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Research Article

Stability Analysis for Impulsive Stochastic Reaction-Diffusion Differential System and Its Application to Neural Networks

Yanke Du,[1] Yanlu Li,[2] and Rui Xu[1]

[1] *Institute of Applied Mathematics, Shijiazhuang Mechanical Engineering College, Shijiazhuang 050003, China*
[2] *Training Department, Shijiazhuang Mechanical Engineering College, Shijiazhuang 050003, China*

Correspondence should be addressed to Yanke Du; yankedu2011@163.com

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This paper is concerned with the stability of impulsive stochastic reaction-diffusion differential systems with mixed time delays. First, an equivalent relation between the solution of a stochastic reaction-diffusion differential system with time delays and impulsive effects and that of corresponding system without impulses is established. Then, some stability criteria for the stochastic reaction-diffusion differential system with time delays and impulsive effects are derived. Finally, the stability criteria are applied to impulsive stochastic reaction-diffusion Cohen-Grossberg neural networks with mixed time delays, and sufficient conditions are obtained for the exponential p-stability of the zero solution to the neural networks. An example is given to illustrate the effectiveness of our theoretical results. The systems we studied in this paper are more general, and some existing results are improved and extended.

1. Introduction

In recent years, impulsive dynamical systems have attracted considerable attention due to its wide applications in the areas of economics, physics, population dynamics, engineering, biology, and so on. These systems arise because they are subject to abrupt state changes at certain moments of time, and these changes may be related to such phenomena as shocks, harvesting, or other faults. Meanwhile, time delays are frequently encountered in real world, which can cause instability and oscillations in a system. A large number of stability criteria of impulsive delay systems have been reported (see [1–6] and references therein).

Stochastic effects are common phenomena due to disturbances or uncertainties in a system. A lot of dynamical systems have variable structures subject to stochastic abrupt changes, which may result from abrupt phenomena such as stochastic failures and repairs of the components, changes in the interconnections of subsystems, and sudden environment switching. Hence, considerable attention has been paid to the study of stochastic systems, and various interesting results have been reported in the literatures; for example, see [2–4, 7, 8] and references therein. In particular, Li et al. [8] considered the following impulsive stochastic differential system with time delay:

$$\begin{aligned} \mathrm{d}y_i(t,x) &= F_i\big(t, y_1(t), \dots, y_n(t), y_1(t-\tau(t)), \\ &\qquad \dots, y_n(t-\tau(t))\big)\,\mathrm{d}t \\ &\quad + G_i\big(t, y_1(t), \dots, y_n(t), y_1(t-\tau(t)), \\ &\qquad \dots, y_n(t-\tau(t))\big)\,\mathrm{d}w_i(t), \quad t \neq t_k, \\ y_i(t_k^+) - y_i(t_k) &= I_{ki}\big(y_1(t_k), \dots, y_n(t_k)\big), \\ &\qquad t = t_k, \quad k \in N, \quad i = 1, 2, \dots, n. \end{aligned} \tag{1}$$

They showed the stability results of system (1) by transforming (1) into an equivalent system without impulses.

Generally speaking, diffusion effects cannot be avoided in systems modeling many real world phenomena. As a representation example in neural networks, when electrons are moving in an asymmetric electromagnetic field, it inevitably leads to diffusion phenomena. In [5, 9–11], the stabilities of the equilibrium points of some types of neural networks with reaction-diffusion terms have been investigated. On the other hand, distributed delay systems can characterize the cumulative effects of the past values of the dynamic and are often used to model the time lag phenomena in thermodynamics, ecology, epidemiology, and neural networks. For example, neural networks usually have a spatial extent due to the presence of a multitude of parallel pathways with a variety of axon sizes and lengths. It is desired to model them by introducing continuously distributed delays over a certain duration of time such that the distant past has less influence compared to the recent behavior of the state.

However, in [8], the authors neglected the effects of diffusion and distributed delays. To the best of our knowledge, there are few results about the stability of impulsive stochastic reaction-diffusion deferential systems (ISRDDSs) with mixed time delays. Motivated by [8] and the previous discussions, we are concerned with the stability of the following ISRDDS with time-varying discrete delays and distributed delays:

$$\begin{aligned}
\mathrm{d}u(t,x) &= \nabla\cdot\left(D(t,x,u(t,x))\circ\nabla u(t,x)\right)\mathrm{d}t \\
&\quad + F\Big(t,u(t,x),u(t-\tau_1(t),x),\dots, \\
&\qquad u(t-\tau_p(t),x),\int_{t-r(t)}^{t}u(s,x)\,\mathrm{d}s\Big)\mathrm{d}t \\
&\quad + G\Big(t,u(t,x),u(t-\tau_1(t),x),\dots, \\
&\qquad u(t-\tau_p(t),x), \\
&\qquad \int_{t-r(t)}^{t}u(s,x)\,\mathrm{d}s\Big)\mathrm{d}w(t),\quad t\neq t_k, \\
u(t_k^+,x)-u(t_k,x) &= I_k(u(t_k,x)), \\
&\qquad t=t_k,\quad k\in N, \\
u(t,x) &= \phi(t,x), \\
&(t,x)\in[-\gamma,0]\times\Omega,\quad \Omega\in\mathbb{R}^m,
\end{aligned}\tag{2}$$

in which

$$\begin{gathered}
u(t,x) = (u_1(t,x),u_2(t,x),\dots,u_n(t,x))^{\mathrm{T}}, \\
u(t-\tau_j(t),x) = \big(u_1(t-\tau_j(t),x), u_2(t-\tau_j(t),x),\dots, u_n(t-\tau_j(t),x)\big)^{\mathrm{T}}\quad (j=1,2,\dots,p), \\
\int_{t-r(t)}^{t}u(s,x)\,\mathrm{d}s = \left(\int_{t-r_1(t)}^{t}u_1(s,x)\,\mathrm{d}s,\int_{t-r_2(t)}^{t}u_2(s,x)\,\mathrm{d}s,\dots,\int_{t-r_n(t)}^{t}u_n(s,x)\,\mathrm{d}s\right)^{\mathrm{T}}, \\
F=(F_1,\dots,F_n)^{\mathrm{T}},\qquad G=(G_1,\dots,G_n)^{\mathrm{T}}, \\
I_k=(I_{k1},\dots,I_{kn})^{\mathrm{T}}\quad (k\in N), \\
\phi=(\phi_1,\dots,\phi_n)^{\mathrm{T}}, \\
D(t,x,u(t,x)) = (D_{il}(t,x,u(t,x)))_{n\times m}, \\
\nabla u=(\nabla u_1,\dots,\nabla u_n)^{\mathrm{T}},\qquad \nabla u_i=\left(\frac{\partial u_i}{\partial x_1},\dots,\frac{\partial u_i}{\partial x_m}\right), \\
D\circ\nabla u=\left(D_{il}(t,x,u(t,x))\frac{\partial u_i}{\partial x_l}\right)_{n\times m}, \\
Y=(Y_1,\dots,Y_n)^{\mathrm{T}},\qquad Y_i=(y_{i1},\dots,y_{im})^{\mathrm{T}}, \\
\nabla\cdot Y_i=\sum_{l=1}^{m}\frac{\partial y_{il}}{\partial x_l},\qquad \nabla\cdot Y=(\nabla\cdot Y_1,\dots,\nabla\cdot Y_n)^{\mathrm{T}}.
\end{gathered}\tag{3}$$

$u_i(t,x)$ is the state variable, x_i is the space variable, $D_{il}(t,x,u(t,x))\geq 0$ is a diffusion operator, $\tau_j(t)$ and $r_i(t)$ are time-varying functions, $w(t)=(w_1(t),w_2(t),\dots,w_n(t))^{\mathrm{T}}\in\mathbb{R}^n$ is a Brownian motion defined on a complete probability space $(S,\mathscr{F},\mathscr{P})$, and I_{ki} is the impulsive function; $\gamma=\max_{1\leq j\leq p,1\leq i\leq n}\{\tau_j,r_i\}$, $\tau_j=\sup_{t\geq 0}\tau_j(t)$, and $r_i=\sup_{t\geq 0}r_i(t)$.

The organization of this paper is as follows. In Section 2, some preliminaries are given. In Section 3, by transforming the solutions of the stochastic reaction-diffusion differential system with delay and impulsive effects into that of the corresponding system without impulses, some stability criteria for the stochastic reaction-diffusion differential system with delay and impulsive effects are derived. In Section 4, the stability criteria are applied to impulsive stochastic reaction-diffusion Cohen-Grossberg neural networks (ISRD-CGNNs) with mixed time delays, and sufficient conditions are obtained for the exponential p-stability of the zero solution to the neural networks. In Section 5, a numerical example is provided to illustrate the effectiveness of the theoretical results. A concluding remark is given in Section 6 to end this work.

2. Preliminaries

For convenience, we introduce several notations. Let $PC([-\gamma,0]\times\Omega,\mathbb{R}^n):=\{u(t,x):[-\gamma,0]\times\Omega\to\mathbb{R}^n\mid u(t,x)$ is continuous at $t\neq t_k$, $u(t_k^-,x)=u(t_k,x)$, and $u(t_k^+,x)$ exists for $t_k\}$. For $u(t,x)=(u_1(t,x),\dots,u_n(t,x))^{\mathrm{T}}\in\mathbb{R}^n$, we define

$$\|u(t,x)\|=\sum_{i=1}^{n}\|u_i(t,x)\|=\sum_{i=1}^{n}\left(\int_{\Omega}|u_i(t,x)|^p\mathrm{d}x\right)^{1/p},\tag{4}$$

and for $\phi(s,x)=(\phi_1(s,x),\dots,\phi_n(s,x))^{\mathrm{T}}\in PC([-\gamma,0]\times\Omega,\mathbb{R}^n)$, we define $\|\phi(s,x)\|=\sup_{-\gamma\leq s\leq 0}\sum_{i=1}^{n}\|\phi_i(s,x)\|$. Denote $PC_{F_0}^{b}([-\gamma,0]\times\Omega,\mathbb{R}^n):=\{u(t,x)\in PC([-\gamma,0]\times\Omega,\mathbb{R}^n)\mid u(t,x)$ is bounded, F_0-measurable, and $E\|u(t,x)\|<\infty\}$, $PC_{F_0}^{b}(\delta):=\{u(t,x)\in PC_{F_0}^{b}([-\gamma,0]\times\Omega,\mathbb{R}^n)\mid E\|u(t,x)\|\leq\delta\}$. Throughout this paper, we always assume that a product equals to unity if the number of factors is zero.

Definition 1. A function $u(t,x) : [-\gamma, 0] \times \Omega \to \mathbb{R}^n$ is said to be the solution of system (2) if the following conditions are satisfied:

(i) $u(t,x)$ is piecewise continuous with the first kind discontinuity at the points $t_k, k \in N$. Moreover, $u(t,x)$ is left continuous at each point,

(ii) $u(t,x)$ satisfies system (2).

Definition 2. The zero solution of system (2) is said to be as follows.

(i) p-stable if, for any $\varepsilon > 0$, there exists a $\delta > 0$ such that the initial function $\phi \in PC^b_{F_0}(\delta)$ implies $E\|u(t,x)\|^p < \varepsilon$ for $(t,x) \in (0,+\infty) \times \Omega$. Especially, when $p = 1$, it is said to be stable.

(ii) Exponentially p-stable if there is a pair of positive constants λ and K such that, for any initial condition $\phi \in PC^b_{F_0}([-\gamma,0] \times \Omega, \mathbb{R}^n)$, there holds $E\|u(t,x)\|^p \le K\|\phi\|^p e^{-\lambda t}$, $t \ge 0$. Here, λ is called the exponential convergence rate. When $p = 2$ especially, it is said to be exponentially stable in mean square.

(iii) Asymptotically stable if it is stable, and there exists a $\delta > 0$ such that the initial function $\phi \in PC^b_{F_0}(\delta)$ implies $\lim_{t\to\infty} E\|u(t,x)\| = 0$.

For system (2), one makes the following assumptions:

(H1) $0 \le t_0 < t_1 < t_2 < \cdots < t_k < \cdots$ are fixed impulsive moments such that $t_k \to \infty$ as $k \to \infty$;

(H2) $F, G : R_+ \times \underbrace{R^n \times \cdots \times R^n}_{p+2} \to R^n$ satisfy $F(t,0,\ldots,0) \equiv 0$ and $G(t,0,\ldots,0) \equiv 0$;

(H3) $I_k : R^n \to R^n$ satisfies $I_k(0) = 0$, $k \in N$;

(H4) except for the zero solution of (2), for any solution $u(t,x)$ of (2), $u(t,x) \ne 0$ and $I_k(u(t_k,x)) \ne -u(t_k,x)$, $k \in N$.

Denoting

$$J_{ki} := \frac{u_i(t_k,x)}{u_i(t_k,x) + I_{ki}(u_1(t_k,x),\ldots,u_n(t_k,x))}, \qquad i = 1,2,\ldots,n, \quad k \in N, \tag{5}$$

we consider the following delayed stochastic reaction-diffusion differential system without impulses:

$$\begin{aligned}
dv(t,x) &= \nabla\cdot\left(\widetilde{D}(t,x,v(t,x)) \circ \nabla v(t,x)\right) dt \\
&\quad + \prod_{0\le t_k<t} J_k \widetilde{F}\Big(t, v(t,x), v(t-\tau_1(t),x), \ldots, v(t-\tau_p(t),x), \int_{t-r(t)}^{t} v(s,x)\,ds\Big) dt \\
&\quad + \prod_{0\le t_k<t} J_k \widetilde{G}\Big(t, v(t,x), v(t-\tau_1(t),x), \ldots, v(t-\tau_p(t),x), \int_{t-r(t)}^{t} v(s,x)\,ds\Big) dw(t), \\
v(t,x) &= \phi(t,x), \quad (t,x) \in [-\gamma,0] \times \Omega,
\end{aligned} \tag{6}$$

where

$$\begin{aligned}
J_k &= \operatorname{diag}(J_{k1}, J_{k2}, \ldots, J_{kn}), \\
\widetilde{D}(t,x,v(t,x)) &= D\Big(t, x, \prod_{0\le t_k<t} J_k^{-1} v(t,x)\Big), \\
\widetilde{F}\Big(t, v(t,x), v(t-\tau_1(t),x), \ldots, v(t-\tau_p(t),x), \int_{t-r(t)}^{t} v(s,x)\,ds\Big)
&= F\Big(t, \prod_{0\le t_k<t} J_k^{-1} v(t,x), \prod_{0\le t_k<t-\tau_1(t)} J_k^{-1} v(t-\tau_1(t),x), \ldots, \\
&\qquad \prod_{0\le t_k<t-\tau_p(t)} J_k^{-1} v(t-\tau_p(t),x), \int_{t-r(t)}^{t} \prod_{0\le t_k<s} J_k^{-1} v(s,x)\,ds\Big), \\
\widetilde{G}\Big(t, v(t,x), v(t-\tau_1(t),x), \ldots, v(t-\tau_p(t),x), \int_{t-r(t)}^{t} v(s,x)\,ds\Big)
&= G\Big(t, \prod_{0\le t_k<t} J_k^{-1} v(t,x), \prod_{0\le t_k<t-\tau_1(t)} J_k^{-1} v(t-\tau_1(t),x), \ldots, \\
&\qquad \prod_{0\le t_k<t-\tau_p(t)} J_k^{-1} v(t-\tau_p(t),x), \int_{t-r(t)}^{t} \prod_{0\le t_k<s} J_k^{-1} v(s,x)\,ds\Big).
\end{aligned} \tag{7}$$

A function vector $v(t,x)$ is a solution of (6) on $[-\gamma,+\infty) \times \Omega$ if it is absolutely continuous on $[-\gamma,+\infty) \times \Omega$ and satisfies (6) almost everywhere for $t \ge 0$ and $x \in \Omega$.

3. Stability Criteria

In this section, we first establish an equivalent relation between the solution of system (2) and that of system (6).

Lemma 3. *Assume that (H1)–(H4) hold. Then $u(t,x)$ is a solution of system (2) if and only if $v(t,x)$ is a solution of system (6), where $u(t,x) = \prod_{0\le t_k<t} J_k^{-1} v(t,x)$ or $v(t,x) = \prod_{0\le t_k<t} J_k u(t,x)$.*

Proof. First, we prove the sufficiency. Letting $v(t,x)$ be a solution of system (6), we derive that, for any $t \ne t_k$,

$$\begin{aligned}
\mathrm{d}u(t,x) &= \mathrm{d}\left(\prod_{0\le t_k<t} J_k^{-1} v(t,x)\right) = \prod_{0\le t_k<t} J_k^{-1}\mathrm{d}(v(t,x)) \\
&= \prod_{0\le t_k<t} J_k^{-1}\nabla\cdot\left(\widetilde{D}(t,x,v(t,x))\circ\nabla v(t,x)\right)\mathrm{d}t \\
&\quad + \widetilde{F}\Bigg(t, v(t,x), v(t-\tau_1(t),x),\ldots, \\
&\qquad v(t-\tau_p(t),x), \int_{t-r(t)}^{t} v(s,x)\,\mathrm{d}s\Bigg)\mathrm{d}t \\
&\quad + \widetilde{G}\Bigg(t, v(t,x), v(t-\tau_1(t),x), \\
&\qquad \ldots, v(t-\tau_p(t),x), \\
&\qquad \int_{t-r(t)}^{t} v(s,x)\,\mathrm{d}s\Bigg)\mathrm{d}w(t) \\
&= \prod_{0\le t_k<t} J_k^{-1}\nabla\cdot\Bigg(D\Bigg(t,x,\prod_{0\le t_k<t} J_k^{-1}v(t,x)\Bigg) \\
&\qquad \circ\nabla\Bigg(\prod_{0\le t_k<t} J_k u(t,x)\Bigg)\Bigg)\mathrm{d}t \\
&\quad + F\Bigg(t, \prod_{0\le t_k<t} J_k^{-1}v(t,x), \\
&\qquad \prod_{0\le t_k<t-\tau_1(t)} J_k^{-1} v(t-\tau_1(t),x),\ldots, \\
&\qquad \prod_{0\le t_k<t-\tau_p(t)} J_k^{-1} v(t-\tau_p(t),x), \\
&\qquad \int_{t-r(t)}^{t} \prod_{0\le t_k<s} J_k^{-1} v(s,x)\,\mathrm{d}s\Bigg)\mathrm{d}t \\
&\quad + G\Bigg(t, \prod_{0\le t_k<t} J_k^{-1}v(t,x), \\
&\qquad \prod_{0\le t_k<t-\tau_1(t)} J_k^{-1} v(t-\tau_1(t),x),\ldots, \\
&\qquad \prod_{0\le t_k<t-\tau_p(t)} J_k^{-1} v(t-\tau_p(t),x), \\
&\qquad \int_{t-r(t)}^{t} \prod_{0\le t_k<s} J_k^{-1} v(s,x)\,\mathrm{d}s\Bigg)\mathrm{d}w(t) \\
&= \nabla\cdot(D(t,x,u(t,x))\circ\nabla u(t,x))\,\mathrm{d}t \\
&\quad + F\Bigg(t, u(t,x), u(t-\tau_1(t),x),\ldots, \\
&\qquad u(t-\tau_p(t),x), \int_{t-r(t)}^{t} u(s,x)\,\mathrm{d}s\Bigg)\mathrm{d}t \\
&\quad + G\Bigg(t, u(t,x), u(t-\tau_1(t),x),\ldots, \\
&\qquad u(t-\tau_p(t),x), \\
&\qquad \int_{t-r(t)}^{t} u(s,x)\,\mathrm{d}s\Bigg)\mathrm{d}w(t).
\end{aligned} \tag{8}$$

On the other hand, for any $k \in N$, we have

$$\begin{aligned}
u(t_k^+,x) &= \lim_{t\to t_k^+}\prod_{0\le t_j<t} J_j^{-1} v(t,x) = \prod_{0\le t_j\le t_k} J_j^{-1} v(t_k^+,x) \\
&= J_k^{-1}\prod_{0\le t_j<t_k} J_j^{-1} v(t_k,x) = J_k^{-1}u(t_k,x) \\
&= u(t_k,x) + I_k(u(t_k,x)),
\end{aligned} \tag{9}$$

$$\begin{aligned}
u(t_k^-,x) &= \lim_{t\to t_k^-}\prod_{0\le t_j<t} J_j^{-1} v(t,x) \\
&= \prod_{0\le t_j<t_k} J_j^{-1} v(t_k^-,x) \\
&= \prod_{0\le t_j<t_k} J_j^{-1} v(t_k,x) = u(t_k,x).
\end{aligned} \tag{10}$$

Further, if $v(t,x)$ is a solution of system (6) with initial condition $v(t,x) = \phi(t,x)$, $(t,x) \in [-\gamma,0]\times\Omega$, then $u(t,x) = \prod_{0\le t_k<t} J_k^{-1} v(t,x) = v(t,x) = \phi(t,x)$, $(t,x) \in [-\gamma,0]\times\Omega$. Therefore, $u(t,x)$ is a solution of system (2) with initial condition $u(t,x) = \phi(t,x)$, $(t,x)\in[-\gamma,0]\times\Omega$, and the sufficiency is proved. Similarly, we can prove the necessity. □

Remark 4. Lemma 3 gives the equivalent relation between the solution of a stochastic reaction-diffusion differential delay system with impulsive effects and the solution of a corresponding system without impulses. Based on the "equivalent method," the existence and uniqueness of the solution of a stochastic reaction-diffusion differential delay system with impulsive effects can be derived by a new way; that is, any conditions that ensure the existence and uniqueness of the solution of system (6) without impulses will also ensure the existence and uniqueness of the system (2) with impulses.

In what follows, we will reduce the stabilities of system (2) to those of corresponding system (6).

Theorem 5. *Under assumptions (H1)–(H4), if there exists a constant $M > 0$ such that, for any $t > 0$,*

$$\left| \prod_{0 \le t_k < t} J_{ki}^{-1} \right| \le M, \quad i = 1, 2, \ldots, n, \tag{11}$$

and the zero solution of (6) *is p-stable (exponentially p-stable, asymptotically stable), then the zero solution of* (2) *is also p-stable (exponentially p-stable, asymptotically stable).*

Proof. Let $u(t,x)$ and $v(t,x)$ be the solutions of systems (2) and (6), respectively. Since the zero solution of (6) is p-stable, we have that, for any $\varepsilon > 0$, there exists a scalar $\delta > 0$ such that the initial condition $\phi \in PC_{F_0}^{b}(\delta)$ implies $E\|v(t,x)\|^p < \varepsilon/M^p$ for $(t,x) \in (0,+\infty) \times \Omega$. By Lemma 3, $u(t,x) = \prod_{0 \le t_k < t} J_k^{-1} v(t,x)$ is a solution of (2) on $(t,x) \in [-\gamma,+\infty) \times \Omega$. Furthermore, it is easy to see that

$$\begin{aligned}
E\|u(t,x)\|^p &= E\left\|\left(u_1(t,x), \ldots, u_n(t,x)\right)^{\mathrm{T}}\right\|^p \\
&= E\left\|\left(\prod_{0 \le t_k < t} J_{k1}^{-1} v_1(t,x), \ldots, \prod_{0 \le t_k < t} J_{kn}^{-1} v_n(t,x)\right)^{\mathrm{T}}\right\|^p \\
&= E\left[\sum_{i=1}^{n}\left(\int_\Omega \left|\prod_{0 \le t_k < t} J_{ki}^{-1} v_i(t,x)\right|^p \mathrm{d}x\right)^{1/p}\right]^p \\
&\le E\left[M\sum_{i=1}^{n}\left(\int_\Omega |v_i(t,x)|^p \mathrm{d}x\right)^{1/p}\right]^p \\
&= M^p E\|v(t,x)\|^p < M^p \cdot \frac{\varepsilon}{M^p} = \varepsilon.
\end{aligned} \tag{12}$$

Hence, the zero solution of (2) is p-stable. Using similar arguments, we can verify that if the zero solution of (6) is exponentially p-stable (asymptotically stable), then the zero solution of (2) is also exponentially p-stable (asymptotically stable). This completes the proof. □

In a similar way, we can derive the following results.

Theorem 6. *Under assumptions (H1)–(H4), if there exists a constant $L > 0$ such that, for any $t > 0$,*

$$\left| \prod_{0 \le t_k < t} J_{ki} \right| \le L, \quad i = 1, 2, \ldots, n, \tag{13}$$

and the zero solution of (2) *is p-stable (exponentially p-stable, asymptotically stable), then the zero solution of* (6) *is also p-stable (exponentially p-stable, asymptotically stable).*

Combining Theorems 5 and 6, one can easily obtain the following results.

Theorem 7. *Assume that (H1)–(H4) hold and inequalities* (11) *and* (13) *are satisfied, then the zero solution of* (2) *is p-stable (exponentially p-stable, asymptotically stable) if and only if the zero solution of* (6) *is also p-stable (exponentially p-stable, asymptotically stable).*

Remark 8. For impulsive neural networks, many researchers supposed that the impulsive operators are linear (e.g., [7, 12–15]); that is,

$$x_i(t_k^+) - x_i(t_k) = -\gamma_{ki} x_i(t_k), \quad 0 < \gamma_{ki} \le 2. \tag{14}$$

From the definition of J_{ki} in Section 2, we have $|J_{ki}^{-1}| = |1 - \gamma_{ki}| < 1$; then $|\prod_{0 \le t_k < t} J_{ki}^{-1}| \le 1$. Obviously, the condition (11) in Theorem 5 is less conservative than (14), in which (11) ensures that the stability of the delayed stochastic reaction-diffusion differential system without impulses can be used to judge the stability of the corresponding system with impulses.

Remark 9. In [2, 8], the authors dealt with the stochastic differential systems with delay and nonlinear impulsive effects, the stability results of which were showed by transforming the system into a corresponding system without impulses. However, the distributed delays and diffusion effects were not taken into account in the previous systems. In this paper, we incorporated stochastic perturbations, reaction-diffusion effects, and mixed time delays into impulsive differential system and derived the stability criteria of the system. It is readily seen that our results are more general than those reported in [2, 8].

4. Application to Impulsive Stochastic Reaction-Diffusion Neural Networks

In this section, we apply our previous stability results to analyze the stability of the following ISRDCGNNs with time delays:

$$\begin{aligned}
\mathrm{d}u_i(t,x) = {} & \sum_{l=1}^{m} \frac{\partial}{\partial x_l}\left(D_{il}(t,x,u_i(t,x)) \frac{\partial u_i(t,x)}{\partial x_l}\right)\mathrm{d}t \\
& - \alpha_i(u_i(t,x)) \left[\beta_i(u_i(t,x)) - \sum_{j=1}^{n} a_{ij} f_j(u_j(t,x)) - \sum_{j=1}^{n} b_{ij} g_j(u_j(t-\tau_{ij}(t),x)) - \sum_{j=1}^{n} c_{ij} h_j\left(\int_{t-r(t)}^{t} u_j(s,x)\mathrm{d}s\right)\right]\mathrm{d}t
\end{aligned}$$

$$+\sum_{j=1}^{n}\sigma_{ij}\Big(t,u_j(t,x),u_j(t-\tau_{ij}(t),x),\int_{t-r(t)}^{t}u_j(s,x)\,\mathrm{d}s\Big)\mathrm{d}w_i(t),\quad t\neq t_k,$$
$$u_i(t_k^+,x)-u_i(t_k,x)=I_{ki}(u_1(t_k,x),\ldots,u_n(t_k,x)),\quad t=t_k,\quad k\in N,\quad i=1,2,\ldots,n,\tag{15}$$

where $u(t,x)=(u_1(t,x),u_2(t,x),\ldots,u_n(t,x))^{\mathrm{T}}\in\mathbb{R}^n$ denotes the state vector associated with the neurons, $x\in\Omega\subset\mathbb{R}^m$, and $\Omega=\{x=(x_1,x_2,\ldots,x_m)^{\mathrm{T}}\|x_k|<l_k,k=1,2,\ldots,m\}$ is a bounded compact set with smooth boundary $\partial\Omega$ and $\operatorname{mes}\Omega>0$, $D_{il}(t,x,u_i(t,x))\geq 0$ denotes the diffusion function, and let $D_{il}=\sup_{t\geq0,x\in\Omega}D_{il}(t,x,u_i(t,x))$; $(a_{ij})_{n\times n}$, $(b_{ij})_{n\times n}$ and $(c_{ij})_{n\times n}$ are the interconnection weight matrices, $\alpha_i(u_i(t,x))$ represents an amplification function, $\beta_i(u_i(t,x))$ is an appropriately behaved function, f_j, g_j, and h_j denote the activation functions, and $(\sigma_{ij})_{n\times n}$ is the diffusion coefficient matrix; $\sigma_i=(\sigma_{i1},\sigma_{i2},\ldots,\sigma_{in})$, $\beta_i(0)=f_i(0)=g_i(0)=h_i(0)=\sigma_{ij}(0)=0$, and $\tau_{ij}(t)\leq\tau$, $r(t)\leq r$.

The boundary condition and the initial value of system (15) are

$$u_i(t,x)|_{\partial\Omega}=0,\quad (t,x)\in[-\delta,+\infty)\times\partial\Omega,\ i=1,2,\ldots,n,$$
$$u_i(s,x)=\phi_i(s,x),\quad (s,x)\in[-\delta,0]\times\Omega,\ i=1,2,\ldots,n,\tag{16}$$

where $\delta=\max\{\tau,r\}$.

Equivalently, we consider the following stochastic reaction-diffusion Cohen-Grossberg neural networks without impulses:

$$\begin{aligned}\mathrm{d}v_i(t,x)&=\sum_{l=1}^{m}\frac{\partial}{\partial x_l}\Big(D_{il}\Big(t,x,\prod_{0\leq t_k<t}J_{ki}^{-1}v_i(t,x)\Big)\frac{\partial v_i(t,x)}{\partial x_l}\Big)\mathrm{d}t\\&-\prod_{0\leq t_k<t}J_{ki}\cdot\alpha_i\Big(\prod_{0\leq t_k<t}J_{ki}^{-1}v_i(t,x)\Big)\\&\times\Big[\beta_i\Big(\prod_{0\leq t_k<t}J_{ki}^{-1}v_i(t,x)\Big)\\&\quad-\sum_{j=1}^{n}a_{ij}f_j\Big(\prod_{0\leq t_k<t}J_{kj}^{-1}v_j(t,x)\Big)\\&\quad-\sum_{j=1}^{n}b_{ij}g_j\Big(\prod_{0\leq t_k<t-\tau_{ij}(t)}J_{kj}^{-1}v_j(t-\tau_{ij}(t),x)\Big)\\&\quad-\sum_{j=1}^{n}c_{ij}h_j\Big(\int_{t-r(t)}^{t}\prod_{0\leq t_k<s}J_{ks}^{-1}v_j(s,x)\,\mathrm{d}s\Big)\Big]\mathrm{d}t\\&+\prod_{0\leq t_k<t}J_{ki}\cdot\sum_{j=1}^{n}\sigma_{ij}\Big(t,\prod_{0\leq t_k<t}J_{kj}^{-1}v_j(t,x),\prod_{0\leq t_k<t-\tau_{ij}(t)}J_{kj}^{-1}v_j(t-\tau_{ij}(t),x),\\&\qquad\int_{t-r(t)}^{t}\prod_{0\leq t_k<s}J_{kj}^{-1}v_j(s,x)\,\mathrm{d}s\Big)\mathrm{d}w_i(t),\quad i=1,2,\ldots,n.\end{aligned}\tag{17}$$

Throughout this section, we make the following assumptions:

(H5) $\alpha_i(u)$ is a continuous function, and $0<\underline{\alpha}_i\leq\alpha_i(u)\leq\overline{\alpha}_i$ for all $u\in\mathbb{R}$, $i=1,2,\ldots,n$;

(H6) there exists a constant $\beta_i>0$ such that

$$\frac{\beta_i(u)-\beta_i(v)}{u-v}\geq\beta_i,\tag{18}$$

for all $u,v\in\mathbb{R}(u\neq v)$, $i=1,2,\ldots,n$;

(H7) there exist positive constants F_i, G_i, and H_i such that

$$F_i=\sup_{u\neq v}\left|\frac{f_i(u)-f_i(v)}{u-v}\right|,\qquad G_i=\sup_{u\neq v}\left|\frac{g_i(u)-g_i(v)}{u-v}\right|,\qquad H_i=\sup_{u\neq v}\left|\frac{h_i(u)-h_i(v)}{u-v}\right|,\tag{19}$$

for all $u,v\in\mathbb{R}(u\neq v)$, $i=1,2,\ldots,n$;

(H8) there exist positive constants $s_{ij}^{(1)}$, $s_{ij}^{(2)}$, and $s_{ij}^{(3)}$ $(i,j=1,2,\ldots,n)$ such that

$$\begin{aligned}&\big(\sigma_i\big(t,u^{(1)},u^{(2)},u^{(3)}\big)-\sigma_i\big(t,v^{(1)},v^{(2)},v^{(3)}\big)\big)\\&\quad\times\big(\sigma_i\big(t,u^{(1)},u^{(2)},u^{(3)}\big)-\sigma_i\big(t,v^{(1)},v^{(2)},v^{(3)}\big)\big)^{\mathrm{T}}\\&\quad\leq\sum_{j=1}^{n}\Big(s_{ij}^{(1)}\big|u_j^{(1)}-v_j^{(1)}\big|^2+s_{ij}^{(2)}\big|u_j^{(2)}-v_j^{(2)}\big|^2+s_{ij}^{(3)}\big|u_j^{(3)}-v_j^{(3)}\big|^2\Big),\end{aligned}\tag{20}$$

for all $u^{(k)}=(u_1^{(k)},u_2^{(k)},\ldots,u_n^{(k)})$, $v^{(k)}=(v_1^{(k)},v_2^{(k)},\ldots,v_n^{(k)})$, $k=1,2,3$.

The following lemmas are useful in proving our main results.

Lemma 10 (see [16]). *Let Q be an $n\times n$ matrix with nonpositive off-diagonal elements; then Q is an M-matrix if and only if there exists a vector $\xi>0$ such that $Q\xi>0$.*

Lemma 11 (see [10]). *Let $p\geq 2$ be a positive integer, l_k $(k=1,2,\ldots,m)$ positive constants, and cube*

$\Omega = \{x = (x_1, x_2, \ldots, x_m)^{\mathrm{T}} \in \mathbb{R}^m \| |x_k| < l_k, k = 1, 2, \ldots, m\}$. *Let $h(x)$ be a real-valued function belonging to $C^1(\Omega)$ which vanishes on the boundary $\partial\Omega$ of Ω, that is, $h(x)|_{\partial\Omega=0}$. Then*

$$\int_{\Omega} |h(x)|^p \,\mathrm{d}x \le \frac{p^2 l_k^2}{4} \int_{\Omega} |h(x)|^{p-2} \left|\frac{\partial h(x)}{\partial x_k}\right|^2 \mathrm{d}x, \quad k = 1, 2, \ldots, m. \tag{21}$$

Lemma 12 (see [17]). *Let $a, b \ge 0$, $p \ge i > 0$; then*

$$a^{p-i} b^i \le \frac{p-i}{p} a^p + \frac{i}{p} b^p. \tag{22}$$

Theorem 13. *Under assumptions (H5)–(H8), if inequalities (11) and (13) are satisfied and $Q + T$ is an M-matrix, where*

$$\begin{aligned}
&Q = (q_{ij})_{n\times n},\\
&q_{ij} = -LM\overline{\alpha}_i \left(|a_{ij}| F_j + |b_{ij}| G_j + r |c_{ij}| H_j\right),\\
&T = (t_{ij})_{n\times n},\\
&t_{ij} = -(p-1) L^2 M^2 \left(s_{ij}^{(1)} + s_{ij}^{(2)} + r^2 s_{ij}^{(3)}\right) \quad (i \ne j),\\
&t_{ii} = \frac{4(p-1)}{p} \sum_{l=1}^{m} \frac{D_{il}}{l_i^2} + p\underline{\alpha}_i \beta_i\\
&\quad - (p-1) LM\overline{\alpha}_i\\
&\quad \times \sum_{j=1}^{n} \left[|a_{ij}| F_j + |b_{ij}| G_j + r |c_{ij}| H_j + \frac{(p-2) LM}{2\overline{\alpha}_i} \left(s_{ij}^{(1)} + s_{ij}^{(2)} + r^2 s_{ij}^{(3)}\right)\right]\\
&\quad - (p-1) L^2 M^2 \left(s_{ii}^{(1)} + s_{ii}^{(2)} + r^2 s_{ii}^{(3)}\right),
\end{aligned} \tag{23}$$

then the zero solution of system (15) is exponentially p-stable.

Proof. Since $Q+T$ is an M-matrix, by Lemma 10, there exists $\xi = (\xi_1, \xi_2, \ldots, \xi_n)^{\mathrm{T}}$ such that $(Q+T)\xi > 0$; that is,

$$\begin{aligned}
&\left[\frac{4(p-1)}{p} \sum_{l=1}^{m} \frac{D_{il}}{l_i^2} + p\underline{\alpha}_i \beta_i - (p-1) LM\overline{\alpha}_i \right.\\
&\quad \times \sum_{j=1}^{n} \left(|a_{ij}| F_j + |b_{ij}| G_j + r |c_{ij}| H_j \right.\\
&\quad \left.\left. + \frac{(p-2) LM}{2\overline{\alpha}_i} \left(s_{ij}^{(1)} + s_{ij}^{(2)} + r^2 s_{ij}^{(3)}\right)\right)\right] \xi_i\\
&\quad - LM\overline{\alpha}_i \sum_{j=1}^{n} \left[|a_{ij}| F_j + |b_{ij}| G_j + r |c_{ij}| H_j \right.\\
&\quad \left. + \frac{(p-1) LM}{\overline{\alpha}_i} \left(s_{ij}^{(1)} + s_{ij}^{(2)} + r^2 s_{ij}^{(3)}\right)\right] \xi_j > 0,\\
&\quad i = 1, 2, \ldots, n.
\end{aligned} \tag{24}$$

We can choose a sufficiently small constant $\epsilon > 0$ such that

$$\begin{aligned}
&\left[\frac{4(p-1)}{p} \sum_{l=1}^{m} \frac{D_{il}}{l_i^2} + p\underline{\alpha}_i \beta_i - \epsilon - (p-1) LM\overline{\alpha}_i \right.\\
&\quad \times \sum_{j=1}^{n} \left(|a_{ij}| F_j + |b_{ij}| G_j + r |c_{ij}| H_j \right.\\
&\quad \left.\left. + \frac{(p-2) LM}{2\overline{\alpha}_i} \left(s_{ij}^{(1)} + s_{ij}^{(2)} + r^2 s_{ij}^{(3)}\right)\right)\right] \xi_i\\
&\quad - LM\overline{\alpha}_i \sum_{j=1}^{n} \left[\left(|a_{ij}| F_j + \frac{(p-1) LM}{\overline{\alpha}_i} s_{ij}^{(1)} \right) \right.\\
&\quad + e^{\epsilon\tau} \left(|b_{ij}| G_j + \frac{(p-1) LM}{\overline{\alpha}_i} s_{ij}^{(2)} \right)\\
&\quad \left. + \frac{e^{\epsilon r - 1}}{\epsilon} \left(|c_{ij}| H_j + \frac{(p-1) LMr}{\overline{\alpha}_i} s_{ij}^{(3)} \right)\right] \xi_j > 0.
\end{aligned} \tag{25}$$

Let $w_i(t,x) = e^{\epsilon t} \int_{\Omega} |v_i(t,x)|^p \mathrm{d}x$, $p \ge 2$, $i = 1, 2, \ldots, n$. By the Itô differential formula, the stochastic derivative of $w_i(t,x)$ along (17) can be derived as follows:

$$\begin{aligned}
&Lw_i(t,x)\\
&\quad = \epsilon e^{\epsilon t} \int_{\Omega} |v_i(t,x)|^p \mathrm{d}x\\
&\qquad + p e^{\epsilon t} \operatorname{sgn}(v_i(t,x)) \int_{\Omega} |v_i(t,x)|^{p-1}\\
&\qquad \times \left[\sum_{l=1}^{m} \frac{\partial}{\partial x_l} \left(D_{il} \left(t, x, \prod_{0 \le t_k < t} J_{ki}^{-1} v_i(t,x) \right) \frac{\partial v_i(t,x)}{\partial x_l} \right) \right.\\
&\qquad - \prod_{0 \le t_k < t} J_{ki} \cdot \alpha_i \left(\prod_{0 \le t_k < t} J_{ki}^{-1} v_i(t,x) \right)\\
&\qquad \times \left[\beta_i \left(\prod_{0 \le t_k < t} J_{ki}^{-1} v_i(t,x) \right) \right.\\
&\qquad - \sum_{j=1}^{n} a_{ij} f_j \left(\prod_{0 \le t_k < t} J_{kj}^{-1} v_j(t,x) \right)\\
&\qquad - \sum_{j=1}^{n} b_{ij} g_j \left(\prod_{0 \le t_k < t - \tau_{ij}(t)} J_{kj}^{-1} v_j(t - \tau_{ij}(t), x) \right)
\end{aligned}$$

$$-\sum_{j=1}^{n} c_{ij} h_j \left(\int_{t-r(t)}^{t} \prod_{0\le t_k<s} J_{ks}^{-1} v_j(s,x)\,\mathrm{d}s \right) \Bigg] \Bigg] \mathrm{d}x$$
$$+ \frac{p(p-1)}{2} e^{\epsilon t} \left(\prod_{0\le t_k<t} J_{ki} \right)^2 \int_\Omega |v_i(t,x)|^{p-2} \sigma_i \sigma_i^{\mathrm{T}} \mathrm{d}x,$$
$$i = 1,2,\ldots,n. \tag{26}$$

By Lemma 11, we derive from (16) that

$$\begin{aligned} & p \operatorname{sgn}(v_i(t,x)) \int_\Omega |v_i(t,x)|^{p-1} \\ & \quad \times \sum_{l=1}^{m} \frac{\partial}{\partial x_l} \left(D_{il}\left(t, x, \prod_{0\le t_k<t} J_{ki}^{-1} v_i(t,x) \right) \times \frac{\partial v_i(t,x)}{\partial x_l} \right) \mathrm{d}x \\ & = p \int_\Omega |v_i(t,x)|^{p-2} v_i(t,x) \nabla \cdot \left(D_{il}\left(t, x, \prod_{0\le t_k<t} J_{ki}^{-1} v_i(t,x) \right) \frac{\partial v_i(t,x)}{\partial x_l} \right)_{l=1}^{m} \mathrm{d}x \\ & \le -p(p-1) \int_\Omega \sum_{l=1}^{m} D_{il} |v_i(t,x)|^{p-2} \left(\frac{\partial v_i(t,x)}{\partial x_l} \right)^2 \mathrm{d}x \\ & \le -\frac{4(p-1)}{p} \sum_{l=1}^{m} \frac{D_{il}}{l_i^2} \int_\Omega |v_i(t,x)|^p \mathrm{d}x. \end{aligned} \tag{27}$$

Applying assumptions (H5)–(H8) and Lemma 12, we can deduce from (11), (13), and (27) that

$$\begin{aligned} L w_i(t,x) \le{} & e^{\epsilon t} \int_\Omega \Bigg[\left(\epsilon - \frac{4(p-1)}{p} \sum_{l=1}^{m} \frac{D_{il}}{l_i^2} - p \underline{\alpha}_i \beta_i \right) \times |v_i(t,x)|^p \\ & + pLM\overline{\alpha}_i |v_i(t,x)|^{p-1} \times \sum_{j=1}^{n} \Bigg(|a_{ij}| F_j |v_j(t,x)| + |b_{ij}| G_j |v_j(t-\tau_{ij}(t),x)| + |c_{ij}| H_j \int_{t-r(t)}^{t} |v_j(s,x)| \mathrm{d}s \Bigg) \\ & + \frac{p(p-1)}{2} L^2 M^2 |v_i(t,x)|^{p-2} \times \sum_{j=1}^{n} \Bigg(s_{ij}^{(1)} v_j^2(t,x) + s_{ij}^{(2)} v_j^2(t-\tau_{ij}(t),x) + r s_{ij}^{(3)} \int_{t-r(t)}^{t} v_j^2(s,x) \mathrm{d}s \Bigg) \Bigg] \mathrm{d}x \\ \le{} & e^{\epsilon t} \int_\Omega \Bigg[\left(\epsilon - \frac{4(p-1)}{p} \sum_{l=1}^{m} \frac{D_{il}}{l_i^2} - p \underline{\alpha}_i \beta_i \right) \times |v_i(t,x)|^p \\ & + (p-1) LM\overline{\alpha}_i |v_i(t,x)|^p \times \sum_{j=1}^{n} \left(|a_{ij}| F_j + |b_{ij}| G_j + r |c_{ij}| H_j \right) \\ & + LM\overline{\alpha}_i \sum_{j=1}^{n} \Bigg(|a_{ij}| F_j |v_j(t,x)|^p + |b_{ij}| \times G_j |v_j(t-\tau_{ij}(t),x)|^p + |c_{ij}| H_j \times \int_{t-r(t)}^{t} |v_j(s,x)|^p \mathrm{d}s \Bigg) \\ & + \frac{(p-1)(p-2)}{2} L^2 M^2 |v_i(t,x)|^p \times \sum_{j=1}^{n} \left(s_{ij}^{(1)} + s_{ij}^{(2)} + r^2 s_{ij}^{(3)} \right) \\ & + (p-1) L^2 M^2 \times \sum_{j=1}^{n} \Bigg(s_{ij}^{(1)} |v_j(t,x)|^p + s_{ij}^{(2)} |v_j(t-\tau_{ij}(t),x)|^p + r s_{ij}^{(3)} \times \int_{t-r(t)}^{t} |v_j(s,x)|^p \Bigg) \mathrm{d}s \Bigg] \mathrm{d}x \\ \le{} & \Bigg[\epsilon - \frac{4(p-1)}{p} \sum_{l=1}^{m} \frac{D_{il}}{l_i^2} - p\underline{\alpha}_i \beta_i + (p-1) LM\overline{\alpha}_i \\ & \times \sum_{j=1}^{n} \Bigg(|a_{ij}| F_j + |b_{ij}| G_j + r |c_{ij}| H_j + \frac{(p-2) LM}{2\overline{\alpha}_i} \times \left(s_{ij}^{(1)} + s_{ij}^{(2)} + r^2 s_{ij}^{(3)} \right) \Bigg) \Bigg] w_i(t,x) \end{aligned}$$

$$+ LM\overline{\alpha}_i \sum_{j=1}^{n} \left[\left(\left| a_{ij} \right| F_j + \frac{(p-1) LM}{\overline{\alpha}_i} s_{ij}^{(1)} \right) \times w_j(t,x) + e^{\epsilon\tau} \left(\left| b_{ij} \right| G_j + \frac{(p-1) LM}{\overline{\alpha}_i} s_{ij}^{(2)} \right) \times w_j\left(t - \tau_{ij}(t), x\right) + \left(\left| c_{ij} \right| H_j + \frac{(p-1) LMr}{\overline{\alpha}_i} s_{ij}^{(3)} \right) \times \int_{t-r(t)}^{t} e^{\epsilon(t-s)} w_j(s,x)\, ds \right]. \tag{28}$$

Further, we can get

$$\begin{aligned} D^{+}\left(Ew_i(t,x)\right) \leq{}& \left[\epsilon - \frac{4(p-1)}{p} \sum_{l=1}^{m} \frac{D_{il}}{l_i^2} - p\underline{\alpha}_i \beta_i + (p-1) LM\overline{\alpha}_i \times \sum_{j=1}^{n} \left(\left| a_{ij} \right| F_j + \left| b_{ij} \right| G_j + r \left| c_{ij} \right| H_j + \frac{(p-2) LM}{2\overline{\alpha}_i} \left(s_{ij}^{(1)} + s_{ij}^{(2)} + r^2 s_{ij}^{(3)} \right) \right) \right] Ew_i(t,x) \\ &+ LM\overline{\alpha}_i \sum_{j=1}^{n} \left[\left(\left| a_{ij} \right| F_j + \frac{(p-1) LM}{\overline{\alpha}_i} s_{ij}^{(1)} \right) Ew_j(t,x) + e^{\epsilon\tau} \left(\left| b_{ij} \right| G_j + \frac{(p-1) LM}{\overline{\alpha}_i} s_{ij}^{(2)} \right) \times Ew_j\left(t - \tau_{ij}(t), x\right) \right. \\ &\left. + \left(\left| c_{ij} \right| H_j + \frac{(p-1) LMr}{\overline{\alpha}_i} s_{ij}^{(3)} \right) \times \int_{t-r(t)}^{t} e^{\epsilon(t-s)} Ew_j(s,x)\, ds \right]. \end{aligned} \tag{29}$$

Denoting $k_0 = \|\phi\|^p / \min_{1 \leq i \leq n}\{\xi_i\}$, we have

$$\begin{aligned} Ew_i(t,x) &= e^{\epsilon t} E\left\| v_i(t,x) \right\|^p \leq E\left\| v_i(t,x) \right\|^p \\ &\leq k_0 \xi_i, \quad t \in [-\delta, 0], i = 1, 2, \ldots, n. \end{aligned} \tag{30}$$

In what follows, we prove that

$$Ew_i(t,x) \leq k_0 \xi_i, \quad t \geq 0, \ i = 1, 2, \ldots, n. \tag{31}$$

In fact, if (31) is not true, then there exist $i_0 \in \{1, 2, \ldots, n\}$ and $t^* \in [0, +\infty)$ such that

$$\begin{gathered} Ew_{i_0}(t^*, x) \leq k_0 \xi_{i_0}, \qquad D^{+} Ew_{i_0}(t^*, x) > 0, \\ Ew_j(t,x) \leq k_0 \xi_j, \quad t \in \left[-\delta, t^*\right], \ j = 1, 2, \ldots, n. \end{gathered} \tag{32}$$

However, (25), (29), and (32) imply that

$$\begin{aligned} D^{+}\left(Ew_{i_0}(t^*, x)\right) \leq{}& k_0 \left\{ \left[\epsilon - \frac{4(p-1)}{p} \sum_{l=1}^{m} \frac{D_{i_0 l}}{l_{i_0}^2} - p\underline{\alpha}_{i_0} \beta_{i_0} + (p-1) LM\overline{\alpha}_{i_0} \times \sum_{j=1}^{n} \left(\left| a_{i_0 j} \right| F_j + \left| b_{i_0 j} \right| G_j + r \left| c_{i_0 j} \right| H_j + \frac{(p-2) LM}{2\overline{\alpha}_{i_0}} \left(s_{i_0 j}^{(1)} + s_{i_0 j}^{(2)} + r^2 s_{i_0 j}^{(3)} \right) \right) \right] \xi_{i_0} \right. \\ &+ LM\overline{\alpha}_{i_0} \times \sum_{j=1}^{n} \left[\left(\left| a_{i_0 j} \right| F_j + \frac{(p-1) LM}{\overline{\alpha}_{i_0}} s_{i_0 j}^{(1)} \right) + e^{\epsilon\tau} \left(\left| b_{i_0 j} \right| G_j + \frac{(p-1) LM}{\overline{\alpha}_{i_0}} s_{i_0 j}^{(2)} \right) \right. \\ &\left.\left. + \frac{e^{\epsilon r} - 1}{\epsilon} \left(\left| c_{i_0 j} \right| H_j + \frac{(p-1) LMr}{\overline{\alpha}_{i_0}} s_{i_0 j}^{(3)} \right) \right] \xi_j \right\} \\ <{}& 0, \end{aligned} \tag{33}$$

which is a contradiction. Hence, (31) holds, which leads to

$$E\left\| v_i(t,x) \right\|^p \leq k_0 \xi_i e^{-\epsilon t}, \quad t \geq 0, \ i = 1, 2, \ldots, n. \tag{34}$$

Therefore,

$$E\| v(t,x) \|^p \leq K \left\| \phi \right\|^p e^{-\epsilon t}, \quad t \geq 0, \tag{35}$$

where $K = (\max_{1 \leq i \leq n}\{\xi_i\} / \min_{1 \leq i \leq n}\{\xi_i\}) n^p$. This means that the zero solution of system (17) is exponentially p-stable and the exponential convergence rate equals ϵ. By Theorem 5, we can obtain that the zero solution of system (15) is exponentially p-stable. The proof is complete. □

Remark 14. The stability of impulsive Cohen-Grossberg neural networks without spacial diffusion or distributed delays or stochastic disturbance, which are special cases of system (15), have been studied in [5, 13, 18–20]. It should be noted that the main result in [13] is a special case of Theorem 13. Further, the stability criteria derived in [5, 18–20] are dependent on the intervals of adjoining impulsive moments, while our results are independent of that. Thus, our results are new, and they effectually complement or improve the previously published results.

Remark 15. In [19, 21–26], the reaction-diffusion neural networks have been investigated. Nevertheless, the diffusion terms were eliminated by inequality analysis techniques, and the derived conditions for the stability of neural networks are the same as those obtained in the cases when there are no reaction-diffusion terms in the systems. Thus, our results including reaction-diffusion terms are less conservative than those in [19, 21–26].

Remark 16. As far as we know, almost all the existing results concerning the stability of neural networks are based on 2-norm (e.g., [5, 11, 19, 21, 22, 26–28]). In this paper, we derived the stability criteria of ISRDCGNNs with mixed time delays in terms of p-norm. Hence, our results generalize and improve the existing results reported in the previous literature.

5. Numerical Example

In this section, we give an example to illustrate the main theoretical results in Sections 3 and 4.

In system (15), let

$$
\begin{aligned}
& n = m = 2, \qquad \alpha_1(x) = 2.5 + 0.5\cos x, \\
& \alpha_2(x) = 3 - \sin x, \qquad \beta_1(x) = 8x, \\
& \beta_2(x) = 10x, \\
& f_j(x) = g_j(x) = h_j(x) \\
& \qquad = \frac{1}{2}(|x+1| + |x-1|), \qquad l_j = 1 \quad (j = 1, 2), \\
& r(t) = 1 + \sin t, \qquad (D_{il}(x)) = \begin{pmatrix} 6 & 6 \\ 8 & 8 \end{pmatrix}, \\
& (a_{ij}) = \begin{pmatrix} 0.1 & -0.2 \\ -0.3 & -0.5 \end{pmatrix}, \qquad (b_{ij}) = \begin{pmatrix} 0.8 & 0.8 \\ 0.4 & 0.2 \end{pmatrix}, \\
& (c_{ij}) = \begin{pmatrix} 0.3 & -0.2 \\ 0.1 & -0.2 \end{pmatrix}, \\
& (\tau_{ij}(t)) = \begin{pmatrix} 0.02\sin^2(t) & 0.01|\cos t| \\ 0.03\cos^2(t) & 0.02|\sin 2t| \end{pmatrix}, \\
& (\sigma_{ij}(x, y, z)) \\
& \qquad = \begin{pmatrix} 0.1x - 0.2y - 0.1z & 0.2x + 0.3y + 0.1z \\ 0.5x + 0.4y - 0.3z & 0.3x + 0.1y + 0.2z \end{pmatrix}, \\
& I_{k1}(u_1(t_k, x), u_2(t_k, x)) \\
& \qquad = \frac{1}{2^{k+2}} u_1(t_k, x) \\
& \qquad\quad + \frac{1}{(2k)^2} u_1(t_k, x)\sin^2(u_2(t_k, x)), \\
& I_{k2}(u_1(t_k, x), u_2(t_k, x)) \\
& \qquad = \frac{1}{2^2 3^k} u_2(t_k, x) \\
& \qquad\quad + \frac{1}{(2k)^2} u_2(t_k, x)\cos^2(u_1(t_k, x)).
\end{aligned}
\tag{36}
$$

By direct calculation, we obtain that

$$
\begin{gathered}
\underline{\alpha}_1 = 2, \qquad \overline{\alpha}_1 = 3, \qquad \underline{\alpha}_2 = 2, \\
\overline{\alpha}_2 = 4, \qquad \beta_1 = 8, \qquad \beta_2 = 10, \\
F_i = G_i = H_i = 1, \qquad \tau = 0.03, \qquad r = 2, \\
(s_{ij}^{(1)}) = \begin{pmatrix} 0.04 & 0.12 \\ 0.6 & 0.18 \end{pmatrix}, \qquad (s_{ij}^{(2)}) = \begin{pmatrix} 0.08 & 0.18 \\ 0.48 & 0.06 \end{pmatrix}, \\
(s_{ij}^{(3)}) = \begin{pmatrix} 0.04 & 0.06 \\ 0.36 & 0.12 \end{pmatrix},
\end{gathered}
\tag{37}
$$

$$
\begin{gathered}
J_{k1}^{-1} = 1 + \frac{1}{2^{k+2}} + \frac{1}{(2k)^2}\sin^2(u_2(t_k, x)), \\
J_{k2}^{-1} = 1 + \frac{1}{2^2 3^k} + \frac{1}{(2k)^2}\cos^2(u_1(t_k, x)), \\
L = 1, \qquad M = 2.
\end{gathered}
\tag{38}
$$

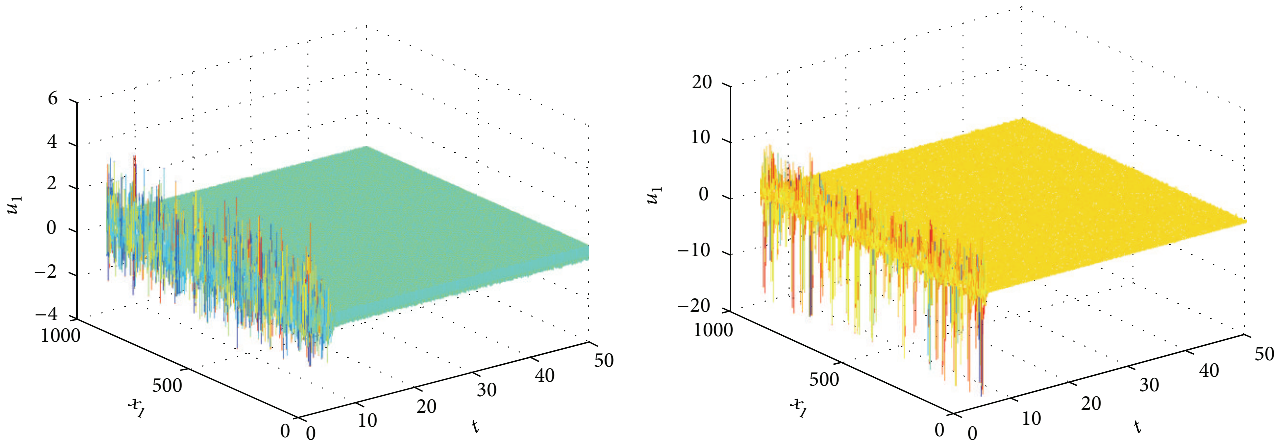
Hence, assumptions (H5)–(H8) and inequalities (11) and (13) are satisfied. Taking $p = 4$, it is not difficult to compute that

$$
\begin{gathered}
Q = \begin{pmatrix} -9.0 & -8.4 \\ -7.2 & -8.8 \end{pmatrix}, \qquad T = \begin{pmatrix} 34.60 & -6.48 \\ -30.24 & 32.48 \end{pmatrix}, \\
Q + T = \begin{pmatrix} 25.60 & -14.88 \\ -37.44 & 23.68 \end{pmatrix},
\end{gathered}
\tag{39}
$$

in which $Q + T$ is an M-matrix, and all the conditions of Theorem 13 are satisfied. From Theorem 13, we know that the zero solution of system (15) with the parameters and functions above is exponentially 4-stable (see Figure 1).

6. Concluding Remark

In this paper, we incorporated stochastic perturbations, reaction-diffusion effects, and mixed time delays into impulsive differential systems. First, an equivalent relation between the solution of a stochastic reaction-diffusion differential system with time delays and impulsive effects and that of corresponding system without impulses was established. Second, some stability criteria for the stochastic reaction-diffusion differential system with time delays and impulsive effects were derived by transforming the solutions of the system to those of corresponding one without impulses. Third, the stability criteria were applied to ISRDCGNNs with mixed time delays, and sufficient conditions were obtained for the exponential p-stability of the zero solution to the neural networks. Lastly, a numerical example was provided to illustrate the effectiveness of our theoretical results. Our



(a) (b)

Figure 1: Transient behaviors of the state variables $u_1(t, x)$ and $u_2(t, x)$ in the example.

stability results provide a new, convenient, and efficient approach to study the stability of stochastic reaction-diffusion differential systems with time delays and impulsive effects, and some previously published results are generalized and improved.

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