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### Research Article

# The Magnetohydrodynamic Boundary Layer Flow of a Nanofluid past a Stretching/Shrinking Sheet with Slip Boundary Conditions

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The magnetohydrodynamic (MHD) boundary layer flow of a nanofluid past a stretching/shrinking sheet with velocity, thermal, and solutal slip boundary conditions is studied. Numerical solutions to the governing equations were obtained using a shooting method. The skin friction coefficient and the local Sherwood number increase as the stretching/shrinking parameter increases. However, the local Nusselt number decreases with increasing the stretching/shrinking parameter. The range of the stretching/shrinking parameter for which the solution exists increases as the velocity slip parameter and the magnetic parameter increase. For the shrinking sheet, the skin friction coefficient increases as the velocity slip parameter and the magnetic parameter increase. For the stretching sheet, it decreases when the velocity slip parameter and the magnetic parameter increase. The local Nusselt number diminishes as the thermal slip parameter increases while the local Sherwood number decreases with increasing the solutal slip parameter. The local Nusselt number is lower for higher values of Lewis number, Brownian motion parameter, and thermophoresis parameter.

#### 1. Introduction

The boundary layer flow over a stretching sheet is significant in applications such as extrusion, wire drawing, metal spinning, and hot rolling [1]. Wang [2, 3], Mandal and Mukhopadhyay [4], P. S. Gupta and A. S. Gupta [5], Andersson [6], Ishak et al. [7], and Makinde and Aziz [8] are among various names whose papers on stretching sheet were published. However, to complement the study of flow over a stretching sheet, Miklavčič and Wang [9] then began the study of flow over a shrinking sheet in which they observed that the vorticity is not confined within a boundary layer and a steady flow cannot exist without exerting adequate suction at the boundary. As the studies of shrinking sheet garnered considerable attention, these findings prove to be crucial to these researches. In response to Miklavčič and Wang [9], numerous studies on these problems have been conducted by

researchers, namely, Wang [10], Fang et al. [11], Bachok et al. [12], Bhattacharyya et al. [13], Zaimi et al. [14], and Roşca and Pop [15], among others.

However, most studies were done in the absence of magnetic field. Ishak et al. [16] studied the magnetohydrodynamic (MHD) stagnation point flow towards a stretching sheet while Merkin and Kumaran [17] studied the unsteady MHD boundary layer flow on a shrinking sheet.

Motivated by the above-mentioned studies, this paper aims at studying the MHD boundary layer flow of a nanofluid over a stretching/shrinking sheet with slip conditions and suction effect at the boundary. The inclusion of nanoparticles enhances thermal conductivity as reported by Masuda et al. [18]. This study will use the model developed by Buongiorno [19] where we pay more attention to the Brownian motion and thermophoresis effects as previously done by Nield and Kuznetsov [20–22]. In addition, we employ the velocity

and thermal and solutal slip conditions because the recent findings that rarefied gas flows with slip boundary conditions are often encountered in the microscale devices and lowpressure situations (Kumaran and Pop [23]). The effects of slip conditions are very important in technological applications such as in the polishing of artificial heart valves and the internal cavities. For the shrinking case, the solution does not exist since vorticity could not be confined within the boundary layer. However, with an added suction effect to confine the vorticity, the solution may exist. The dependency of the local skin friction coefficient, Nusselt number, and Sherwood number on seven parameters, namely, the stretching/shrinking, velocity slip, thermal slip, concentration slip, magnetic, Brownian motion, and thermophoresis parameters, is the main focus of the present investigation. Numerical solutions are presented graphically and in tabular forms to show the effects of these parameters on the local Nusselt number and the local Sherwood number.

#### 2. Mathematical Formulation

Consider a steady, two-dimensional boundary layer flow of a nanofluid over a permeable stretching/shrinking sheet. It is assumed that the stretching/shrinking velocity is in the form  $U_w = ax$ , where a is a positive constant and x is the coordinate measured along the stretching/shrinking surface. It is also assumed that the constant mass flux velocity is  $v_0$  with  $v_0 < 0$  for suction and  $v_0 > 0$  for injection or withdrawal of the fluid. The nanofluid is confined to y > 0, where y is the coordinate measured normal to the stretching/shrinking surface. The flow is subjected to the transverse magnetic field of strength  $B_0$ , which is assumed to be applied in the positive y-direction. The induced magnetic field is also assumed to be small compared to the applied magnetic field; hence, it is neglected.

The governing equations for the steady conservation of mass, momentum, thermal energy, and nanoparticle volume fraction equations in the presence of magnetic field can be written as [19, 20, 22, 24]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,\tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_f}\frac{\partial p}{\partial x} + \omega\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\lambda B_0^2}{\rho_f}u,\tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_f}\frac{\partial p}{\partial y} + \omega\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{\lambda B_0^2}{\rho_f}v, \quad (3)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}$$

$$= \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \tau \left\{ D_B \left( \frac{\partial \varphi}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial \varphi}{\partial y} \frac{\partial T}{\partial y} \right) + \frac{D_T}{T_\infty} \left[ \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right] \right\},$$
(4)

$$u\frac{\partial \varphi}{\partial x} + v\frac{\partial \varphi}{\partial y} = D_B \left( \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} \right) + \frac{D_T}{T_{\infty}} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right), \tag{5}$$

where u and v are the velocity components along the x-and y-axes respectively, p is the fluid pressure, T is the fluid temperature,  $\alpha$  is the thermal diffusivity,  $\omega$  is the kinematic viscosity,  $\lambda$  is the electrical conductivity,  $B_0$  is the magnetic field,  $D_B$  is the Brownian diffusion coefficient,  $D_T$  is the thermophoresis diffusion coefficient, and  $\varphi$  is the nanoparticle volume fraction. Furthermore,  $\tau = (\rho c)_p/(\rho c)_f$  is the ratio between the effective heat capacity of the fluid with  $\rho_f$  and  $\rho_p$  being the density of the fluid and the density of the particles, respectively, and  $c_f$  and  $c_p$  denote the specific heat of the fluid and the particle at constant pressure, respectively. The subscript  $\infty$  represents the values at large values of y (outside the boundary layer). Details of the derivation of (4) and (5) are given in the papers by Buongiorno [19] and Nield and Kuznetsov [21].

Equations (1)–(5) are subjected to the following boundary conditions [24]:

$$v = v_0, \quad u = \sigma U_w + L \frac{\partial u}{\partial y}, \quad T = T_w + K_1 \frac{\partial T}{\partial y},$$

$$\varphi = \varphi_w + K_2 \frac{\partial \varphi}{\partial y} \quad \text{at } y = 0,$$
(6)

$$u \longrightarrow 0, \quad v \longrightarrow 0, \quad T \longrightarrow T_{\infty}, \quad \varphi \longrightarrow \varphi_{\infty} \quad \text{as } y \longrightarrow \infty,$$
 (7)

where  $\sigma$  is a constant with  $\sigma>0$  for stretching and  $\sigma<0$  for shrinking. The subscript w denotes the values at the solid surface. Furthermore, L,  $K_1$ , and  $K_2$  are the velocity, thermal, and solutal slip factors, respectively and when  $L=K_1=K_2=0$ , the no-slip condition is recovered. The governing equations (1)–(5) subjected to the boundary conditions (6) and (7) can be expressed in a simpler form by introducing the following transformation:

$$\psi = (U_w \omega x)^{1/2} f(\eta), \qquad \eta = \left(\frac{U_w}{\omega x}\right)^{1/2} y,$$

$$\theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \qquad \beta(\eta) = \frac{\varphi - \varphi_\infty}{\varphi_w - \varphi_\infty},$$
(8)

where  $\eta$  is the similarity variable and  $\psi$  is the stream function defined as  $u = \partial \psi / \partial y$  and  $v = -\partial \psi / \partial x$ , which identically satisfy (1). By employing the boundary layer approximations and the similarity variables (8), (2)–(5) reduce to the following nonlinear ordinary differential equations:

$$f''' + ff'' - f'^2 - Mf' = 0, (9)$$

$$\frac{1}{\Pr}\theta'' + f\theta' + \operatorname{Nb}\beta'\theta' + \operatorname{Nt}\theta'^2 = 0, \tag{10}$$

$$\beta'' + \frac{Nt}{Nb}\theta'' + Lef\beta' = 0$$
 (11)

and the boundary conditions (6) and (7) become

$$f(0) = S,$$
  $f'(0) = \sigma + Af''(0),$   
 $\theta(0) = 1 + B\theta'(0),$   $\beta(0) = 1 + C\beta'(0),$  (12)

$$f' = 0$$
,  $\theta = 0$ ,  $\beta = 0$  as  $\eta \longrightarrow \infty$ , (13)

where primes denote differentiation with respect to  $\eta$ . Further, Pr is the Prandtl number, Nb is the Brownian motion parameter, Nt is the thermophoresis parameter, Le is the Lewis number, S is the mass flux parameter with S>0 for suction and S<0 for injection, M is the magnetic parameter, and A, B, and C are velocity, thermal, and solutal slip parameters respectively, which are defined as

$$Pr = \frac{\omega}{\alpha}, \qquad Nb = \frac{\tau D_B (\varphi_w - \varphi_\infty)}{\omega},$$

$$Nt = \frac{\tau D_T (T_w - T_\infty)}{\omega T_\infty}, \qquad Le = \frac{\omega}{D_B},$$

$$S = -\frac{v_0}{\sqrt{a\omega}}, \qquad M = \frac{\lambda B_0^2}{\rho_f a}, \qquad A = L\sqrt{\frac{a}{\omega}},$$

$$B = K_1 \sqrt{\frac{a}{\omega}}, \qquad C = K_2 \sqrt{\frac{a}{\omega}};$$
(14)

when Nb = Nt = 0, the present problem reduces to a regular viscous fluid, and the nanoparticle volume fraction equation (11) becomes ill-posed and is of no physical significance.

The physical quantities of interest are the skin friction coefficient  $C_f$ , the local Nusselt number  $Nu_x$ , and the local Sherwood number  $Sh_x$  which are defined as [6]

$$C_f = -\frac{\tau_w}{\rho U_w^2}, \qquad \text{Nu}_x = \frac{xq_w}{k(T_f - T_\infty)},$$

$$\text{Sh}_x = \frac{xq_m}{D_B(\varphi_w - \varphi_\infty)},$$
(15)

where  $\tau_w$ ,  $q_w$ , and  $q_m$  are the surface shear stress, the heat and mass fluxes, respectively, which are given by [25]

$$\tau_{w} = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0}, \qquad q_{w} = -k \left(\frac{\partial T}{\partial y}\right)_{y=0},$$

$$q_{m} = -D_{B} \left(\frac{\partial \varphi}{\partial y}\right)_{y=0}.$$
(16)

Using the similarity variables (8), we obtain

$$C_f \operatorname{Re}_x^{1/2} = -f''(0), \qquad \operatorname{Nu}_x \operatorname{Re}_x^{-1/2} = -\theta'(0),$$

$$\operatorname{Sh}_x \operatorname{Re}_x^{-1/2} = -\beta'(0), \qquad (17)$$

where  $Re_x = U_w x/\omega$  is the local Reynolds number.

#### 3. Results and Discussions

The set of ordinary differential equations (9)–(11) subjected to boundary conditions (12) and (13) were solved numerically

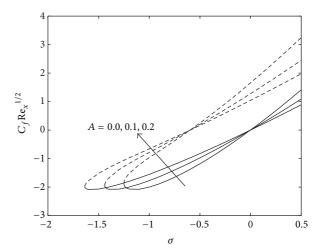


FIGURE 1: Variation of the skin friction coefficient with  $\sigma$  for different values of A when S=3 and M=1.

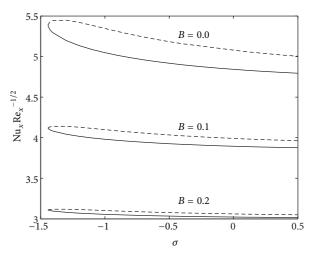


FIGURE 2: Variation of the local Nusselt number with  $\sigma$  for different values of B when S=3, M=1, Pr=6.8, A=C=0.1, Nt=Nb=0.5, and Le=2.

using a shooting method. In this method, the dual solutions are obtained by setting different initial guesses for the values of f''(0),  $-\theta'(0)$  and  $-\beta'(0)$ . The asymptotic boundary conditions (13) at  $\eta = \infty$  are replaced by  $\eta = 15$  which is sufficient for all the velocity, temperature, and concentration profiles to vanish asymptotically.

Figures 1–3 show the variations of the local skin friction coefficient  $C_f \operatorname{Re}_x^{1/2}$ , the local Nusselt number  $\operatorname{Nu}_x \operatorname{Re}_x^{-1/2}$  (representing the heat transfer rate at the surface), and the local Sherwood number  $\operatorname{Sh}_x \operatorname{Re}_x^{-1/2}$  (representing the mass transfer rate at the surface) with  $\sigma$  for different values of the velocity slip parameter A, thermal slip parameter B, and solutal slip parameter C, respectively. As can be seen, there are more than one solution obtained for a fixed value of  $\sigma$ . When  $\sigma$  is equal to a certain value  $\sigma = \sigma_c$  where  $\sigma_c(<0)$  is the critical value of  $\sigma$ , there is only one solution, and when  $\sigma < \sigma_c$ , there is no solution. Here, the computations have

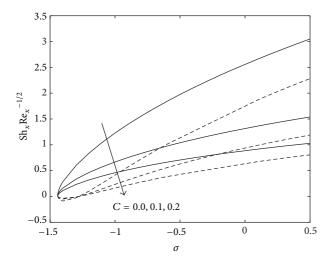


FIGURE 3: Variation of the local Sherwood number with  $\sigma$  for different values of C when S=3, M=1, Pr=6.8, A=B=0.1, Nt=Nb=0.5, and Le=2.

TABLE 1: Values of  $\sigma_c$  for several values of S and A.

S	A	$\sigma_c$
3	0.1	-1.4400
4		-2.6128
5		-3.3948
3	0.0	-1.2500
	0.2	-1.6368

been performed until the point where the solution does not converge. Based on our computations, the critical values  $\sigma_c$  are presented in Table 1. The range of  $\sigma$  for which the solution exists increases as A and S increase. For an increasing slip parameter at the boundary, the generation of vorticity for shrinking velocity is slightly reduced [13]. Therefore, with the imposed suction, that vorticity remains confined in the boundary layer region for larger shrinking velocity (i.e.,  $\sigma < 0$ ). Hence, the steady solution is possible for some large values of  $\sigma$ . Although only the first solution is stable and physically realizable [15, 26, 27], the second solution is of mathematical interest as the differential equations are concerned.

From Figures 1–3, the skin friction coefficient, the local Nusselt number, and the local Sherwood number change accordingly with the change of shrinking/stretching parameter  $\sigma$ , velocity slip A, thermal slip B, and solutal slip C. As  $\sigma$  increases, the skin friction coefficient and the local Sherwood number increase. On the other hand, the local Nusselt number decreases as  $\sigma$  increases. Furthermore, the change occurring in the local Nusselt number is very small as compared to the skin friction coefficient and the local Sherwood number.

Figure 1 portrays the skin friction coefficient for different values of A, the velocity slip parameter. The graph shows that the differences are uniform for all values of A and  $\sigma$ . It is interesting to see the different characteristics possessed by the shrinking and stretching sheets. We can see that for

Table 2: Variations of the skin friction coefficient -f''(0), local Nusselt number  $-\theta'(0)$ , and local Sherwood number  $-\beta'(0)$  for (a) stretching and (b) shrinking sheets at A=0.1 for different values of S and M.

	(a)			
S	M	-f''(0)		
3	0.1	1.180642		
4		1.444545		
5		1.676866		
3	0.0	1.189619		
	1.0	1.087972		
	1.5	1.022694		
(b)				
S	M	-f''(0)		
3	0.1	-2.105103		
4		-2.743993		
5		-3.261509		
3	0.0	-2.131948		
	1.0	-1.771988		
	1.5	-0.956086		

shrinking sheet ( $\sigma$  < 0), the skin friction coefficient increases with A. Yet, the opposite is true for the stretching sheet ( $\sigma$  > 0) where the skin friction coefficient decreases as A increases. This trait is again seen in Table 2 where we computed values of the skin friction coefficient with different values of S and M. For stretching sheet, the skin friction increases as S increases while the increment in M lowers the skin friction coefficient. On the other hand, for the shrinking sheet, the skin friction coefficient decreases as S increases and increases with increasing M. The changes occurring for a shrinking sheet is more pronounced than those of a stretching sheet. For example, this observation can be seen when M is increased, where the difference in the skin friction coefficient is approximately 10% for a stretching sheet and 35% for a shrinking sheet.

Figure 2 shows the variations of the local Nusselt number for different values of *B*. The surface heat transfer rate is consistently lower for higher thermal slip parameter *B*. This phenomenon agrees with the findings of Aman et al. [28]. The diminishing rate may be caused by the increase in thermal boundary layer thickness. Although the change in the local Nusselt number is uniform as *B* is increased, it is more pronounced than the change occurring in the skin friction coefficient and the local Sherwood number (Figure 3).

Through Figure 3, we can see the effect of the solutal slip parameter C on the local Sherwood number. As C increases, the local Sherwood number decreases consistently. It is worth to note that as we apply the solutal slip condition to a previously no-slip condition (C=0), the values reduce abruptly where the change is approximately 50%. However as C increases from 0.1 to 0.2, the difference between the resulting surface mass transfer rates is lower where the difference is about 40%.

Table 3: Variations of the local Nusselt number  $-\theta'(0)$  and local Sherwood number  $-\beta'(0)$  at Le = 2, M = 1, S = 3, A = B = C = 0.1, Pr = 6.8, and  $\sigma = -1$  for different values of Nt and Nb.

Nt = Nb	Le	- heta'(0)	$-\beta'(0)$
0.1	2	6.236334	-0.328127
0.3		5.139931	0.355463
0.5		3.885173	1.139644
0.1	3	6.156869	1.350187
	5	6.099435	3.461077
	7	6.093712	4.736331

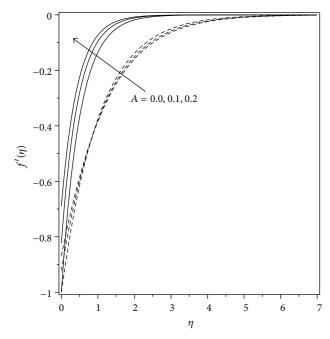


FIGURE 4: The velocity profiles for different values of *A* when S = 3, M = 1, and  $\sigma = -1$ .

Table 3 presents the variations of the local Nusselt and Sherwood numbers as we manipulate both thermophoresis and Brownian motion parameters (Nt = Nb) and Lewis number Le. As Nt and Nb increase, the local Nusselt number decreases while the local Sherwood number increases. The negative values of the local Sherwood number imply that the surface is losing mass. The local Nusselt number also reacts negatively when Le is increased. However, the mass transfer rate at the surface increases with Le. Although not shown, the thermal boundary layer thickness increases while the concentration boundary layer thickness decreases as Le is increased which causes this effect.

The samples of velocity, temperature, and concentration profiles for shrinking sheet are included in Figures 4–8. These profiles satisfy the far field boundary condition (13) asymptotically, which support the numerical results obtained besides supporting the existence of dual solutions shown in Figures 1–3. Figures 4 and 5 show the velocity profiles for different values of A and M, respectively. As A is increased, the velocity also increases. However, as M increases, the

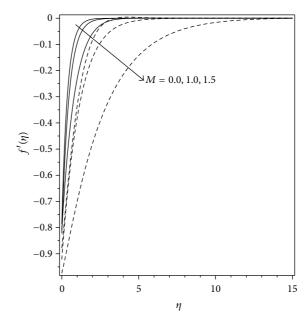


FIGURE 5: The velocity profiles for different values of M when S=3, A=0.1, and  $\sigma=-1$ .

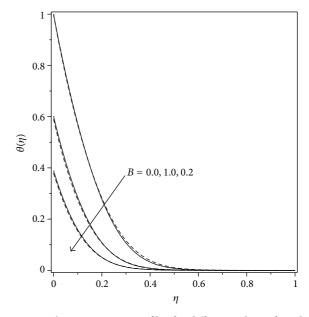


FIGURE 6: The temperature profiles for different values of B when S = 3, M = 1, Pr = 6.8, A = C = 0.1, Nt = Nb = 0.5, Le = 2, and  $\sigma = -1$ .

velocity decreases due to the retarding effect resulted from the presence of transverse magnetic field [24]. Hence, as *M* increases, the retarding force increases and thus the velocity decreases.

Through Figures 6 and 7, we can see the effect of *B* and Nt = Nb on the temperature profiles. The temperature lowers as *B* is increased. However, it increases as both Nt and Nb increase due to the fact that Nt is directly proportional to the heat transfer coefficient associated with the fluid [24]. It is also observed that the boundary layer thickness decreases with

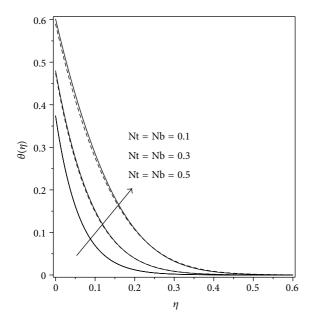


FIGURE 7: The temperature profiles for different values of Nt(= Nb) when S = 3, M = 1, Pr = 6.8, A = B = C = 0.1, Le = 2, and  $\sigma = -1$ .

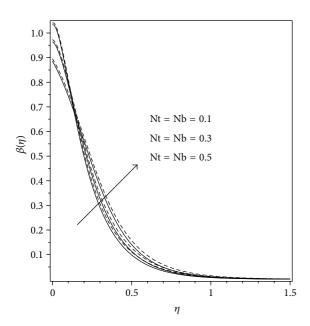


FIGURE 8: The concentration profiles for different values of Nt(= Nb) when S = 3, M = 1, Pr = 6.8, A = B = C = 0.1, Le = 2, and  $\sigma = -1$ .

increasing *B*. On the other hand, the boundary layer thickness increases as both Nt and Nb are increased. This may be the probable cause of the diminishing the local Nusselt number in Figure 2 and Table 3.

The concentration profiles for different values of Nt (= Nb) are shown in Figure 8. Due to the dependency of the concentration on the temperature field, we expect that a higher thermophoresis would allow a deeper penetration of the concentration [29]. As a result, we can see from Figure 8 that the concentration increases as we increase both Nt and Nb. In addition, we can also note that at the surface and

its surrounding area, the concentration decreases with the increasing Nt and Nb.

#### 4. Conclusions

The MHD boundary layer flow of a nanofluid past a stretching/shrinking sheet with hydrodynamic, thermal, and solutal slip boundary conditions was studied. Numerical solutions to the governing equations were obtained using a shooting method. A few observations have been made in this paper as follows.

- (1) The range of the stretching/shrinking parameter for which the solution exists increases as velocity slip parameter *A* increases.
- (2) For a shrinking sheet, the skin friction coefficient increases with the velocity slip parameter *A* and magnetic parameter *M*. For a stretching sheet, it decreases when the velocity slip parameter and magnetic parameter are increased. The changes occurring in shrinking sheet are more pronounced than those in the stretching sheet. As *M* increases, the difference in the skin friction coefficient is approximately 10% for a stretching sheet and 35% for a shrinking sheet.
- (3) The local Nusselt number diminishes as thermal slip parameter *B* increases while the local Sherwood number decreases with increasing values of the solutal slip parameter *C*.
- (4) Increasing the Lewis number and both thermophoresis parameter and Brownian motion parameter is to decrease the local Nusselt number and the local Sherwood number.

#### **Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

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