(x, y, \theta)u(y)dyd\theta\in C^\infty(U),$$

with $a(x, y, \theta)\in S^m_{\rho,\delta}(U\times U\times\boldsymbol{R}^n)$.

The operator $P\in L^m_{\rho,\delta}(U)$ is said to be a pseudo-differential operator of order m, type (ρ, δ).

For $P\in L^m_{\rho,\delta}(U)$ we denote by $K_P\in\mathcal{D}'(U\times U)$ the kernel distribution corresponding to the operator P, that is,

$$\langle K_P, u\rangle=\iiint e^{i<x-y,\theta>}a(x, y, \theta)u(x, y)dxdyd\theta,\ u\in C^\infty_0(U\times U).$$

Let $P\in L^m_{\rho,\delta}(U)$ be such that the corresponding kernel distribution K_P has the following property:

> the projections p_1 and $p_2(p_1(x, y)=x,\ p_2(x, y)=y)$ are proper, if both maps are restricted to the support of K_P.

In this case the operator P is called, properly supported. A properly supported operator $P\in L^m_{\rho,\delta}(U)$ is represented in the form $\iint e^{i<x-y,\theta>}a(x, y, \theta)u(y)dyd\theta$ with such an $a\in S^m_{\rho,\delta}(U\times U\times \boldsymbol{R}^n)$ that the projections p_1 and p_2 restricted to the set $\{(x, y)\,;\, 0\neq {}^\exists\theta\in \boldsymbol{R}^n,\ (x, y, \theta)\in \mathrm{supp}[a]\}$ are proper. Hence properly supported operators can act on the space $C^\infty(U)$.

Also a properly supported pseudo-differential operator $P\in L^m_{\rho,\delta}(U)$ can be written in the following form:

let $\tau_P(x, \theta)=e^{-i<x,\theta>}P(e^{i<\cdot,\theta>})(x)$, then

$P(u)(x)=(2\pi)^{-n}\int e^{i<x,\theta>}\tau_P(x, \theta)\hat{u}(\theta)\mathrm{d}\theta$,

where $\hat{u}(\theta)=\int_{\boldsymbol{R}^n} e^{-i<x,\theta>}u(x)\mathrm{d}x$.

The function τ_P is called the total symbol of P, belongs to the class $S^m_{\rho,\delta}(U\times \boldsymbol{R}^n)$ and has the following asymptotic expansion:

$$\tau_P(x, \theta)\sim(2\pi)^n \sum (iD_\theta)^\alpha D^\alpha_y a(x, y, \theta)_{|y=x}/\alpha\,!,$$

where $D_y=(-i)\dfrac{\partial}{\partial y}$, and $\sim$ means that

$$\tau_P(x, \theta)-(2\pi)^n \sum_{|\alpha|<N}((iD_\theta)^\alpha D^\alpha_y a)_{|y=x}/\alpha\,!\in S^{m-\rho N+\delta N}_{\rho,\delta}(U\times \boldsymbol{R}^n)$$

for all integers N.

Every $P\in L^m_{\rho,\delta}(U)$ can be written in the form $P=P_c+P_s$, where $P_c\in L^m_{\rho,\delta}(U)$ is properly supported and P_s has smooth kernel. The correspondence $P\mapsto \tau_{P_c}$ (we denote τ_{P_c} simply by τ_P) defines the isomorphism:

$$\tau:\ L^m_{\rho,\delta}(U)/L^{-\infty}(U)\cong S^m_{\rho,\delta}(U\times \boldsymbol{R}^n)/S^{-\infty}(U\times \boldsymbol{R}^n),$$

where $S^{-\infty}(U\times \boldsymbol{R}^n)=\bigcap_{m\in \boldsymbol{R}} S^m_{\rho,\delta}(U\times \boldsymbol{R}^n)=\bigcap_{m\in \boldsymbol{R}} S^m_{1,0}(U\times \boldsymbol{R}^n)$ and $L^{-\infty}(U)=\bigcap_{m\in \boldsymbol{R}} L^m_{\rho,\delta}(U)=\bigcap_{m\in \boldsymbol{R}} L^m_{1,0}(U)$ is the space of all operators with smooth kernel. For a general $P\in L^m_{\rho,\delta}(U)$ we call a function $\tau_P(\mathrm{mod}\ S^{-\infty}(U\times \boldsymbol{R}^n))$ a total symbol of P.

Owing to the condition for ρ and $\delta(0\leqq 1-\rho\leqq\delta<\rho\leqq 1)$ we can define the space $S^m_{\rho,\delta}(T^*X)$ for a manifold X and also the space of pseudo-differential operators on X, that is, a continuous linear operator P: $C^\infty_0(X)\to C^\infty(X)$ belongs to $L^m_{\rho,\delta}(X)$, if and only if, on each coordinate neighborhood $U\ni x=(x_1, \cdots, x_n)$

$$e^{-i<x,\theta>}P(u\cdot e^{i<\cdot,\theta>})(x)\in S^m_{\rho,\delta}(U\times \boldsymbol{R}^n)\ (n=\dim X),$$

where u runs over $C^\infty_0(U)$ and $\langle x, \theta\rangle=\sum x_i\theta_i$ (see [4] Theorem 2.16, p. 151).

The total symbol cannot be invariantly defined in this case, but the correspondence $P\mapsto\tau_P$ (in each local coordinate) defines the isomorphism:

$$\sigma:\ L^m_{\rho,\delta}(X)/L^{m-\rho+\delta}_{\rho,\delta}(X)\cong S^m_{\rho,\delta}(T^*X)/S^{m-\rho+\delta}_{\rho,\delta}(T^*X).$$

A function $\sigma(P)\ \mathrm{mod}(S^{m-\rho+\delta}_{\rho,\delta}(T^*X))$ for $P\in L^m_{\rho,\delta}(X)$ is said to be a principal symbol of P. If there exists a limit $\lim_{t\to\infty}\sigma(P)(x, t\theta)/t^m$, $0\neq\theta\in T^*_xX$, this limit is denoted by $\sigma_m(P)$ and said to be the homogeneous principal symbol of P. This is a globally defined smooth function on $T^*X\backslash X$ and homogeneous of degree m.

Definition 1. Let P be in $L^m_{\rho,\delta}(X)$. P is said to be elliptic if for every non-zero cotangent vector $(x, \theta)\in T^*_xX$

$$\varliminf_{t\to\infty}|\sigma(P)(x, t\theta)|/t^m\neq 0.$$

Of course this definition does not depend on the choice of a principal symbol and if P has the homgeneous principal symbol $\sigma_m(P)$ then the ellipticity of P is equivalent to say that $\sigma_m(P)$ never vanishes on $T^*X\backslash X$.

Definition 2. For $P\in L^m_{\rho,\delta}(X)$ we say the set $Ch(P)=\{(x, \theta)\in T^*X;\ \theta\neq 0, \varliminf_{t\to\infty}|\sigma(P)(x, t\theta)|/t^m=0\}$ the characteristic set of P.

§2. Proof of Theorem 1.

We can apply $P\circ\pi^*$ to all $f\in C^\infty_0(Y)$, if one of the conditions (i) or (ii) in the statement of Theorem 1 is satisfied.

Let $x=(x_1, \cdots, x_n)$ and $y=(y_1, \cdots, y_p)$ be local coordinate systems on $U\subset X$ and $V\subset Y$ such that $\pi^{-1}(V)\supset U$. Let (x, θ) and (y, η) be the corresponding coordinate systems on T^*U and T^*V respectively.

Let $\phi_\eta(y)\in C^\infty(V)$ be $\phi_\eta(y)=\langle y, \eta\rangle=\sum y_i\eta_i$. Then $\mathrm{d}(\pi^*\phi_\eta)_x=(\mathrm{d}\pi)^*_x(\sum\eta_i\mathrm{d}y_i)$, where $\mathrm{d}\pi_x:\ T_xX\to T_{\pi(x)}Y$, and $(\mathrm{d}\pi)^*_x:\ T^*_{\pi(x)}Y\to T^*_xX$.

Let $f\in C^\infty_0(V)$ and $g\in C^\infty_0(U)$ be such that $f\equiv 1$ on a neighborhood of $y_0\in V$ and $g\equiv 1$ on a neighborhood of $x_0\in U$ and $\pi(x_0)=y_0$. The supports of f and g are taken to be sufficiently small, if necessary.

With these notations we show Theorem 1.

Step 1. If $\mathrm{d}\pi\equiv 0$ for all $x\in X$, that is, π is a constant mapping, then

$$\begin{aligned}&e^{-i\langle\pi(x_0),\eta\rangle}Q(f\cdot e^{i\phi_\eta})(\pi(x_0))\\&\quad=e^{-i\langle\pi(x_0),\eta\rangle}P(\pi^*(f)\cdot\pi^*(e^{i\phi_\eta}))(x_0)=P(1)(x_0).\end{aligned}$$

By the ellipticity of Q and $m_2>0$, we have

$$0\neq\varliminf_{t\to\infty}|Q(f\cdot e^{it\phi_\eta})(\pi(x_0))|/t^{m_2}=\varliminf_{t\to\infty}|P(1)(x_0)|/t^{m_2}=0,$$

and this is a contradiction. Hence π is not a constant mapping.

Step 2. Assume that $(\mathrm{d}\pi)^*_{x_0}(\sum\eta_i\mathrm{d}y_i)\neq 0$, and $\pi(x_0)=y_0$. We have an equality:

$$\begin{aligned}&\pi^*(e^{-i\phi_\eta})Q(f\cdot e^{i\phi_\eta})(x_0)\\&\quad=e^{-i\pi^*(\phi_\eta)}P(\pi^*(f)\cdot\pi^*(e^{i\phi_\eta}))(x_0)\\&\quad=e^{-i\pi^*(\phi_\eta)}P((1-g)\cdot\pi^*(f)\cdot\pi^*(e^{i\phi_\eta}))(x_0)\\&\qquad+e^{-i\pi^*(\phi_\eta)}P(g\cdot\pi^*(f)\cdot\pi^*(e^{i\phi_\eta}))(x_0)\end{aligned}$$

Let $K_P(x, z)$ be the kernel distribution of the operator P, then we may write

$$\begin{aligned}&P((1-g)\cdot\pi^*(f)\cdot\pi^*(e^{i\phi_\eta}))(x_0)\\&\quad=\int_X K_P(x_0, z)\cdot(1-g)(z)\cdot\pi^*(f)(z)\cdot\pi^*(e^{i\phi_\eta})(z)\mathrm{d}z.\end{aligned}$$

By the reason that the singular support of the distribution K_P is contained in the diagonal of $X\times X$, the above integral can be taken in the usual sense, if we take the supports of the functions f and g suitably. Hence we have

$$\lim_{t\to\infty} e^{-it\phi_\eta(\pi(x_0))}\cdot P((1-g)\cdot\pi^*(f)\cdot\pi^*(e^{it\phi_\eta}))(x_0)/t^{m_1}=0.$$

Since $d\pi^*(\phi_\eta)_{x_0}\neq 0$, we can take $\pi^*(\phi_\eta)$ as a first coordinate in a neighborhood of x_0 by a coordinate change.

From these considerations we have

$$\varliminf_{t\to\infty}|P(g\cdot\pi^*(f)\cdot\pi^*(e^{it\phi_\eta}))(x_0)|/t^{m_1}$$

$$=\varliminf_{t\to\infty}|(\pi^*e^{-it\phi_\eta})(x_0)Q(f\cdot e^{it\phi_\eta})(x_0)|/t^{m_1}\neq 0.$$

Hence $m_1=m_2$. In the following we put $m_1=m_2=m$.

Slep 3. We may assume $x_0=0$ and $\pi(x_0)=y_0=0$. We denote $g\cdot\pi^*(f)$ simply by u, and $\pi^*(\phi_\eta)$ by ϕ. Assume that $d\phi_{x_0}=0$, i.e., there exists an $\eta=\sum\eta_i dy_i \in T^*_{y_0}Y$, $\eta\neq 0$ such that $(d\pi)^*_{x_0}(\eta)=0$. Then we may write $\phi(x)=\sum h_{ij}(x)x_ix_j$, where h_{ij} are smooth functions defined in a neighborhood U_0 of x_0.

Let k_0 and N be integers such that $mk_0>(n+2\rho)/(2\rho-1)$ and $(mk_0+n)/\rho< 2N\leqq(mk_0+n)/\rho+2$.

Instead of P and Q, assume that we are given P^{k_0} and Q^{k_0}, then all the arguments above are valid. Hence from the beginning we can suppose that the order of $P(=m_0)>(n+2\rho)/(2\rho-1)$.

Put $e^{it\phi}-\sum_{0\leq n<N}(it\phi)^n/n\,!=(it\phi)^N A_N(z,t)$, then

$$P(u\cdot e^{it\phi})(x_0)$$

$$=\sum_{n=0}^{N-1}\iint_{U_0\times R^n} e^{-i<z,\theta>}(it\phi(z))^n/n\,!\cdot a(x_0,z,\theta)u(z)dzd\theta$$

$$+(it)^N\iint_{U_0\times R^n} e^{-i<z,\theta>}(\sum_{i,j}h_{ij}(z)z_iz_j)^N A_N(z,t)a(x_0,z,\theta)u(z)dzd\theta$$

$$=\sum_{n<N}c_nt^n$$

$$+(-it)^N\iint e^{-i<z,\theta>}u(z)\cdot A_N(z,t)\Big(\sum h_{ij}\frac{\partial^2}{\partial\theta_i\partial\theta_j}\Big)^N a(x_0,z,\theta)dzd\theta,$$

where $a(x,z,\theta)\in S^{m_0}_{\rho,\delta}(U_0\times U_0\times \boldsymbol{R}^n)$.

Since $m_0-2\rho N+n<0$,

$$\Big|\iint e^{-i<z,\theta>}u(z)\cdot A_N(z,t)\Big(\sum h_{ij}\frac{\partial^2}{\partial\theta_i\partial\theta_j}\Big)^N a(x_0,z,\theta)dzd\theta\Big|$$

$$\leqq C_1\iint|u(z)A_N(z,t)(1+|\theta|)^{m_0-2\rho N}|dzd\theta$$

$$\leqq C_2\int(1+|\theta|)^{m_0-2\rho N}d\theta<+\infty,$$

where we use the inequality: $|A_N(z,t)|\leqq 1/N!$. Hence we have

$$P(u\cdot e^{it\phi})(x_0)=O(t^N).$$

On the other hand, $2(N-m_0)<(m_0+n)/\rho+2-2m_0=(m_0(1-2\rho)+2\rho+n)/\rho<0$, because $m_0>(n+2\rho)/(2\rho-1)$. Therefore we have

$$\lim_{t\to\infty} P(u\cdot e^{it\phi})(x_0)/t^{m_0}=0.$$

This contradicts the ellipticity of the operator Q. Here note that the order of Q is m_0=order of P. Consequently there exists no $\eta\in T^*Y$ such that $\eta\neq 0$ and $(d\pi)^*(\eta)=0$, which shows the map π is a submersion.

Remark. If the order of $P>(n+2\rho)/(2\rho-1)$ from the beginning, then we may assume

(iii) π is a proper mapping, instead of the assumption (i) or (ii) in the statement of Theorem 1. Because in this case we need not take the iteration P^{k_0} of P in the above proof.

§ 3. An application.

In this section we give an application of Theorem 1.

Let X be a compact Riemannian manifold and Δ its Laplace operator. Then Δ is a second order elliptic differential operator and its spectrum consists only of isolated positive eigenvalues with finite multiplicities. Let $\sum=\{0=\lambda_0<\lambda_1\leqq\lambda_2\leqq\lambda_3\leqq\cdots\leqq\lambda_n\leqq\cdots\}$ be the eigenvalues of Δ counted with multiplicities. The following asymptotic property of eigenvalues is fundamental for this section (for the proof see [6], p. 305):

$$\lambda_j=cj^{2/n}+o(j^{2/n}),\quad n=\dim X \quad\text{and}\quad c=\mathrm{Vol}(X).$$

For a complex number s, the complex power $\Delta^s=\int_0^\infty\lambda^s dE_\lambda$ is defined by the spectral resolution of $\Delta=\int_0^\infty\lambda dE_\lambda$, and we know by the result in [3] that this operator is also a pseudo-differential operator in $L^m_{1,0}(X)$, where $m=2\cdot\boldsymbol{Re}(s)$.

Theorem 2. *Let X and Y be compact Riemannian manifolds, Δ_X and Δ_Y Laplace operators on X and Y respectively. Assume that there exist a smooth map $\pi: X\to Y$ and complex numbers s_1 and s_2 such that $\boldsymbol{Re}(s_i)>0$ $i=1, 2$ and $\Delta_X^{s_1}\circ\pi^*=\pi^*\circ\Delta_Y^{s_2}$, then $s_1=s_2$, $\Delta_X\circ\pi^*=\pi^*\circ\Delta_Y$ and π is a Riemannian submersion.*

Proof. Put $s_i=\sigma_i+\sqrt{-1}\,\tau_i$, σ_i, τ_i: real, $i=1, 2$. From Theorem 1 we know at once $\sigma_1=\sigma_2$, so we show $\tau_1=\tau_2$.

Let $u\in C^\infty(Y)$ be an eigenfunction of Δ_Y: $\Delta_Y u=\lambda u$, $\lambda>0$, $u\neq 0$. Then there exists a positive number μ such that

$$\pi^*\circ\Delta_Y^{s_2}(u)=\pi^*(\lambda^{s_2}u)=\Delta_X^{s_1}\circ\pi^*(u)=\mu^{s_1}\pi^*(u).$$

Hence we see that $\mu^{s_1}=\lambda^{s_2}$. Therefore $\mu=\lambda$ and for every eigenvalue λ_k of Δ_Y we have $\lambda_k^{i\tau_1}=\lambda_k^{i\tau_2}$.

Suppose that $\tau_1 \neq \tau_2$, then $(|\tau_1-\tau_2|/2\pi)\cdot\log\lambda_k = n_k$ must be a positive integer for every sufficiently large eigenvalue λ_k. Take a sequence $\{a(l)\}_{l=1}^{\infty}$ of integers such that $n_{a(l)}-n_{a(l)-1} \geqq 1$, then

$$1 \leqq n_{a(l)}-n_{a(l)-1} = \frac{|\tau_1-\tau_2|}{2\pi}\cdot\log(\lambda_{a(l)}/\lambda_{a(l)-1})$$

$$= \frac{|\tau_1-\tau_2|}{2\pi}\cdot\log\left(\frac{c\cdot a(l)^{2/m}+c_{a(l)}\cdot a(l)^{2/m}}{c(a(l)-1)^{2/m}+c_{a(l)-1}\cdot(a(l)-1)^{2/m}}\right) \longrightarrow 0,$$

as $a(l)\to\infty$, where we put $\lambda_k = ck^{2/m}+c_k k^{2/m}$, $c_k \to 0\,(k\to\infty)$, $m=\dim Y$. Hence τ_1 must be equal to τ_2. So we have $\Delta_X^s \circ \pi^* = \pi^* \circ \Delta_Y^s\,(s=s_1=s_2)$, which implies simultaneously $\Delta_X \circ \pi^* = \pi^* \circ \Delta_Y$, because both operators Δ_X and Δ_Y are positive definite. The rest of the proof follows from the result in [1].

Corollary 4. *If $X=Y$ in Theorem 2, then the map $\pi: X\to Y$ is an isometry.*

Proof. It is enough to show that the map π is injective. For the proof of this see [2, Theorem 1].

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