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Research Article

Dufour and Soret Effects on Melting from a Vertical Plate Embedded in Saturated Porous Media

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Thermal-diffusion and diffusion-thermo effects on combined heat and mass transfer in mixed convection boundary layer flow with aiding and opposing external flows from a vertical plate embedded in a liquid saturated porous medium with melting are investigated. The resulting system of nonlinear ordinary differential equations is solved numerically using Runge Kutta-Fehlberg with shooting techniques. Numerical results are obtained for the velocity, temperature, and concentration distributions, as well as the Nusselt number and Sherwood number for several values of the parameters, namely, the buoyancy parameter, melting parameter, Dufour effect, Soret effect, and Lewis number. The obtained results are presented graphically and in tabular form and the physical aspects of the problem are discussed.

1. Introduction

The range of free convective flows that occur in nature and in engineering practice is very large and has been extensively considered by many researchers [1, 2]. When heat and mass transfer occur simultaneously between the fluxes, the driving potentials are of more intricate nature. An energy flux can be generated not only by temperature gradients but by composition gradients as well. When mass transfer takes place in a fluid at rest, the mass is transferred purely by molecular diffusion resulting from concentration gradients. For low concentration of the mass in the fluid and low mass transfer rates, the convective heat and mass transfer process are similar in nature. A number of investigations have already been carried out on combined heat and mass transfer under the assumption of different physical situations. Thermal radiation in free convection has also been studied by many authors because of its applications in many engineering and industrial processes. Examples include nuclear power plant, solar power technology, steel industry, fossil fuel combustion, and space sciences applications. Kinyanjui et al. [3] analyzed simultaneous heat and mass transfer in unsteady free convection flows with radiation absorption past an impulsively started infinite vertical porous plate subjected to a strong magnetic field. Hayat et al. [4] analyzed a mathematical model in order to study the heat and mass transfer characteristics in the mixed convection boundary layer flow about a linearly stretching vertical surface in a porous medium filled with a viscoelastic fluid, by taking into account the diffusion-thermo (Dufour) and thermal diffusion (Soret) effects. Li et al. [5] took an account of the thermal diffusion and diffusion-thermo effects to study the properties of the heat and mass transfer in a strongly endothermic chemical reaction system for a porous medium. Gaikwad et al. [6] investigated the onset of the double diffusive convection in a two-component couple stress fluid layer with the Soret and Dufour effects using both linear and nonlinear stability analyses. Osalusi et al. [7] investigated the thermo-diffusion and diffusion-thermo effects on combined heat and mass transfer of a steady hydromagnetic convective and slip flow due to a rotating disk in the presence of viscous dissipation and Ohmic heating. Shateyi [8] investigated the thermal radiation and buoyancy effects on heat and mass transfer over a semi-infinite stretching surface with suction and blowing. Ambethkar [9] studied numerical solutions of heat and mass transfer effects of an unsteady MHD free convective flow past an infinite vertical plate with constant suction. Alam et al. [10] studied the Dufour and Soret effects on a steady MHD combined free-forced convective and mass transfer flow past a semi-infinite vertical plate, while Alam and Rahman [11] investigated the Dufour and Soret effects on the mixed convection flow past a vertical porous flat plate with variable suction. Abreu et al. [12] discussed boundary layer flows with the Dufour and Soret effects. Postelnicu [13] discussed the influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in a porous media considering the Soret and Dufour effects. Motsa [14] investigated both the Soret and Dufour effects on the onset of double diffusive convection. Moorthy and Senthilvadivu [15] investigated the effects of the heat and mass transfer characteristics of natural convection on a vertical surface embedded in a saturated porous medium subject to variable viscosity by taking into account the diffusion-thermo (Dufour) and thermal-diffusion (Soret) effects.

Convective heat transfer in porous media in the presence of melting effect has received some attention in recent years. This stems from the fact that this topic has significant direct applications in permafrost melting, frozen ground thawing, and casting and welding processes as well as phase change material (PCM). This has been shown [16] to be of special interest in the permafrost research in which the melting effect plays an important role in problems of permafrost melting and frozen ground thawing. According to the analysis of Walker [17], the phenomenon of permafrost degradation in Arctic Alaska is very critical due to global warming, and this result accelerates the greenhouse effect. Many studies have been reported to study the melting process by heat convection mechanism such as Gorla et al. [18]; Bakier [19]; Cheng and Lin [20]; Tashtoush [21]; Bakier et al. [22], Zongqin and Bejan [23]; Chang and Yang [24]; those of Cheng and Lin [25].

Motivated by the works mentioned above, it is observed that and melting in porous medium plays a significant role in modeling different physical situations. However, introducing some chemical species into the flow has tendency to affect the buoyancy due to mass transfer. In addition, the influence of heat flux on concentration as well as that of mass flux on temperature cannot be neglected in such situation. Therefore, the main objective of this work is to study thermal-diffusion and diffusion-thermo effects on combined heat and mass transfer on mixed convection boundary layer with aiding and opposing external flow from vertical plate embedded in a liquid saturated porous medium with melting.

2. Mathematical Formulation

Consider the mixed convection heat and mass transfer flow in a liquid-saturated porous medium adjacent to the vertical plate, with uniform wall temperature, that constitutes the interface between an incompressible Newtonian fluid and solid phases during melting inside the porous matrix at steady state. Figure 1 shows the coordinates and the flow model. The x-coordinate is measured along the plate while the y-coordinate is normal to it. This work will designate the flow condition sketched in Figure 1(a), as an aiding external flow, where the gravitational acceleration (g) is in the direction parallel to x-coordinate. On the other hand, the flow condition sketched in Figure 1(b) is opposing external flow where the buoyancy force has component parallel to the x-direction and free stream velocity. The temperature and concentration on the porous vertical plate, T_m, C_m, are the melting temperature and concentration of the material occupying the porous matrix. The liquid phase far from the plate is maintained at constant temperature T_∞ ($T_\infty > T_m$) and concentration C_∞ ($C_\infty > C_m$). In addition, the temperature and concentration of the solid porous medium far from the interface are constant and denoted by T_s ($T_s < T_m$) and C_s ($C_s < C_m$). Also the convective fluid and the liquid-saturated porous media are everywhere in local thermodynamics equilibrium. Properties of the fluid and the porous media such as viscosity (μ), thermal conductivity (κ), specific heat (c_p), thermal expansion coefficient (β), and permeability (K) are constant; and the Darcy's flow [26] associated with the Boussinesq approximation [27] can be applied. Therefore, the continuity, momentum, and energy transfer equations are, respectively, given as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial y} = \pm \frac{Kg}{\nu}\left(\beta_\gamma \frac{\partial T}{\partial y} + \beta_c \frac{\partial C}{\partial y}\right), \tag{2}$$

$$\begin{gathered} u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + D_1\frac{\partial^2 C}{\partial y^2}, \\ u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + D_2\frac{\partial^2 T}{\partial y^2}, \end{gathered} \tag{3}$$

where u and v are Darcy's velocities in the x and y directions; T is the temperature in the thermal boundary layer; C is the concentration; ν is the kinematic viscosity; and α is the equivalent thermal diffusivity. Additionally, it should be noted that the positive (+) and negative (−) in (2) indicate cases of aiding and opposing external flows, respectively. The boundary conditions necessary to complete the problem formulations are

$$\begin{gathered} y = 0, \qquad T = T_m, \\ k\frac{dT}{dy} = \rho_f\left[\lambda + c_s\left(T_m - T_s\right)\right]\nu, \qquad C = C_m, \\ y \longrightarrow \infty, \qquad T \longrightarrow T_\infty, \qquad u \longrightarrow U_\infty, \qquad C \longrightarrow C_\infty, \end{gathered} \tag{4}$$

where λ and c_s are latent heat of the solid and specific heat capacity of the solid phase, respectively. Particularly, the boundary condition (4) means that the temperature on the plate is uniform; and the thermal flux of heat conduction to the melting surface is equal to the heat of melting plus the sensible heat required raising the temperature of solid T_s to its melting temperature T_m [28, 29].

The continuity equation is automatically satisfied by defining a stream function $\psi(x, y)$ such that

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}. \tag{5}$$

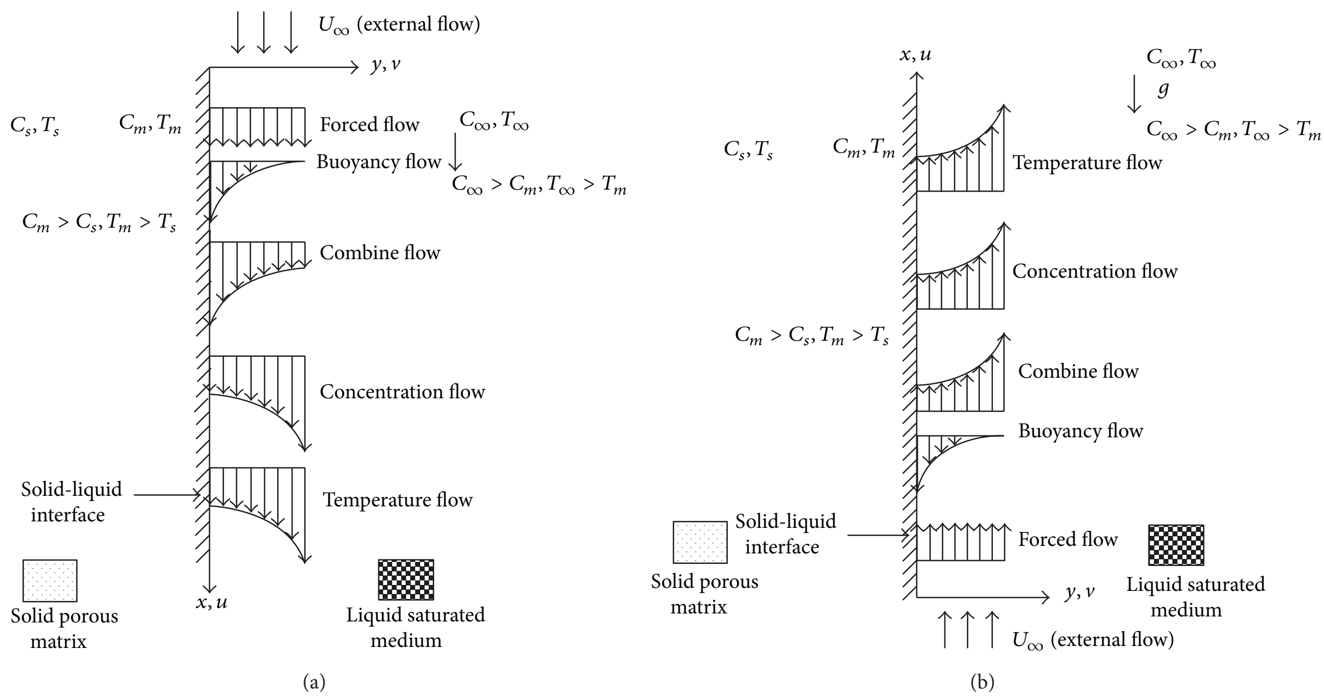

FIGURE 1: Physical model investigated in this study: (a) aiding external flow and (b) opposing external flow.

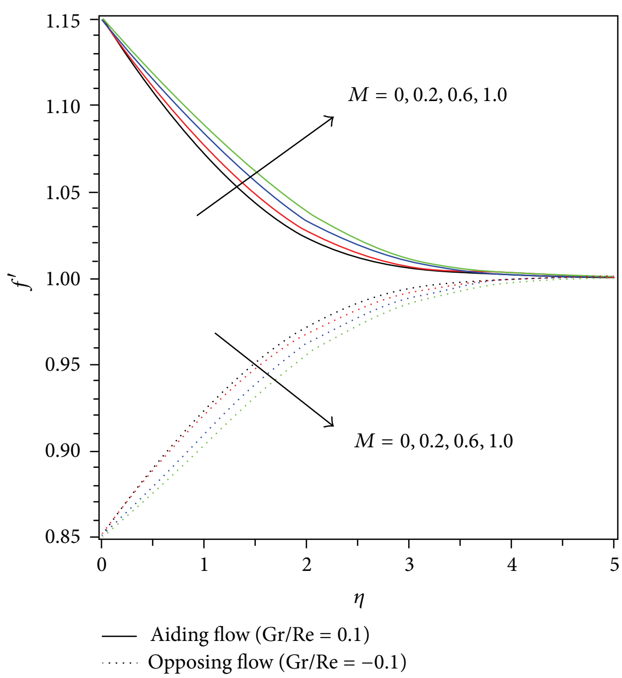

FIGURE 2: Velocity profile for both aiding and opposing external flows for different values of M, $N = 0.5$, $\mathrm{Df} = 0.02$, $\mathrm{Sr} = 0.2$, and $\mathrm{Le} = 1.0$.

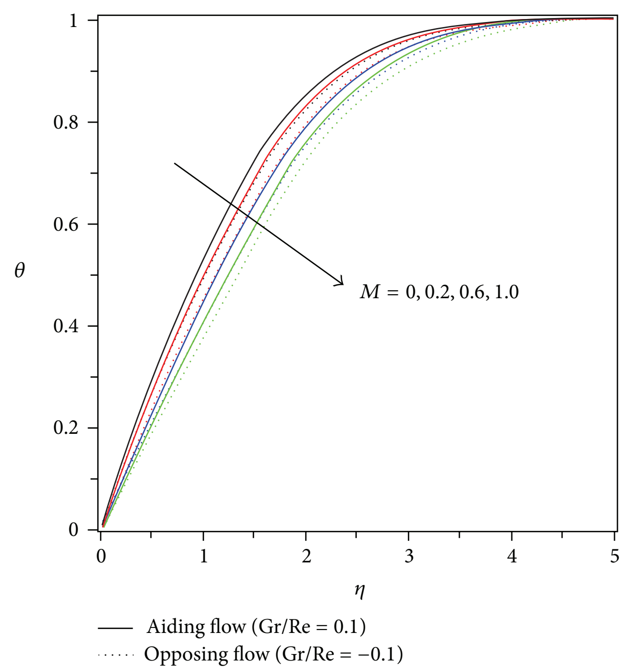

FIGURE 3: Temperature profile for both aiding and opposing external flows for different values of M, $N = 0.5$, $\mathrm{Df} = 0.02$, $\mathrm{Sr} = 0.2$, and $\mathrm{Le} = 1.0$.

We introduce the following similarity transformation:

$$\eta = \mathrm{Pe}_x^{0.5}\frac{y}{x}, \qquad \psi = \alpha \mathrm{Pe}_x^{0.5} f(\eta),$$
$$\theta(\eta) = \frac{T - T_m}{T_\infty - T_m}, \qquad \phi(\eta) = \frac{C - C_m}{C_\infty - C_m}, \tag{6}$$

where $\mathrm{Pe} = u_\infty x/\alpha$ is the Peclet number and α is the thermal diffusivit upon substituting (5) and (6) in (2) and (3), we obtain the following transformed governing equations:

$$f'' \pm \frac{\mathrm{Gr}}{\mathrm{Re}}\left(\theta' + N\phi'\right) = 0, \tag{7}$$

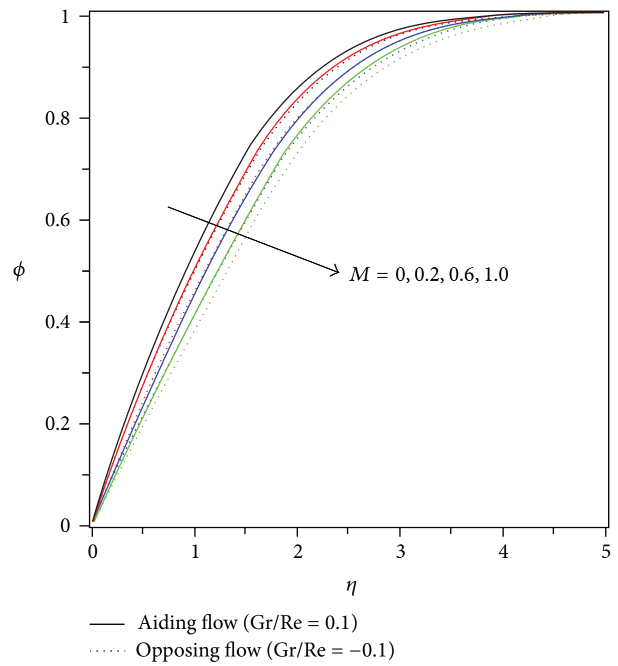

FIGURE 4: Concentration profile for both aiding and opposing external flows for different values of M, $N = 0.5$, $\mathrm{Df} = 0.02$, $\mathrm{Sr} = 0.2$, $\mathrm{Gr} = 0.1$, and $\mathrm{Le} = 1.0$.

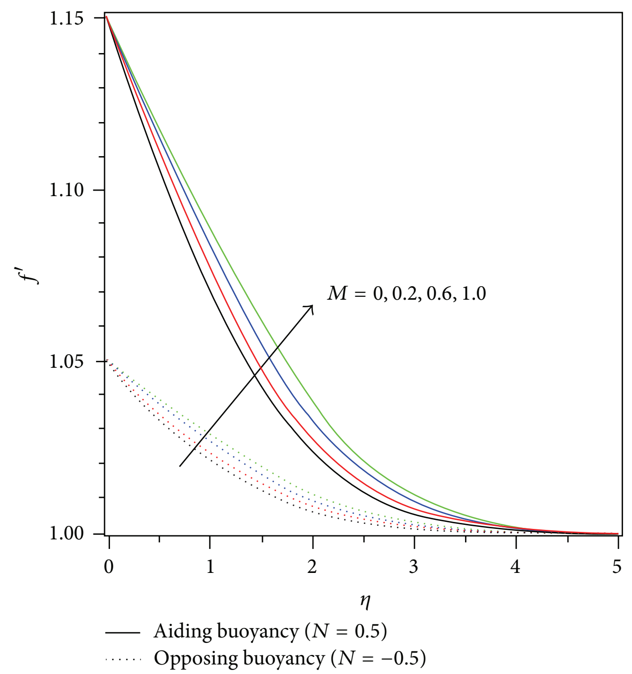

FIGURE 5: Velocity profile for both aiding and opposing buoyancy for different values of M, $\mathrm{Df} = 0.02$, $\mathrm{Sr} = 0.2$, $\mathrm{Gr/Re} = 0.1$, and $\mathrm{Le} = 1.0$.

$$\theta'' + \frac{1}{2} f\theta' + Df\phi'' = 0,$$
$$\frac{1}{\mathrm{Le}}\phi'' + \frac{1}{2} f\phi' + \mathrm{Sr}\theta'' = 0, \tag{8}$$

where the primes denote differentiation with respect to the similarity variable η; and the ratio

$$\mathrm{Gr} = \frac{kg\beta_T (T_\infty - T_m) x}{\nu^2} \quad \text{to} \quad \mathrm{Re} = \frac{U_\infty x}{\nu} \tag{9}$$

is a measurement of mixed convective flow, whose limiting case of $\mathrm{Gr/Re} = 0$ expresses the pure forced convection. The corresponding boundary conditions are

$$\eta = 0, \quad \theta = 0, \quad f(0) + 2M\theta'(0) = 0, \quad \phi = 0$$
$$\eta \longrightarrow \infty, \quad \theta = 1, \quad f' = 1, \quad \phi = 1, \tag{10}$$

where $M = C_f(T_\infty - T_m)/(1 + C_s(T_m - T_s))$ is the melting parameter, $\mathrm{Le} = \alpha/D$ is the Lewis number, $\mathrm{Df} = D_1\Delta C/\alpha\Delta T$ is the Dufour number, and $\mathrm{Sr} = D_2\Delta T/\alpha\Delta C$ is the Soret number. The buoyancy ratio is $N = \beta_c\Delta C/\beta_T\Delta T$. The parameter $N > 0$ represents the aiding buoyancy and $N < 0$ represents the opposing buoyancy.

The nondimensional heat and mass transfer coefficients are defined as

$$\mathrm{Nu} = \frac{hx}{k_{\mathrm{eff}}} = \frac{q_w x}{(T_\infty - T_m) k_{\mathrm{eff}}} = \theta'(0)\,\mathrm{Pe}^{0.5}, \tag{11}$$

$$\mathrm{Sh} = \frac{hx}{D} = \frac{q_m x}{(C_\infty - C_m) K_{\mathrm{eff}}} = \phi'(0)\,\mathrm{Pe}^{0.5}. \tag{12}$$

In practical applications, the rates of heat and mass transfer are usually expressed as the Nusselt and Sherwood numbers, where h denotes the local heat and mass transfer coefficient; $q_x = -k_{\mathrm{eff}}[\partial T/\partial y]_{y=0}$ is the wall heat flux; and $q_m = -D[\partial C/\partial y]_{y=0}$ is the wall mass flux.

3. Numerical Methods

The above equations (7) and (8) with the boundary condition (10) are coupled and nonlinear in nature. It is therefore difficult to get a closed-form solution for this system of equations. It also depends on aiding and opposing mixed convection parameter ($\pm\mathrm{Gr/Re}$), melting parameter M, Dufour effect (Df), Soret effect (Sr), and Lewis number (Le). In this section, a Runge Kutta-Fehlberg method combined with Newton's iteration methods is employed to obtain the solutions as function of the strength of melting phenomena.

4. Result and Discussion

Here, (7)-(8) subject to the boundary condition (10) were solved numerically using maple 16. This software uses a Runge Kutta-Fehlberg method as the default method to solve the boundary value problems numerically. This method has been proven to be adequate and gives the accurate results for boundary layer equations. Figures 2–6 illustrate the influence of melting parameter M on the velocity, temperature, concentration, the Nusselt number, and Sherwood number, respectively. To validate the results in the present problem, the governing parameters are chosen in the absence of mass transfer, we set $N = 0$, $\mathrm{Df} = 0$, and $\mathrm{Sr} = 0$, and the numerical

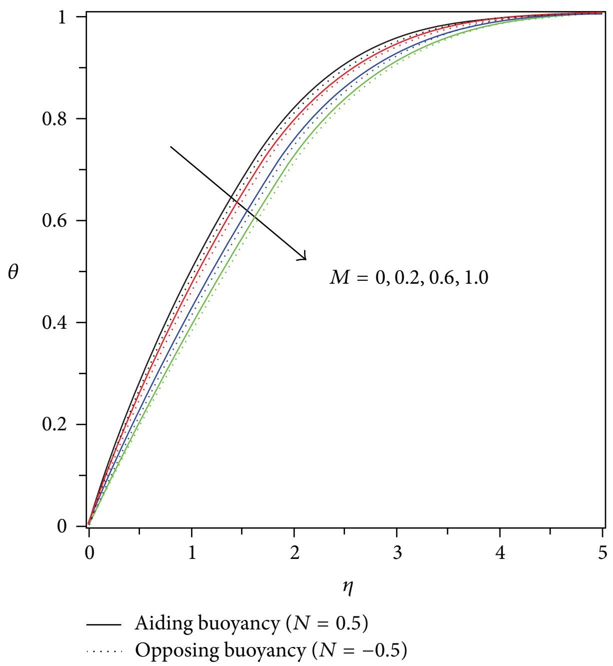

FIGURE 6: Temperature profile for both aiding and opposing buoyancy for different values of M, Df = 0.02, Sr = 0.2, Gr/Re = 0.1, and Le = 1.0.

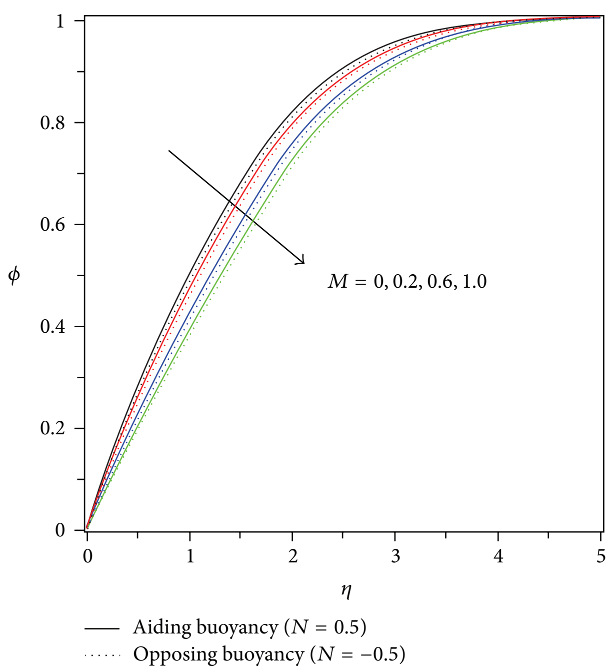

FIGURE 7: Concentration profile for both aiding and opposing buoyancy for different values of M, Df = 0.02, Sr = 0.2, Gr = 0.1, and Le = 1.0.

values are compared with existing literature in Tables 1 and 2. The comparison shows an excellent agreement between this work and that of Cheng and Lin [20]. The Dufour number and Soret number are chosen in such a way that their product is constant according to definition provided that the mean temperature T_m is kept constant (Kafoussias and Williams [30]). Figure 2 shows the dependence of dimensionless velocity profile on the melting strength Gr/Re = 0.1, respectively, for aiding and opposing external flows (±Gr/Re). In the figure, the velocity gradient is reduced with the increase in the melting strength. Figures 3 and 4 display the melting effect on the temperature and concentration distribution for aiding and opposing external flows, respectively. As observed from the figures, the temperature and concentration gradients are reduced with increasing melting strength because convective heat and mass transfer are inhibited from the liquid-saturated porous medium to the solid plate for cases of aiding and opposing external flows, but the thickness of thermal and concentration boundary layer can be reduced and thickened by increasing the mixed convective strength for heat and mass transfer in a liquid saturated porous medium with aiding and opposing external flow respectively, in the presence of melting. The effects of Dufour and Soret numbers on the velocity field, temperature field, and concentration field are listed in Tables 3 and 4 for both aiding and opposing external flows. In the velocity profile, we observed that quantitatively when $\eta = 0.5$, the response of velocity to growing Dufour and Soret is dependent on the range of values considered. For instance, in the interval $0 < \mathrm{Df}/\mathrm{Sr} < 1$, growing Dufour and Soret leads to a decrease in fluid velocity for aiding external flow while fluid velocity increases in the case of opposing external flow. On the other hand, for $\mathrm{Df}/\mathrm{Sr} > 1$, fluid velocity increases in the case of aiding external flow while it decreases in the case of opposing external flow. When Df/Sr increases, fluid temperature decreases for both aiding and opposing external flows while concentration increases in both cases.

Figures 5, 6, and 7 present the influence of melting parameter (M) on velocity, temperature, and concentration profiles for both aiding and opposing buoyancy, respectively. It is obvious that increasing the melting parameter causes higher acceleration to the fluid flow which, in turn, increases its motion and causes decrease in temperature and concentration for both aiding and opposing buoyancy. Figure 8 depicts the effect of buoyancy ratio (N) on velocity distribution. It is observed that increasing the values of (N) has a tendency to increase the slip velocity on the plate for aiding external flow while it decreases the slip velocity in the case of opposing external flow.

The impacts of the melting on local heat and mass transfer rate on the plate are sketched in Figures 9 and 10 with the help of Nusselt and Sherwood numbers in (11) and (12). As found in Figures 9 and 10, increasing the value of M significantly decreases the local heat and mass transfer rate for both aiding and opposing external flows. We also observed that the local heat and mass transfer rate grow with the increase in mixed convection parameter Gr/Re for aiding and decrease for opposing external flow.

5. Conclusion

In this paper, the thermal-diffusion and diffusion-thermo effects on combined heat and mass transfer on mixed convection boundary layer flow from a vertical plate embedded in a liquid saturated porous medium with melting have been

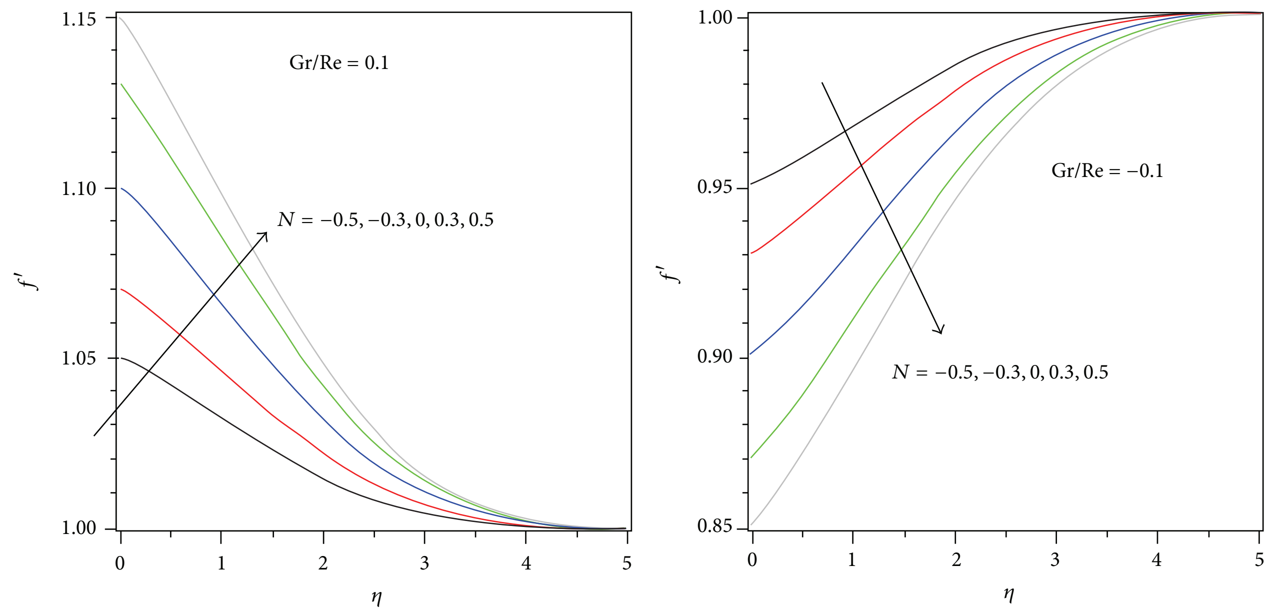

FIGURE 8: Velocity profile for both aiding and opposing external flows for different values of N, $M = 2$, $\mathrm{Df} = 0.02$, $\mathrm{Sr} = 0.2$, and $\mathrm{Le} = 1.0$.

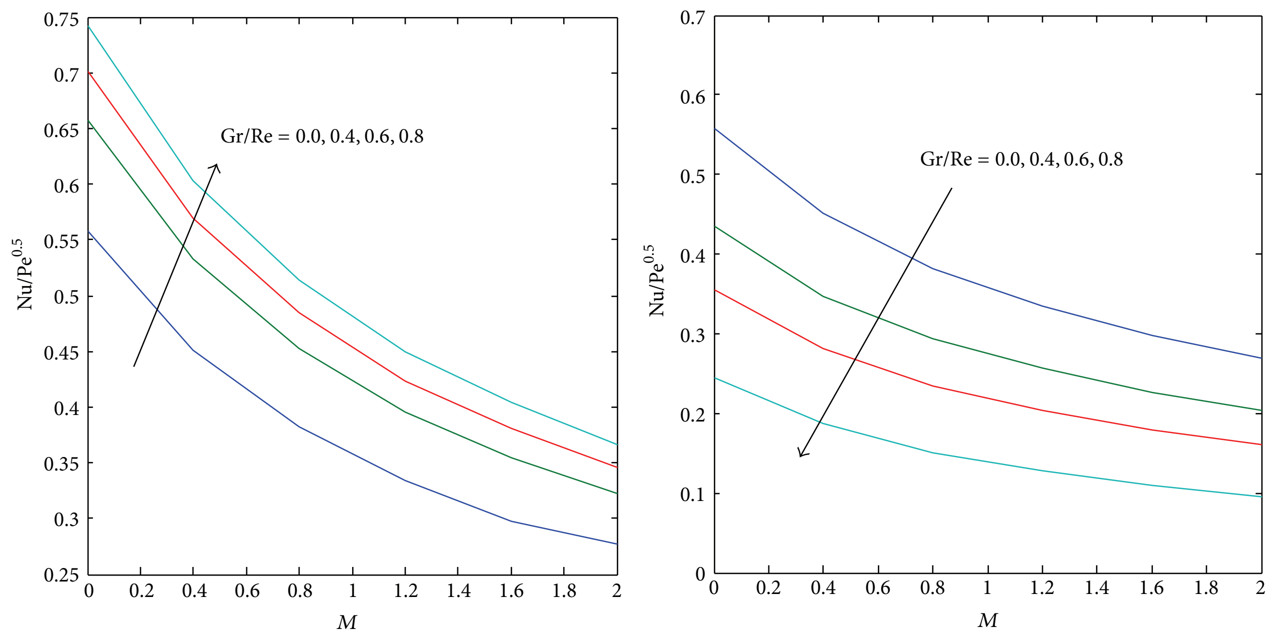

FIGURE 9: The Nusselt number varying with the melting parameter M for aiding and opposing external flows and in the different mixed convection strength, $N = 0.5$, $\mathrm{Df} = 0.03$, $\mathrm{Sr} = 0.3$, and $\mathrm{Le} = 1.0$.

comprehensively studied in the presence of aiding and opposing external flows. The governing equation was derived by the boundary layer and Boussinesq approximation. A boundary condition to account for melting was used at the interface between the solid and liquid phases. These equations were then transformed using similarity transformation and solved by the Runge Kutta-Fehlberg algorithm associated with Newton's iteration. Graphical results regarding the velocity temperature and concentration distributions as well as the Nusselt number and Sherwood number were presented and discussed for different parameters. Comparison with previously published work was performed, and the results were found to be in good agreement.

Nomenclature

c_s: Specific heat of solid phase (J/(kg K))
f: Dimensionless stream function
g: Acceleration due to gravity (m/s^2)
Gr: Grashof number

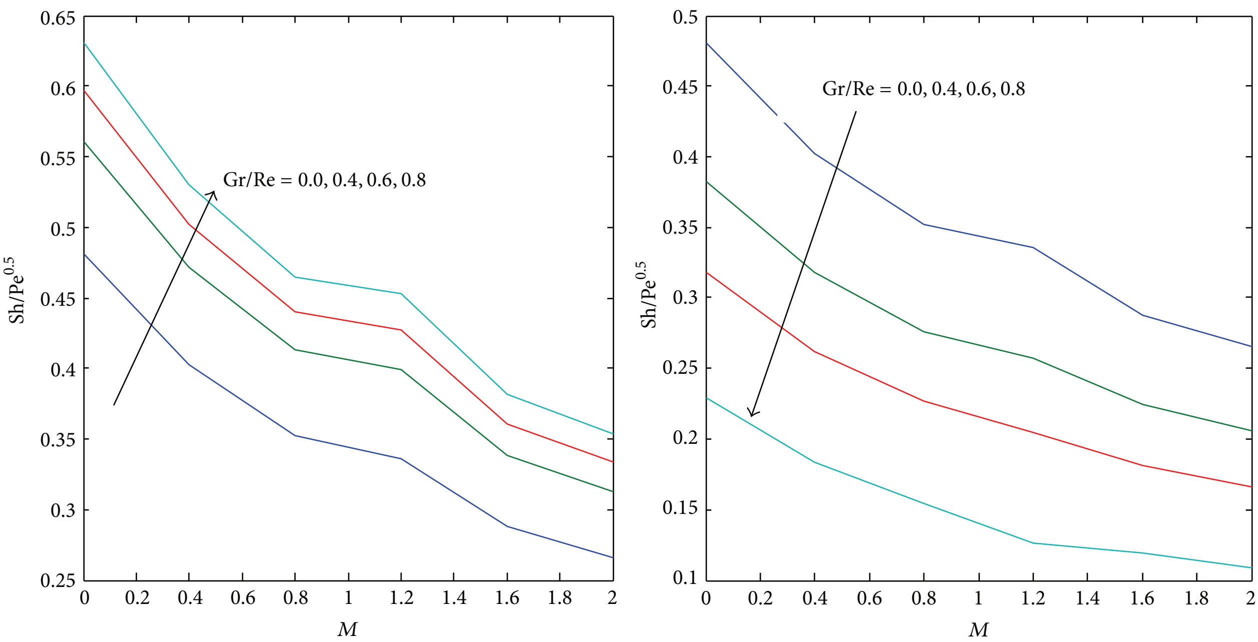

Figure 10: The Sherwood number varying with the melting parameter M for aiding and opposing external flows and in the different mixed convection strength, $N = 0.5$, $\mathrm{Df} = 0.03$, $\mathrm{Sr} = 0.3$, and $\mathrm{Le} = 1.0$.

Table 1: Comparison of present results with values obtained by Cheng and Lin [20] for the melting strength $M = 2.0$, $N = 0$, $\mathrm{Df} = 0$, $\mathrm{Sr} = 0$, and $\mathrm{Le} = 1.0$ in the mixed convective strength with an aiding external flow.

M	Gr/Re	Cheng and Lin [20]		Present work	
		η_T	$\theta'(0)$	η_T	$\theta'(0)$
2.0	0.0	1.000	0.2706	1.000	0.2706
	1.4	2.400	0.3801	2.400	0.3800
	3.0	4.000	0.4745	4.000	0.4745
	8.0	9.000	0.6902	9.000	0.6902
	10.0	11.00	0.7594	11.00	0.7594
	20.0	21.00	1.0383	21.00	1.0383
	50.0	51.00	1.6066	51.00	1.6066

Table 2: Comparison of present results with values obtained by Cheng and Lin [20] for the melting strength $M = 0.0$, $N = 0$, $\mathrm{Df} = 0$, $\mathrm{Sr} = 0$, and $\mathrm{Le} = 1.0$ in the mixed convective strength for the opposing external flow.

M	Gr/Re	Cheng and Lin [20]		Present work	
		η_T	$\theta'(0)$	η_T	$\theta'(0)$
0.0	0.2	3.8	0.5270	3.8	0.5269
	0.4	3.9	0.4866	3.9	0.4866
	0.6	4.2	0.4421	4.2	0.4421
	0.8	4.5	0.3917	4.5	0.3917
	1.0	4.9	0.3331	4.9	0.3331

Table 3: Comparison of the values of velocity, temperature, and concentration for different values of Df and Sr with $\eta = 0.5$ and $N = 0.5$ for aiding external flow ($\mathrm{Gr/Re} = 0.1$).

Df, Sr	Df/Sr	$f'(\eta)$	$\theta(\eta)$	$C(\eta)$
(0.02, 2.0)	0.01	1.4758	0.3013	0.0184
(0.04, 1.0)	0.04	1.4498	0.2960	0.1592
(0.2, 0.2)	1.0	1.4359	0.2734	0.2734
(1.0, 0.04)	25	1.4723	0.1636	0.3111
(2.0, 0.02)	100	1.5315	0.0026	0.3373
(0.08, 2.0)	0.04	1.4757	0.3048	0.0114
(0.1, 1.6)	0.06	1.4653	0.3009	0.0717
(0.4, 0.4)	1.00	1.4462	0.2563	0.2563
(1.6, 0.1)	16	1.5075	0.0667	0.3291
(2.0, 0.08)	25	1.5342	0.0034	0.3436

Table 4: Comparison of the values of velocity, temperature, and concentration for different values of Df and Sr with $\eta = 0.5$ and $N = 0.5$ for opposing external flow ($\mathrm{Gr/Re} = -0.1$).

Df, Sr	Df/Sr	$f'(\eta)$	$\theta(\eta)$	$C(\eta)$
(0.02, 2.0)	0.01	0.4846	0.1907	0.0416
(0.04, 1.0)	0.04	0.5011	0.1930	0.1194
(0.2, 0.2)	1.0	0.5100	0.1834	0.1834
(1.0, 0.04)	25	0.4865	0.1179	0.1966
(2.0, 0.02)	100	0.4503	0.0261	0.1993
(0.08, 2.0)	0.04	0.4846	0.1924	0.0380
(0.1, 1.6)	0.06	0.4911	0.1924	0.0707
(0.4, 0.4)	1.00	0.5031	0.1719	0.1719
(1.6, 0.1)	16	0.4647	0.06177	0.1997
(2.0, 0.08)	25	0.4486	0.0201	0.2026

h: Local heat transfer coefficient (J/(s m^2 K))
C_ρ: Specific heat of convective fluid (J/(kg K))
κ: Permeability of the porous medium (m^2)
M: Melting parameter
Df: Dufour number
Sr: Soret number
Le: Lewis number
$N > 0$: Aiding buoyancy
$N < 0$: Opposing buoyancy
$N = 0$: Absent of mass transfer
±Gr/Re: Aiding and opposing external flow in (7)
Nu: Nusselt number defined in (11)
Sh: Sherwood number defined in (12)
Pe: Local Peclet number defined in (6)
q_w: Wall heat flux (J/(s m^2))
k_{eff}: Effective thermal conductivity (J/(s m K))
Re: Local Reynolds number
T: Temperature in thermal boundary layer (K)
C: Temperature in concentration boundary layer
u: Darcy's velocity in x-direction (m/s)
u_∞: Velocity of external flow (m/s)
v: Darcy's velocity in y-direction (m/s)
x: Coordinate along the melting plate (m)
y: Coordinate normal to melting plate (m).

Greek Symbols

α: Equivalent thermal diffusivity (m^2/s)
β: Coefficient of thermal expansion (1/K)
η: Dimensionless similarity variable defined in Equations
η_T: Value of η at the edge of the thermal boundary layer
θ: Dimensionless temperature in (6)
ϕ: Dimensionless concentration in (6)
λ: Latent heat of melting of solid (J/kg)
μ: Dynamic viscosity of fluid (kg/(s m))
υ: Kinematic viscosity of fluid (m^2/s)
ρ_f: Density of convective fluid (kg/m^3)
ψ: Stream function (m^2/s).

Subscripts

m: Melting point
∞: Condition at infinity
s: Condition at solid.

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