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Foliations and foliated cobordisms of spheres in codimension one

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§ 0. Introduction.

We have shown in [**21**], that there is a codimension one foliation on each $(4k+3)$-dimensional sphere, which is foliated cobordant to zero. The main purpose of the present paper is to prove the following theorem:

THEOREM. *On each* $(4k+1)$*-dimensional homotopy sphere, there exists a codimension one foliation which is not foliated cobordant to zero but twice of which is foliated cobordant to zero.*

We shall prove this in Section 3 (Theorem 2).

Most of the codimension one foliations of spheres so far known, are ones which are constructed from spinnable structures of spheres [**4**], [**9**], [**16**].**) Thus nice extensions of spinnable structures mean foliated cobordisms of foliations of spheres. In fact, we can construct null-cobordisms of codimension one foliations of S^3 and S^7 in this way [**21**]. From this view point, it is an interesting problem to ask when two spinnable structures are "spinnable cobordant". Concerning this problem, we shall prove "Relative Spinnable Structure Theorem" in the Appendix, which is a generalization of Tamura [**17**] and Winkelnkemper [**24**].

In Section 1, we shall state some basic definitions and notations.

In Section 2, we shall construct a spinnable structure of S^{4n+1} $(n \geqq 2)$ with axis $S^{2n-1} \times S^{2n}$ which is slightly different from Tamura's construction [**16**].

In Section 4, we obtain a codimension one foliation of S^5 with a single compact leaf which is diffeomorphic to $T^2 \times S^2$. This leads us to new foliations of higher dimensional spheres and highly connected manifolds.

Throughout the paper, foliations will be smooth, of codimension one and transversely orientable unless otherwise stated.

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§ 1. Definitions and notations.

Let $R^n=\{(x_1, x_2, \cdots, x_n)\in R\times\cdots\times R\}$ be an n-dimensional Euclidean space with standard codimension one foliation whose leaves are defined by $x_n=$ constant. Given a smooth manifold M^n without boundary, a codimension one *foliation of* M^n is defined to be a maximal set of charts

$$\{(U_\lambda, h_\lambda),\ U_\lambda \text{ is open in } M^n,\ h_\lambda: U_\lambda\to R^n,\ \lambda\in\Lambda\}$$

of M^n such that

$$h_\lambda\circ h_\mu^{-1}: h_\mu(U_\lambda\cap U_\mu)\to h_\lambda(U_\lambda\cap U_\mu)$$

preserves the leaves of foliations which are induced on $h_\mu(U_\lambda\cap U_\mu)$ and $h_\lambda(U_\lambda\cap U_\mu)$ from that of R^n. Similarly, if M has a boundary, *a codimension one foliation of M tangent to the boundary* is defined by using a half space $H^n=\{(x_1, x_2, \cdots, x_n)\in R^n,\ x_n\geqq 0\}$ with a standard foliation whose leaves are defined by $x_n=$ constant. Also, *a codimension one foliation of M transverse to the boundary* is defined by uisng H^n with a standard foliation whose leaves are defined by $x_{n-1}=$constant. More generally, we shall consider *foliations of a manifold with corner*. In this case, a codimension one foliation of M is defined to be a maximal set of charts of M modelled on a quadrant

$$Q^n=\{(x_1, x_2, \cdots, x_{n-1}, x_n)\in R^n,\ x_{n-1}\geqq 0,\ x_n\geqq 0\}$$

with a standard foliation defined by $x_{n-1}=$constant, such that the coordinate transformations preserve the leaves of this foliation of Q^n.

If M is a foliated manifold, we denote by $(M, \mathcal{F})$ the oriented diffeomorphism class of a foliation of M. $\mathcal{F}$ stands for the set of all the leaves of the foliation of M. Thus, two foliations $(M_0, \mathcal{F}_0)$ and $(M_1, \mathcal{F}_1)$ are identified if and only if there exists an orientation preserving diffeomorphism $h: M_0\to M_1$, which maps each leaf of $\mathcal{F}_0$ into a leaf of $\mathcal{F}_1$. By $-(M, \mathcal{F})$, we mean the same foliation as $(M, \mathcal{F})$ such that only the orientation of the underlying manifold is reversed, i.e., $-(M, \mathcal{F})=(-M, \mathcal{F})$.

DEFINITION 1. Two foliations of closed manifolds $(M_0, \mathcal{F}_0)$ and $(M_1, \mathcal{F}_1)$ are called *foliated cobordant* if there exists a foliation of a compact manifold $(W, \mathcal{F})$ which is transverse to the boundary such that $\partial W=M_0\cup -M_1$ and $\mathcal{F}|M_0=\mathcal{F}_0$, $\mathcal{F}|-M_1=\mathcal{F}_1$, in short, $\partial(W, \mathcal{F})=(M_0, \mathcal{F}_0)-(M_1, \mathcal{F}_1)$.

A spinnable structure of a closed manifold was defined in Tamura [**17**] and Winkelnkemper [**24**]. We extend the definition to a manifold with boundary.

DEFINITION 2. A smooth manifold W is said to have a *spinnable structure* if,

(1) There exists a codimension two submanifold A of W, having the trivial

normal bundle, which we call the *axis*.

(2) Let $A\times D^2$ denote the tubular neighbourhood of A, then $W-A\times \mathrm{Int}\, D^2$ has a structure of a smooth fibre bundle over the circle. We call this bundle the *spinning bundle* and the fibre of this bundle the *generator* of the spinnable structure.

(3) The following diagram commutes:

$$\begin{array}{ccc} W-A\times \mathrm{Int}\, D^2 & \xleftarrow{\ \iota\ } & A\times S^1=A\times\partial D^2 \\ & {\scriptstyle p}\searrow \qquad \swarrow{\scriptstyle pr_2} & \\ & S^1 & \end{array}$$

where ι is an inclusion map, pr_2 is a projection onto the second factor and p is the bundle projection of the spinning bundle.

This definition is equivalent to the following:

Let (F, A) be a pair of manifolds such that A is a submanifold of ∂F of the same dimension. Then a spinnable structure is a pair $\{h, (F, A)\}$ where h is a diffeomorphism of the pair $h:(F, A)\to(F, A)$ such that $h|A=\mathrm{id}_A$.

To obtain W from $\{h, (F, A)\}$, one has only to consider the mapping torus $M(h)$ of h and define $W=M(h)\cup A\times D^2$ where the identification is an obvious one.

The following lemma is useful for construction of codimension one foliations.

LEMMA 1. *Let $\{h, (F, A)\}$ be a spinnable structure and let Q and Q' be the mapping tori of h and $h|A$ respectively. We consider Q is a manifold having corners along $\partial Q'$. Then Q has a codimension one foliation which satisfies*

(1) *Q' is a union of leaves of the foliation.*

(2) *The other leaves of the foliation are transverse to $\partial Q-Q'$ and they are diffeomorphic to $Q-Q'$.*

PROOF. Consider a (relative) collar neighbourhood of Q' and identify Q with $Q\cup Q'\times[0, 1]$ where $Q'\subset Q$ and $Q'\times\{0\}$ are identified. On $S^1\times[0, 1]$, there exists a foliation $\mathcal{V}$ which satisfies: (a) the leaves of $\mathcal{V}$ are the trajectories of a vectorfield. (b) $S^1\times\{1\}$ is a leaf. (c) the leaves of $\mathcal{V}$ intersect normally with $S^1\times\{0\}$.

Let $p: Q\to S^1$ be the bundle projection map, then the fibres of p and the pull-back of $\mathcal{V}$ under the projection, $p|_{Q'}\times\mathrm{id}: Q'\times[0, 1]\to S^1\times[0, 1]$ define a foliation of $Q\cup Q'\times[0, 1]=Q$. It is easily verified that this foliation satisfies the conditions (1), (2) of Lemma 1.

§2. A construction of a spinnable structure of S^{4n+1} ($n \geqq 2$) with axis $S^{n-1} \times S^n$.

I. Tamura **[16]** constructed a spinnable structure of S^{4n+1} ($n \geqq 2$) with axis $S^{n-1} \times S^n$ and used it to prove that every odd dimensional sphere has a foliation. In this section, we shall construct such a spinnable structure of S^{4n+1}, whose generator is a simpler manifold.

First, we review briefly Tamura's construction.

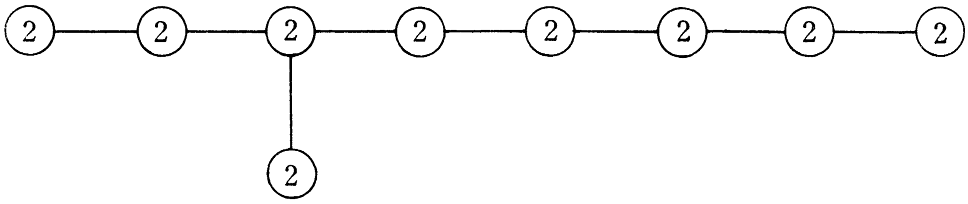
Decompose S^{4n+1} as follows:

$$S^{4n+1} = (S_1^{2n} \times D_1^{2n+1} \natural \cdots \natural S_{17}^{2n} \times D_{17}^{2n+1}) \cup (D_1^{2n+1} \times S_1^{2n} \natural \cdots \natural D_{17}^{2n+1} \times S_{17}^{2n}),$$

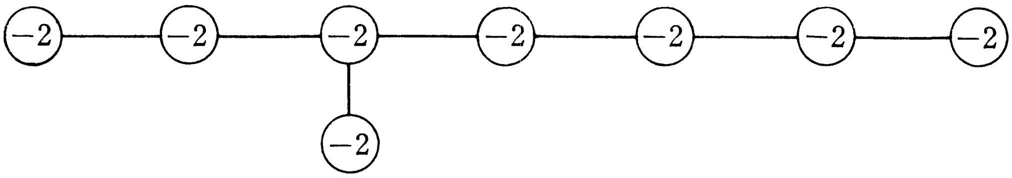
where the linking numbers $Lk(S_i^{2n} \times (0), (0) \times S_i^{2n}) = 1$ for $i = 1, \cdots, 17$, and other linking numbers of $S_i^{2n} \times (0)$'s and $(0) \times S_j^{2n}$'s are all zero. ((0) denotes the center of D^{2n+1}.)

Let $N(\Delta_i)$ denote a tubular neighbourhood of the diagonal of $S_i^{2n} \times \partial D_i^{2n+1}$ and $N(\bar{\Delta}_i)$ denote a tubular neighbourhood of 'anti-diagonal' of $S_i^{2n} \times \partial D_i^{2n+1}$ that is, $N(\bar{\Delta}_i)$ is a tubular neighbourhood of $S_i^{2n} \# (-\partial D_i^{2n+1})$ in $S_i^{2n} \times \partial D_i^{2n+1}$. The self-intersection number of Δ_i (resp. $\bar{\Delta}_i$) is equal to 2 (resp. -2).

Denote by E_9 the tree manifold which is obtained by making plumbings of $N(\Delta_1), \cdots, N(\Delta_9)$ according to the diagram;



and denote by $-E_8$ the tree manifold which is obtained from $N(\bar{\Delta}_{10}), \cdots, N(\bar{\Delta}_{17})$, in the same way, according to the diagram;



Performing all these plumbings in the boundary of $S_1^{2n} \times D_1^{2n+1} \natural \cdots \natural S_{17}^{2n} \times D_{17}^{2n+1}$, we may consider $E_9 \natural (-E_8)$ is a submanifold of $\partial(S_1^{2n} \times D_1^{2n+1} \natural \cdots \natural S_{17}^{2n} \times D_{17}^{2n+1}) = \partial(D_1^{2n+1} \times S_1^{2n} \natural \cdots \natural D_{17}^{2n+1} \times S_{17}^{2n})$.

The inclusion maps;

$$E_9 \natural (-E_8) \rightarrow S_1^{2n} \times D_1^{2n+1} \natural \cdots \natural S_{17}^{2n} \times D_{17}^{2n+1}$$

$$E_9 \natural (-E_8) \rightarrow D_1^{2n+1} \times S_1^{2n} \natural \cdots \natural D_{17}^{2n+1} \times S_{17}^{2n}$$

are verified to be homotopy equivalences. Therefore, by (relative) h-cobordism theorem [**15**], we have,

$$S^{4n+1}=(E_9\natural(-E_8))\times I\cup(E_9\natural(-E_8))\times I\,,$$

and consequently, we obtain a spinnable structure of S^{4n+1} with $E_9\natural(-E_8)$ as generator. The axis or the boundary of $E_9\natural(-E_8)$ is proved to be diffeomorphic to $S^{n-1}\times S^n$.

This is what Tamura constructed. See [**19**] for more details.

On the other hand, M. Kato [8] proved; to a unimodular integral matrix, there corresponds a spinnable structure of S^{2n+1} $(n\geqq 3)$. He called this matrix "*Seifert matrix*" of the spinnable structure. See also K. Sakamoto [**14**].

If we use his theorem, we have a very simple spinnable structure of S^{4n+1} with axis $S^{2n-1}\times S^{2n}$. Namely, we take $\begin{pmatrix}1&-1&0\\0&0&1\\0&-1&0\end{pmatrix}$ as a Seifert matrix. According to Kato, the rank of $H_{2n}(F,Z)$ of the generator F of the corresponding spinnable structure is equal to 3 and the intersection matrix of F is $\begin{pmatrix}2&-1&0\\-1&0&0\\0&0&0\end{pmatrix}$.

To prove $\partial F=S^{2n-1}\times S^{2n}$, we re-construct such a spinnable structure more geometrically.

Let $S^{4n+1}=W_0\cup W_1$,

$$W_0=S_1^{2n}\times D_1^{2n+1}\natural S_2^{2n}\times D_2^{2n+1}\natural S_3^{2n}\times D_3^{2n+1}$$

$$W_1=D_1^{2n+1}\times S_1^{2n}\natural D_2^{2n+1}\times S_2^{2n}\natural D_3^{2n+1}\times S_3^{2n}$$

be a decomposition of S^{4n+1}, which satisfies, $Lk(S_1^{2n}\times(0),S_2^{2n}\times(0))=1$, $Lk(S_i^{2n}\times(0),S_j^{2n}\times(0))=0$ for $(i,j)\neq(1,2),(2,1)$ and $Lk(S_i^{2n}\times(0),(0)\times S_j^{2n})=\delta_{ij}$ for $i,j=1,2,3$.

Instead of $E_9\natural(-E_8)$ in Tamura's construction, we take a submanifold F_3 of $\partial W_0=\partial W_1$ as follows.

Let A_i denote the sphere $S_i^{2n}\times(*)$ in W_0, where $(*)$ stands for a point in ∂D_i^{2n+1}, and let $N(A_i)$ be its tubular neighbourhood in ∂W_0.

Define

$$F_3=N(A_1)\natural N(A_2)\veebar N(\varDelta_3)\,,$$

where we have taken A_1 and A_2 so that they link once each other in S^{4n+1}, $X\veebar Y$ is a plumbing of disk bundles X and Y, and $\varDelta_3$ stands for a diagonal sphere in $S_3^{2n}\times D_3^{2n+1}$ as before.

Again performing connected sum and plumbing in $\partial W_0=\partial W_1$, we may consider F_3 is a submanifold of $\partial W_0=\partial W_1$ (Fig. 1).

It is easily verified that F_3 is simply connected and the homomorphisms, $H_*(F_3,Z)\to H_*(W_0,Z)$ and $H_*(F_3,Z)\to H_*(W_1,Z)$ which are induced by inclusion maps are isomorphisms. Since $n \geqq 2$, by h-cobordism theorem,

$$W_0 = F_3 \times I \quad \text{and} \quad W_1 = F_3 \times I .$$

From these, we have a spinnable structure of S^{4n+1}, whose generator is F_3. Clearly,

$$\partial F_3 = \partial(N(A_1)) \# \partial(N(A_2) \veebar N(\Delta_3)) = \partial N(A_1) \# S^{4n-1} = S^{2n} \times S^{2n-1} .$$

This finishes our construction.

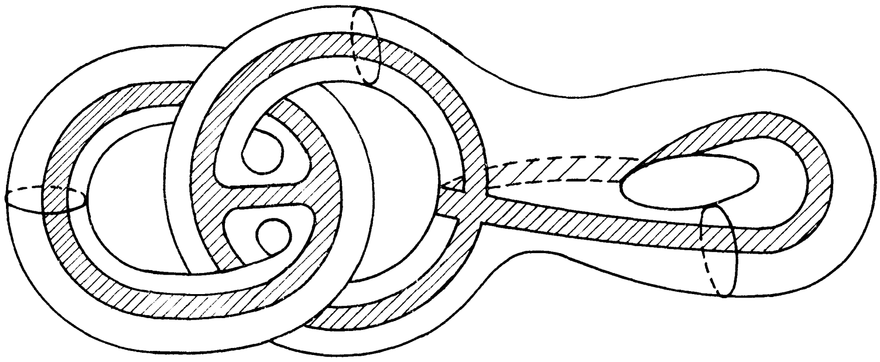


Fig. 1.

For $n=1$, the above argument can not be used since we can not use h-cobordism theorem. We shall prove however, the theorem in case $n=1$ in the next section.

For $(4n+3)$-dimensional spheres, the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ available and by the same method as above, we can construct a spinnable structure of S^{4n+3} whose axis is $S^{2n} \times S^{2n+1}$ for each $n \geqq 1$. For $n=0$, the assertion is obvious. Thus we have,

THEOREM 1 (Tamura [**16**, **19**]). *S^{2n+1} has a spinnable structure with $S^{n-1} \times S^n$ as axis ($n \geqq 0$). Further, for $n \geqq 1$, we can choose a generator to be diffeomorphic to $S^n \times D^n \natural S^n \times D^n \veebar \tau_D(S^n)$, where $\tau_D(S^n)$ denotes a tangent disk bundle of S^n.*

COROLLARY. *$S^{2n-1} \times D^2$ ($n \geqq 1$) has a spinnable structure with axis $S^{n-1} \times S^n$ which is an extension of the obvious spinnable structure of $S^{2n-1} \times \partial D^2$ i.e. the bundle $S^{2n-1} \times \partial D^2 \to \partial D^2 = S^1$.*

PROOF. Let h be the diffeomorphism of $S^n \times D^n \natural S^n \times D^n \veebar \tau_D(S^n)$ which defines the spinnable structure of S^{2n+1}. We can assume h is an identity on a small disk D^{2n} which is contained $\mathrm{Int}(S^n \times D^n \natural S^n \times D^n \veebar \tau_D(S^n))$. Deleting the subbundle $S^1 \times D^{2n}$ from the mapping torus of h, we obtain a desired spinnable structure.

§ 3. Main Theorem.

In this section, we shall prove our main theorem.

THEOREM 2. *On every* $(4n+1)$*-dimensional homotopy sphere* $(n \geqq 0)$, *there exists a codimension one foliation which is not foliated cobordant to zero but twice of which is foliated cobordant to zero.*

For $n=0$, the theorem is easily proved and so from now on we always assume $n \geqq 1$.

First we remark that no foliations on a $(4n+1)$-dimensional homotopy sphere are foliated cobordant to zero. In fact, suppose that a codimension one foliation on S^{4n+1} extended to one of $W^{4n+2}(\partial W=S^{4n+1})$. Then the Euler number of W^{4n+2} should vanish. The Euler number of the closed $(PL\text{-})$ manifold $W^{4n+2} \cup D^{4n+2}$ should be equal to one. This is a contradiction because Euler numbers of $(4n+2)$-dimensional closed $(PL\text{-})$ manifolds are all even.

Now, we will discuss a certain kind of diffeomorphisms of $S^{2n} \times S^{2n}$.

Let $\mathcal{S}$ be the set of all diffeomorphisms that satisfy the following two conditions (1) and (2).

(1) Let S^{2n-1} be a $(2n-1)$-dimensional sphere which is imbedded in a small disk $D_1^{4n} \subset S^{2n} \times S^{2n}$. Then each element $f \in \mathcal{S}$ is an identity map on a tubular neighbourhood of S^{2n-1}.

(2) Let $S^{2n-1} \times \operatorname{Int} D^{2n+1}$ be the tubular neighbourhood of S^{2n-1} on which f is an identity and let F be the deleted manifold $S^{2n} \times S^{2n} - S^{2n-1} \times \operatorname{Int} D^{2n+1}$. Then F is diffeomorphic to $S_1^{2n} \times D_1^{2n} \natural S_2^{2n} \times D_2^{2n} \natural S_3^{2n} \times D_3^{2n}$. We denote the homology classes $[S_1^{2n} \times (0)]$, $[S_2^{2n} \times (0)]$ and $[S_3^{2n} \times (0)]$ by a, b and c respectively. The second condition is that the restriction $\bar{f}$ of each $f \in \mathcal{S}$ to F gives homology isomorphism such that $\bar{f}_*(a)=b+c$, $\bar{f}_*(b)=a$ and $\bar{f}_*(c)=c$.

Let f be an element of $\mathcal{S}$ and $\bar{f}$ be a diffeomorphism of F defined above, then we have the following lemma.

LEMMA 2. $\bar{f}:(F, \partial F) \rightarrow (F, \partial F)$ *gives a spinnable structure of a* $(4n+1)$*-dimensional homotopy sphere.*

PROOF. Let $M(\bar{f})$ be the mapping torus of $\bar{f}$ and put $\Sigma = M(\bar{f}) \cup S^{2n} \times S^{2n-1} \times D^2$, where the attaching map is "identity map", this means we glue the two manifolds so that the product structure of $\partial(M(\bar{f}))=S^{2n} \times S^{2n-1} \times S^1$ extends to $S^{2n} \times S^{2n-1} \times D^2$.

We have only to prove Σ is a homotopy sphere. It is easily seen by Van-Kampen's theorem that Σ is simply connected including the case when $k=1$.

Further, from Wang sequence we have,

$$H_i(M(\bar{f})) \cong \begin{cases} Z & \text{for } \quad i=0,1,2n \text{ and } 2n+1 \\ 0 & \text{otherwise}. \end{cases}$$

The generator of $H_{2n}(M(\bar{f}))$ is the image of a under the inclusion map $F \to M(\bar{f})$ and the generator of $H_{2n+1}(M(\bar{f}))$ is identified with $[S^1]\times c$ where $[S^1]$ is the generator of $H_1(M(\bar{f}))$. Applying Mayer-Vietoris exact sequence to the triple $(\Sigma, M(\bar{f}), S^{2n}\times S^{2n}\times D^2)$, we can see $H_i(\Sigma)=0$ for all i, $i\neq 0$, $4n+1$ and $H_0(\Sigma) = H_{4n+1}(\Sigma)\cong Z$. Thus Σ is a homotopy sphere. This completes the proof.

The following lemma shows $\mathcal{S}$ is a non-empty set.

LEMMA 3. *Let $T: S^{2n}\times S^{2n}\to S^{2n}\times S^{2n}$ denote the involution defined by $T(x, y)=(y, x)$, $(x, y)\in S^{2n}\times S^{2n}$. Then T is isotopic to a diffeomorphism f belonging to $\mathcal{S}$.*

PROOF. There exists a diffeomorphism ρ_1 of $S^{2n}\times S^{2n}$ which is isotopic to the identity such that $\rho_1\circ T$ is the identity on a disk $D_0^{4n}\subset S^{2n}\times S^{2n}$. Take a $(2n-1)$-dimensional sphere imbedded in D_0^{4n} and denote it by S_0^{2n-1}. Let $S_0^{2n-1}\times D_0^{2n+1}(\varepsilon)$ be a closed tubular neighbourhood of S_0^{2n-1} (ε denotes the radius of the disk for some metric). We take ε so small that $S_0^{2n-1}\times D_0^{2n+1}(\varepsilon)$ is contained in Int D_0^{4n}. By an ambient isotopy, there exists a diffeomorphism ρ_2 such that $\rho_2\circ\rho_1\circ T(S_1^{2n}\times(*))=(*)\times S_2^{2n}\,\#\,\partial D_0^{2n+1}(\varepsilon)$, where S_1^{2n} (S_2^{2n}) denotes the sphere of $S^{2n}\times S^{2n}$ in the first (second) factor, $\partial D_0^{2n+1}(\varepsilon)$ denotes a $2n$-dimensional sphere which is the boundary of a fibre of $S_0^{2n-1}\times D_0^{2n+1}(\varepsilon)$ and $(*)$ stands for a fixed point in S_1^{2n} (or S_2^{2n}). Take a regular neighbourhood K of $S_1^{2n}\times(*)\vee(*)\times S_2^{2n}$. If K is sufficiently small, $S_0^{2n-1}\times D_0^{2n+1}(\varepsilon/2)$ is outside $\rho_2\circ\rho_1\circ T(K)$. Thus, both S_0^{2n-1} and $\rho_2\circ\rho_1\circ T(S_0^{2n-1})$ are in a disk D_1^{4n} which is contained in $S^{2n}\times S^{2n}-\rho_2\circ\rho_1\circ T(K)=\text{Int}\,D^{4n}$. Since S_0^{2n-1} and $\rho_2\circ\rho_1\circ T(S_0^{2n-1})$ are isotopic in D_1^{4n}, there exists a diffeomorphism ρ_3 of $S^{2n}\times S^{2n}$ which is isotopic to the identity such that the restriction $\rho_3\circ\rho_2\circ\rho_1\circ T|S_0^{2n-1}$ is an identity map. Let $f=\rho_3\circ\rho_2\circ\rho_1\circ T$. To prove f satisfies the Condition (1), we have to check the trivializations of the tubular neighbourhoods $f(S_0^{2n-1}\times D_0^{2n+1})$ and $S_0^{2n-1}\times D_0^{2n+1}$ will coincide. This can be done as follows. Let μ_t (ν_t) be an isotopy between ρ_2 (ρ_3) and the identity map of $S^{2n}\times S^{2n}$. μ_t and ν_t define the imbeddings:

$$\rho_1\circ T(S_0^{2n-1})\times[0, 1]\to S^{2n}\times S^{2n}\times[0, 1] \quad \text{given by } (x, t)\to(\mu_t(x), t)$$

and

$$\rho_2\circ\rho_1\circ T(S_0^{2n-1})\times[0, 1]\to S^{2n}\times S^{2n}\times[0, 1] \quad \text{given by } (y, t)\to(\nu_t(y), t)\,.$$

Since μ_1 and ν_0 (μ_0 and ν_1) define the same imbedding, these give an imbedding $S^{2n-1}\times S^1\to S^{2n}\times S^{2n}\times S^1$.

The normal bundle of this imbedding is trivial because both manifolds are (stably) parallelizable and the normal bundle is $(2k+1)$-dimensional. This means there is no difference between the two trivializations in question. From this we can see f satisfies Condition (1).

As for Condition (2), we can easily see, from the construction, f has the desired homological property. Thus we have proved f belongs to $\mathcal{S}$, completing the proof. We are going to prove Theorem 2.

PROOF OF THEOREM 2. By Lemma 3, there exists a diffeomorphism H of $S^{2n}\times S^{2n}\times[0,1]$ such that $H|S^{2n}\times S^{2n}\times\{0\}$ is the involution T and $H|S^{2n}\times S^{2n}\times\{1\}$ is a diffeomorphism f belonging to $\mathcal{S}$, which fixes a closed tubular neighbourhood $N(S^{2n-1})$ of an imbedded S^{2n-1}. Let Q be the mapping torus of H. By Lemma 1 (Put $F=S^{2n}\times S^{2n}\times I$ and $A=S^{2n}\times S^{2n}\times\{0\}\cup(N(S^{2n-1})\times\{1\})$), Q has a codimension one foliations with properties, (a) the mapping torus of $H|S^{2n}\times S^{2n}\times\{0\}=T$ is a compact leaf, (b) the mapping torus of $H|N(S^{2n-1})$ is a compact leaf (with boundary), (c) other leaves are diffeomorphic to $S^{2n}\times S^{2n}\times(0,1]-N(S^{2n-1})\times\{1\}$.

On the other hand, there is a codimension one foliation of $S^{2n-1}\times D^{2n+1}\times D^2$ which is a pull-back of a foliation of $S^{2n-1}\times D^2$ whose boundary is a compact leaf [**16**].

By identifying the mapping torus of $H|N(S^{2n-1})$ and $S^{2n-1}\times D^{2n+1}\times\partial D^2$, and by smoothing the corners, we obtain a smooth foliated manifold $W^{4n+2}=Q\cup S^{2n-1}\times D^{2n+1}\times D^2$. The boundary of W is a union of two disjoint closed manifolds; the mapping torus of T and a homotopy sphere Σ (see, Lemma 2). The foliation of W is transverse to Σ and Σ is foliated by using the spinnable structure which was described in Lemma 2.

To complete the proof of Theorem 2, we need the following two lemmas.

LEMMA 4. *Given a homotopy sphere $\tilde{\Sigma}^{4n+1}$, we can modify W^{4n+2} into $\tilde{W}^{4n+2}$ so that*

(1) *$\tilde{W}^{4n+2}$ is a cobordism between the mapping torus of T and $\tilde{\Sigma}^{4n+1}$.*

(2) *$\tilde{W}^{4n+2}$ has a foliation which has the properties* (a) *and* (b) *described above.*

LEMMA 5. *The mapping torus of T has an orientation reversing differentiable involution.*

Assume, for a moment, Lemma 4 and Lemma 5. Then we can prove Theorem 2 as follows. Given a homotopy sphere $\tilde{\Sigma}^{4n+1}$, we take two copies of $\tilde{W}^{4n+2}$ which is obtained by Lemma 4. Glue them along the mapping torus of T by the orientation reversing involution which is obtained by Lemma 5. The resulting foliated manifold is a desired foliated cobordism. This completes the proof of Theorem 2.

Now, we must prove Lemma 4 and Lemma 5.

PROOF OF LEMMA 4. Let g and h be diffeomorphisms of S^{4n}, which correspond to Σ^{4n+1} and $\tilde{\Sigma}^{4n+1}$ respectively. Then $h\circ g^{-1}$ corresponds to the homotopy sphere $-\Sigma^{4n+1}\#\tilde{\Sigma}^{4n+1}$. By a theorem of H. Winkelnkemper [**23**], $h\circ g^{-1}$ extends to a diffeomorphism G of a $(4n+1)$-dimensional manifold V^{4n+1} whose boundary is S^{4n}. We can assume G is an identity on a small half disk D^{4n+1}_+ in V^{4n+1}. Consider the manifold; $X=S^{2n}\times S^{2n}\times I\natural V^{4n+1}$ where the boundary connected sum is made along D_+^{4n+1} and a half disk in $S^{2n}\times S^{2n}\times I$, which is disjoint from $S^{2n}\times S^{2n}\times\{0\}\cup S^{2n-1}\times \mathrm{Int}\, D^{2n+1}\times\{1\}$ and where H is an identity map. We can define a diffeomorphism $H\natural G$ of X in an obvious fashion. Using $(H\natural G, X)$ instead of $(H, S^{2n}\times S^{2n}\times I)$ in the proof of Theorem 2, we obtain a desired foliated manifold $\widetilde{W}^{4n+2}$ as in the case of W^{4n+2}.

PROOF OF LEMMA 5. The mapping torus of T is the manifold obtained from $S^{2n}\times S^{2n}\times[0, 1]$ by identifying $(x, 0)$ and $(T(x), 1)$ for $x\in S^{2n}\times S^{2n}$. It is easily verified that the reflection of the interval $[0, 1]$ defined by $r(t)=1-t$, $t\in[0, 1]$ is compatible with the identification. From this Lemma 5 follows.

§4. A remark on foliations of highly connected manifolds.

In this section, we shall consider the foliations of highly connected manifolds. In [**19**], Tamura proved that every $(n-1)$-connected $(2n+1)$-manifold $(n\geqq 3)$ has a codimension one foliation. A similar result for even dimensional manifolds was obtained in [**13**]. We shall describe more explicitly the foliations of these manifolds.

LEMMA 6. *S^5 has a spinnable structure whose axis is diffeomorphic to $S^1\times S^2$.*

This is an immediate consequence of Lemma 2 and Lemma 3 of Section 3. It can be proved this spinnable structure can not be obtained as a Milnor fibering of an isolated sigularity.

A smooth manifold is said to be specially spinnable if it admits a spinnable structure whose axis is a sphere.

LEMMA 7. *Let M^{2n+1} be a specially spinnable manifold then M^{2n+1} has a spinnable structure whose axis is diffeomorphic to a product of S^1 and even dimensional spheres.*

PROOF. First we shall prove $S^{2n-1}\times D^2$ has a spinnable structure whose axis is a product of S^1 and even dimensional spheres and its restriction to $S^{2n-1}\times\partial D^2$ is a trivial bundle over $S^1=\partial D^2$. For $n=1$, this assertion is clearly true. For $n=2$, Lemma 6 means $S^3\times D^2$ has such a spinnable structure (see Corollary to Theorem 1). Suppose we have proved the assertion for $S^{2k-1}\times D^2$, $1\leqq k<n$. By Corollary to Theorem 1, $S^{2n-1}\times D^2$ has a spinnable structure with axis $S^{n-1}\times S^n$. But our hypothesis says $S^{n-1}\times S^n\times D^2$ has a spinnable structure with desired axis, which is induced from the one of $S^{n-1}\times D^2$ $(S^n\times D^2)$ when n is even (odd), by projections.

Gluing the spinning bundle of $S^{2n-1}\times D^2$ and that of $S^{n-1}\times S^n\times D^2$ along $S^{n-1}\times S^n\times\partial D^2$, we have a new spinnable structure of $S^{2n-1}\times D^2$ whose axis is a desired one. Thus the assertion is true for any $S^{2n-1}\times D^2$, $n\geqq 1$.

Given a specially spinnable manifold M^{2n+1}, consider the spinning bundle

$M - S^{2n-1} \times \operatorname{Int} D^2 \to S^1$. Glue $M - S^{2n-1} \times \operatorname{Int} D^2$ and the spinning bundle of $S^{2n-1} \times D^2$ just obtained, along the boundaries in an obvious way. Thus, M has a new spinnable structure with a desired axis. This completes the proof.

In [**13**], we have proved that an $(n-2)$-connected $2n$-manifold has a spinnable structure with axis which is diffeomorphic to $S^{\text{odd}} \times S^{\text{odd}}$ if $n \geqq 3$ and if its Euler number and signature vanish. Therefore, by the same argument as above, such an even dimensional manifold also has a spinnable structure whose axis is diffeomorphic to a product of S^1 and higher dimensional spheres.

Applying Lemma 1 to these spinnable structures we have the following theorem.

THEOREM 3. *Every $(n-1)$-connected $(2n+1)$-manifold and every $(n-2)$-connected $2n$-manifold with vanishing Euler number and signature $(n \geqq 3)$, has a foliation with single compact leaf which is diffeomorphic to a product of T^2 and higher dimensional spheres.*

REMARK. There are only two kinds of non-compact leaves of the above foliations: One is diffeomorphic to the interior of the generator of a spinnable structure and the other is diffeomorphic to a product of R^2 and higher dimensional spheres.

Appendix.

In this appendix, we shall prove the following "Relative Spinnable Structure Theorem". Our proof is essentially a modification of those of [**13**], [**17**], [**19**], [**24**] to the relative case.

THEOREM. *Let W^{n+1} be a compact, simply connected smooth manifold of dimension $n+1$ $(n \geqq 6)$. Suppose the boundary ∂W admits a spinnable structure whose generator F is simply connected and ∂F is connected. If $n+1=4k$, suppose further the signature of the intersection pairing*

$$H_{2k}(W, F; Z)/\mathrm{Tor} \otimes H_{2k}(W, F; Z)/\mathrm{Tor} \to Z$$

vanishes (Tor stands for the torsion subgroup). Then W^{n+1} has a spinnable structure which is an extension of the given spinnable structure of ∂W.

PROOF. (In this proof, we use only homology and cohomology groups with integer coefficient.) Since ∂W has a spinnable structure with generator F, ∂W is decomposed into $(F \times I)_0 \cup (F \times I)_1$, where $(F \times I)_i$ $(i=0, 1)$ is a copy of $F \times I$ (round the corners if necessary) and the pasting map is one which is determined by the monodromy of the spinnable structure of ∂W.

Using the relative Hurewicz theorem, we can proceed as Smale [**15**] and obtain a handle decomposition of W relative to $(F \times I)_0$, which is minimal with respect to the homology structure of $(W, (F \times I)_0)$. Hereafter, we will fix one of such a handle decomposition of W.

We separate the proof into two cases.

CASE I. When $n+1=2m+1$ $(m\geqq 3)$.

Let V_0 be the submanifold of W, which is obtained by attaching to $(F\times I)_0\times[0, \varepsilon]$ (a collar neighbourhood of $(F\times I)_0$) all the handles of W whose indices are less than $m+1$ and put $V_1=\overline{W-V_0}$.

Since F and W are simply connected, V_0, V_1 are also simply connected. We will denote by $\partial_0 V_0$ (resp. $\partial_0 V_1$) the manifold $\partial V_0-(F\times \mathrm{Int}\, I)_0$ (resp. $\partial V_1-(F\times \mathrm{Int}\, I)_1$). Clearly $\partial_0 V_0=\partial_0 V_1$ in W and $\partial_0 V_0$ is a manifold whose boundary is the double of F (if F is closed, the disjoint union of two copies of F). Since $n\geqq 6$ and F and V_0 are simply connected, $\partial_0 V_0$ is also verified to be simply connected by using the homotopy exact sequence of $(V_0, \partial_0 V_0)$.

We have the following homology exact sequence of the triple $(V_0, \partial_0 V_0, F)$

$$\longrightarrow H_{i+1}(V_0, \partial_0 V_0)\longrightarrow H_i(\partial_0 V_0, F)\longrightarrow H_i(V_0, F)\longrightarrow H_i(V_0, \partial_0 V_0)\longrightarrow .$$

By Poincaré-Lefschetz duality we have

$$H_i(V_0, \partial_0 V_0)=H^{2m+1-i}(V_0, (F\times I)_0)\,.$$

But V_0 is a handlebody obtained from $(F\times I)_0$ by attaching the handles of indices less than $m+1$, so

$$H^{2m+1-i}(V_0, (F\times I)_0)=0 \qquad \text{for} \quad i\leqq m\,.$$

Therefore the inclusion map $(\partial_0 V_0, F)\to(V_0, F\times I)_0)$ induces homomorphisms $H_i(\partial_0 V_0, F)\to H_i(V_0, (F\times I)_0)$ which are bijective for $i<m$, and surjective for $i=m$. If we choose a handle decomposition of $\partial_0 V_0$ relative to F, then we have a submanifold G_0 of $\partial_0 V_0$ which consists of the handles of indices less than $m+1$ such that the homomorphisms $H_i(G_0, F)\to H_i(V_0, (F\times I)_0)$ induced by inclusion map are isomorphisms for $i\leqq m$ (use "handle addition theorem" for m-handles of G_0).

Consider the dual handle decomposition of W. V_1 is regarded as a handlebody relative to $(F\times I)_1$ which is obtained by attaching the handles of indices less than $m+1$ to $(F\times I)_1$ and minimal with respect to the homology structure of $(W, (F\times I)_1)$.

We have the following diagram where all the homomorphisms are bijective for $i<m$.

$$\begin{array}{ccccc} & & H_i(V_0, (F\times I)_0) & & \\ & \nearrow & & \searrow & \\ H_i(\partial_0 V_0, F) & & & & H_i(W, F) \\ & \searrow & & \nearrow & \\ & & H_i(V_1, (F\times I)_1) & & \end{array}$$

Also as before, we have a surjective homomorphism

$$H_m(\partial_0 V_1, F) \to H_m(V_1, (F\times I)_1)\,. \tag{2}$$

Now, we are going to modify G_0 in order to obtain a generator of a spinnable structure of W.

Let p denote the rank of $H_m(G_0, F)$ ($=\operatorname{rank} H_m(V_0, (F\times I)_0)=\operatorname{rank} H_m(V_1, (F\times I)_1)$) and take a natural decomposition of S^{2m+1};

$$S^{2m+1}=(S_1^m\times D_1^{m+1}\natural\cdots\natural S_p^m\times D_p^{m+1})\cup(D_1^{m+1}\times S_1^m\natural\cdots\natural D_p^{m+1}\times S_p^m)\,.$$

Set

$$\tilde{V}_0=V_0\natural S_1^m\times D_1^{m+1}\natural\cdots\natural S_p^m\times D_p^{m+1}$$

$$\tilde{V}_1=V_1\natural D_1^{m+1}\times S_1^m\natural\cdots\natural D_p^{m+1}\times S_p^m\,.$$

Clearly, $\tilde{V}_0\cup\tilde{V}_1=W\# S^{2m+1}=W$.

Let α_i $(i=1,\cdots,p)$ denote the generators of $H_m(G_0, F)\subset H_m(\partial_0 V_0, F)$ and let β_i $(i=1,\cdots,p)$ be the generators of $H_m(\partial_0 V_0, F)$ which are mapped onto the generators of $H_m(V_1, F\times I)_1)$ under the homomorphism (2) above and let a_i, b_i $(i=1,\cdots,p)$ denote the homology classes of $\partial_0\tilde{V}_0=\partial_0 V_0\# S_1^m\times\partial D_1^{m+1}\#\cdots\# S_p^m\times\partial D_p^{m+1}$ which are represented by $S_i^m\times$(point) and (point)$\times\partial D_i^{m+1}$ respectively.

Define $\tilde{G}$ to be the handlebody in $\partial_0\tilde{V}_0$ relative to F satisfying the following:

(1) The handles of indices less than m and the handles of index m which generate the relations in $H_{m-1}(\tilde{G}, F)$ are the same ones as those of G_0.

(2) The handles of index m which are the generators of $H_m(\tilde{G}, F)$, are the handles representing the homology classes α_i+b_i, β_i+a_i $(i=1,\cdots,p)$ (we consider α_i, β_i are naturally the elements of $H_m(\partial_0\tilde{V}_0, F)$). Such a handlebody is obtained by virtue of handle addition theorem.

By the construction of $\tilde{G}$ and the diagram (2) above, we can see the inclusion maps $\tilde{G}\to\tilde{V}_0$ and $\tilde{G}\to\tilde{V}_1$ induce isomorphisms of homology groups and hence they are homotopy equivalences.

By duality, we can also see that $\tilde{G}'=\partial_0\tilde{V}_0-\operatorname{Int}\tilde{G}\to\tilde{V}_0$ and $\tilde{G}'\to\tilde{V}_1$ are also homotopy equivalences.

Thus $(\tilde{V}_0, \tilde{G}, \tilde{G}')$ and $(\tilde{V}_1, \tilde{G}, \tilde{G}')$ are considered as relative h-cobordisms. Therefore by relative h-cobordism theorem, we have $W=\tilde{V}_0\cup\tilde{V}_1=(\tilde{G}\times I)\cup(\tilde{G}\times I)$. This shows that W has a spinnable structure with $\tilde{G}$ as generator. This finishes the proof in Case I.

CASE II. When $n+1=2m$ $(m\geqq 4)$.

We will proceed in almost the same way as in Case I and will not repeat the details which are stated in Case I.

Again we fix a handlebody decomposition of W relative to $(F\times I)_0$, which is minimal with respect to the homology structure of $(W, (F\times I)_0)$.

Let V_0 be the submanifold of W which consists of the following handles:

(1) The handles of indices less than m.

(2) The handles of index m which represent the torsion generators of $H_m(W, (F\times I)_0)$.

(3) The handles of index m which represent free generators e_i $(i=1, \cdots, r, r=1/2 \operatorname{rank} H_m(W, F))$ in $H_m(W, (F\times I)_0)$ where e_i, f_i $(i=1, \cdots, r)$ are the basis of $H_m(W, (F\times I)_0)/\mathrm{Tor}$ whose intersections satisfy $(e_i, e_j)=0$, $(e_i, f_j)=\pm\delta_{ij}$, $(i, j=1, \cdots, r)$ (such a basis do exist since the signature of the intersection form is zero for m even).

Set $V_1=\overline{W-V_0}$ and $\partial_0 V_0=\partial V_0-(F\times \operatorname{Int} I_0)$. As in case I, the homomorphisms induced by the inclusion map $H_i(\partial_0 V_0, F)\to H_i(V_0, (F\times I)_0)$ are bijective for $i<m-1$ and surjective for $i=m-1$. The homomorphism $H_{m-1}(\partial_0 V_1, F)\to H_{m-1}(V_1, (F\times I)_1)$ is also surjective as in case I.

We need the following lemma which we will prove later.

LEMMA. *For $i\leqq m$, $\operatorname{rank} H_i(V_0, (F\times I)_0)=\operatorname{rank} H_i(V_1, (F\times I)_1)$ and the images of inclusion maps $H_i(V_0, F)\to H_i(W, F)$ and $H_i(V_1, F)\to H_i(W, F)$ coincide. In particular, the intersection forms of $H_m(V_0, (F\times I)_0)$ and $H_m(V_1, (F\times I)_1)$ are zero.*

From the last statement of this lemma, we have surjective homomorphisms

$$H_m(\partial_0 V_0, F)\to H_m(V_0, (F\times I)_0), \quad H_m(\partial_0 V_1, F)\to H_m(V_1, (F\times I)_1).$$

Choose a minimal handlebody decomposition of $\partial_0 V_0$ relative to F. From the above observation, we have a handlebody G_0 in $\partial_0 V_0$ consisting of handles of indices less than $m+1$ such that the homomorphisms induced by the inclusion map

$$H_i(G_0, F)\to H_i(V_0, (F\times I)_0)$$

are bijective for $i<m-1$ and surjective for $i=m-1, m$. (In order to attach m-handles to $(m-1)$-skeleton, we use the simply-connectedness of W and F and the condition $m\geqq 4$.)

Let p_1 and p_2 denote the rank of $H_{m-1}(V_0, F)$ and the rank of $H_m(V_0, F)$ respectively and put $p=\max\{p_1, p_2\}$.

Take a natural decomposition

$$S^{2m}=A_0\cup A_1, \qquad \text{where}$$

$$A_0=S_1^{m-1}\times D_1^{m+1}\natural\cdots\natural S_p^{m-1}\times D_p^{m+1}\natural S_1^m\times D_1^m\natural\cdots\natural S_p^m\times D_p^m$$

$$A_1=D_1^m\times S_1^m\natural\cdots\natural D_p^m\times S_p^m\natural D_1^{m+1}\times S_1^{m-1}\natural\cdots\natural D_p^{m+1}\times S_p^{m-1}.$$

Set $\tilde{V}_0=V_0\natural A_0$ and $\tilde{V}_1=V_1\natural A_1$.

Let α_i (resp. β_i) $(i=1, \cdots, p_1)$ be the homology classes of $H_{m-1}(\partial_0 V_0, F)=H_{m-1}(\partial_0 V_1, F)$ whose images by inclusion homomorphism $H_{m-1}(\partial_0 V_0, F)\to H_{m-1}(V_0, (F\times I)_0)$ (resp. $H_{m-1}(\partial_0 V_1, F)\to H_{m-1}(V_1, (F\times I)_1)$) form a basis of

$H_{m-1}(V_0, (F\times I)_1)$ (resp. $H_{m-1}(V_1, (F\times I)_1)$). Similarly, let ξ_i (resp. η_i) ($i=1, \cdots, p_2$) be the homology class of $H_m(\partial_0 V_0, F)=H_m(\partial_0 V_1, F)$ whose images by inclusion homomorphism $H_m(\partial_0 V_1, F)\rightarrow H_m(V_0, (F\times I)_0)$ (resp. $H_m(\partial_0 V_1, F)\rightarrow H_m(V_1, (F\times I)_1)$) form a basis of $H_m(V_0, (F\times I)_0)$ (resp. $H_m(V_1, (F\times I)_1)$). Further, let a_i (resp. b_i) ($i=1, \cdots, p$) be the homology classes of $\partial\tilde{V}_0$ represented by $S_i^{m-1}\times$(point) (resp. (point)$\times S_i^{m-1}$) and denote by x_i (resp. y_i) ($i=1, \cdots, p$) the homology classes represented by $S_i^m\times$(point) (resp. (point)$\times S_i^m$).

Now define the handlebody $\tilde{G}$ relative to F as follows.

(1) $(m-1)$-handles of $\tilde{G}$ are the handles corresponding to the homology classes α_i+b_i, β_i+a_i, a_j+b_j ($i=1, \cdots, p_1$, $j=p_1+1, \cdots, p$).

(2) m-handles of G are the handles corresponding to the homology classes ξ_k+y_k, η_k+x_k, x_l+y_l ($k=1, \cdots, p_2$, $l=p_2+1, \cdots, p$).

(3) Other handles are the same as those of G_0.

By the construction, the inclusion map $(\tilde{G}, F)\rightarrow(\tilde{V}_0, (F\times I)_0)$ (resp. $(\tilde{G}, F)\rightarrow(\tilde{V}_1, (F\times I)_1)$) induces an isomorphism of the homology groups, hence it is a homotopy equivalence.

By the same argument as in Case I, we can conclude that W has a spinnable structure with $\tilde{G}$ as generator, which is an extension of the given spinnable structure of ∂W.

To complete the proof, we must prove Lemma.

PROOF OF LEMMA. For $i\leqq m-1$, the assertion is clear if we consider the dual handlebody decomposition of W.

Consider the following diagram

$$\begin{array}{ccccc} & & H_m(V_0, F) & & \\ & & \downarrow i_0 & \searrow^{j_1\circ i_0} & \\ H_m(V_1, F) & \xrightarrow{i_1} & H_m(W, F) & \xrightarrow{j_1} & H_m(W, V_1) \end{array}$$

where all the homomorphism are induced by inclusion maps.

By Poincaré-Lefschetz duality, we have

$$H_m(V_1, F)\cong H^m(W, V_0)$$

and by the universal coefficient theorem

$$\mathrm{Tor}\, H_m(V_1, F)=\mathrm{Tor}\, H_{m-1}(W, V_0)=0$$

$$\mathrm{rank}\, H_m(V_1, F)=\mathrm{rank}\, H_m(W, V_0)\,.$$

But from the homology exact sequence for the triple (W, V_0, F), it is easily seen $\mathrm{rank}\, H_m(W, V_0)=r+$the number of torsion generators (of $H_{m-1}(W, F)$), which is equal to $\mathrm{rank}\, H_m(V_0, F)$.

It is well-known that the map $j_1\circ i_0$ is determined by the intersection matrix of $H_m(V_0, F)$. But every element of $H_m(V_0, F)$ is mapped to zero under $j_1 \circ i_0$ because $H_m(V_0, F)$ is generated by e_i' $(i=1, \cdots, r)$ and by t_j' $(j=1, \cdots, s$, $s=\operatorname{rank} H_m(V_0, F)-r)$ such that $i_0(e_i')=e_i$ and $i_0(t_j')$'s are the torsion generators in $H_m(W, F)$ and hence their intersections are all zero. Therefore, $\operatorname{Im} i_0 \subset \operatorname{Ker} j_1 = \operatorname{Im} i_1$.

Since $H_m(V_1, F)$ is free abelian of rank $r+s$, $H_m(V_1, F)$ contains the generators e_i'' $(i=1, \cdots, r)$ and t_j'' $(j=1, \cdots, s)$ such that $i_1(e_i'')=e_i$ and $i_1(t_j'') \subset \operatorname{Tor} H_m(W, F)$.

From this we can see the intersection matrix of $H_m(V_1, F)$ is zero. (Consider the intersection matrix of (V_1, F) in (W, F)). Thus by the same argument as above, we have $\operatorname{Im} i_1 \subset \operatorname{Im} i_0$. This completes the proof.

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Added in proof.

**) W. Thurston has succeeded in constructing codimension one foliations of arbitrary closed manifold with vanishing Euler number. His method does not use the spinnable structures of manifolds.

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