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Research Article

Thermal Diffusion and Diffusion Thermo Effects on MHD Thermosolutal Marangoni Convection Boundary Layer Flow over a Permeable Surface

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The problem of thermal diffusion and diffusion thermo effects on thermosolutal Marangoni convection flow of an electrically conducting fluid over a permeable surface is investigated. Using appropriate similarity transformations, the governing system of partial differential equation is transformed to a set of nonlinear ordinary differential equations, then solved numerically using the Runge-Kutta-Fehlberg method. The effects of thermal diffusion and diffusion thermo, magnetic field parameter, thermosolutal surface tension ratio, and suction/injection parameter on the flow field, heat transfer characteristic, and concentration are thoroughly examined. Numerical results are obtained for temperature and concentration profiles as well as the local Nusselt and Sherwood numbers are presented graphically and analyzed. It is found that these governing parameters affect the variations of the temperature and concentration and also the local Nusselt and Sherwood numbers.

1. Introduction

The study of Marangoni convection has received great consideration in recent years in view of its application in industries. Marangoni convection is predictable to be very useful in wide area especially in crystal growth melts and semiconductor processing. The Marangoni boundary layer term was first initiated by Napolitano [1, 2] when studied the existence of the steady dissipative layers which occur along the liquid-liquid or liquid-gas interfaces. Marangoni convection induced by the surface tension gradient can be due to gradients of temperature (thermal convection) and/or concentration (solutal convection). A lot of analyses in Marangoni convection have been discovered in various geometries and conditions. Some of experimental works linked to Marangoni convection were discussed in several papers by Arafune and Hirata [3], Arafune et al. [4], Galazka and Wilke [5], Neumann et al. [6], Arendt and Eggers [7], and Xu et al. [8].

The related works to this present study were done by Al-Mudhaf and Chamkha [9] who obtained the similarity solution for MHD thermosolutal Marangoni convection over a flat surface in the presence of heat generation or absorption with fluid suction and injection. Christopher and Wang [10] have analyzed the effects of Prandtl number on Marangoni convection flow over a flat surface. Later, Pop et al. [11] studied numerically the problem of thermosolutal Marangoni forced convection over a permeable surface and this study continued by Hamid et al. [12] who obtained dual solutions of the problem. Chen [13] investigated the flow and the heat transfer characteristics on the forced convection in a power law liquid film under an applied Marangoni convection over a stretching sheet. Magyari and Chamkha [14] found solution for steady MHD thermosolutal Marangoni convection and present analytical solutions for velocity, temperature, and concentration field. Arifin et al. [15] added new dimension to the Marangoni convection problem by considering the steady thermosolutal marangoni mixed convection boundary layer flow under an external pressure gradient. The problem is solved using the shooting method. Most lately, Hamid et al. [16] studied the two-dimensional Marangoni convection flow past a flat plate in the presence of thermal radiation, suction, and injection effects.

Several papers that deal with flows in the presence Dufour or diffusion thermo effect and Soret or thermal diffusion effect are now presented. A brief literature on existence and development of Dufuor and Soret effects can be found in the papers by Kafoussias and Williams [17] and Puvi Arasu et al. [18]. Puvi Arasu et al. [18] investigated the impact of thermophoresis particles deposition on two-dimensional flow over a vertical stretching surface in the presence of chemical reaction and also Dufour and Soret effects taking place in the flow. The temperature gradients and concentration gradients play vital role in producing Dufour and Soret effects. The concentration gradient has generated the heat flux, namely, Dufour effect while mass flux is created by temperature gradients and is known as Soret effects. It is seem that the Charles Soret in 1879 is the first who found that a salt solution contained in a tube with two ends did not remain uniform in composition at different temperature. By this pioneering discovering, the term “Soret effect” officially introduced regarded his contribution on study of this particular effect. Later, the fundamental study on Soret effects remarkably grow over century (Osalusi et al. [19]).

The effects of thermal diffusion and diffusion thermo have been studied widely by several researchers due to its importance contribution in theory and practical. Some numerical studies on thermal diffusion and diffusion thermo effects include Afify [20] who studied the effects of thermal diffusion and diffusion thermo with suction and injection parameter on MHD free convection heat and mass transfer past a stretching sheet. Kafoussias and Williams [17] considered the mixed forced convection boundary layer flow with the effects of thermal diffusion and diffusion thermo in the presence of variable viscosity effect. This similar work continued by Eldabe et al. [21] for non-Newtonian power law fluid with the temperature dependent viscosity in the flow. Later, El-Aziz [22] considered the MHD three-dimensional free convection boundary layer flows past a stretching sheet with suction or injection and radiation in presence of Dufour and Soret effects. Next, Osalusi et al. [19] numerically studied the effects of thermal diffusion and diffusion thermo on combined heat and mass transfer of MHD convective and slip flow due to a rotating disk with the inclusion of viscous dissipation and Ohmic heating while Rashidi et al. [23] found its analytical solution using the homotopy analysis method (HAM). Most recently, Hayat et al. [24] obtained the series solutions for MHD two-dimensional axisymmetric flow of a second grade fluid with the existence of thermal diffusion and diffusion thermo effects, Joule heating and the chemical reaction effects.

The aim of this paper is to discuss the MHD thermosolutal Marangoni convection boundary layer over a permeable flat surface considering the effects of the thermal diffusion and diffusion thermo. The set of governing equations and boundary equation of the problem that are transformed into a set of nonlinear ordinary differential equation with assisting of similarity transformations are solved using the Runge-Kutta-Fehlberg method. The effects of different physical parameters on the temperature and concentration profiles as well as the local Nusselt and Sherwood numbers are presented. To verify the obtained results, we have compared the present numerical results with previous work by Al-Mudhaf and Chamkha [9]. The comparison results show a good agreement and we are confident that our present numerical results are accurate.

2. Mathematical Formulation

We consider the laminar boundary layer flow of an electrically conducting fluid over a permeable flat surface in the presence of Dufour and Soret effects. It is assumed that the mass flux velocity is v_w with $v_w < 0$ for suction and $v_w > 0$ for injection, respectively. It is also assumed that a uniform magnetic field, B_0 is imposed in the direction normal to the surface. Then, the basic governing equation of the proposed problem (see Al-Mudhaf and Chamkha [9] and Afify [20]):

$$\begin{gathered}\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,\\ u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma^* B_0^2}{\rho}u,\\ u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{D_m k_T}{c_s c_p}\frac{\partial^2 h}{\partial y^2},\\ u\frac{\partial h}{\partial x}+v\frac{\partial h}{\partial y}=D_m\frac{\partial^2 h}{\partial y^2}+\frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2}.\end{gathered} \tag{2.1}$$

The surface tension σ is assumed to vary linearly with the temperature T and concentration h as well as the wall temperature T_w and concentration h_w are presumed to be in quadratic functions of x. Hence, the boundary conditions of (2.1) is (see Al-Mudhaf and Chamkha [9])

$$u=0,\quad v=v_w,\quad T=T_\infty+Ax^2,\quad h=h_\infty+A^* x^2,\quad \mu\frac{\partial u}{\partial y}=\sigma_T\frac{\partial T}{\partial x}+\sigma_h\frac{\partial h}{\partial x}\ \text{ on }\ y=0, \tag{2.2}$$

$$u\longrightarrow 0,\ T\longrightarrow T_\infty,\ h\longrightarrow h_\infty \text{ as } y\longrightarrow\infty, \tag{2.3}$$

where u, v are the components of velocity, respectively, in the x and y directions, ν is the kinematic viscosity, σ^* is the fluid electrical conductivity, ρ is the fluid density, and α is the thermal diffusivity. Besides, D_m, k_T, c_s, c_p, and T_m are the diffusion coefficient, thermal-diffusion ratio, concentration susceptibility, specific heat at constant pressure, and mean fluid temperature, respectively (see Puvi Arasu et al. [18]). Moreover, μ is the dynamic viscosity, σ_T and σ_h are the rates of change of surface tension with temperature and solutal concentration while A and A^* are the temperature and concentration gradient coefficients, respectively.

The surface tension is defined as follow:

$$\sigma = \sigma_0\left[1 - \gamma_T(T - T_\infty) - \gamma_h(h - h_\infty)\right], \tag{2.4}$$

where

$$\gamma_T = -\frac{\partial\sigma}{\partial T}, \quad \gamma_h = -\frac{\partial\sigma}{\partial h}. \tag{2.5}$$

In order to find the similarity solutions of (2.1) subject to boundary conditions (2.2)-(2.3), we introduced the similarity variables (see Al-Mudhaf and Chamkha [9])

$$\eta = C_1 y, \quad f(\eta) = \frac{C_2\psi(x,y)}{x}, \quad \theta(\eta) = \frac{(T - T_\infty)}{Ax^2}, \quad H(\eta) = \frac{(h - h_\infty)}{A^* x^2}, \tag{2.6}$$

and $\psi(x,y)$ is the stream function defined in usual way as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ where

$$C_1 = \sqrt[3]{\frac{\rho A (d\sigma/dT)|_h}{\mu^2}}, \quad C_2 = \sqrt[3]{\frac{\rho^2}{\mu A (d\sigma/dT)|_h}}, \tag{2.7}$$

are the two similarity transformation coefficients.

Substituting (2.6)-(2.7) into (2.1), we obtained the following nonlinear ordinary differential equations:

$$f''' + ff'' - f'^2 - M^2 f' = 0, \tag{2.8}$$

$$\frac{1}{Pr}\theta'' + f\theta' - 2f'\theta + D_f H'' = 0, \tag{2.9}$$

$$\frac{1}{Sc}H'' + fH' - 2f'H + S_r\theta'' = 0, \tag{2.10}$$

where a prime denotes a differentiation with respect to η, $D_f = D_m k_T(h_w - h_\infty)/c_s c_p \nu(T_w - T_\infty)$ and $S_r = D_m k_T(T_w - T_\infty)/T_m \nu(h_w - h_\infty)$ are the Dufour and Soret numbers, respectively. Here, M is the magnetic field parameter, Pr is the Prandtl number, and Sc is the Schmidt number.

It is important to mention that f is the stream function similarity variable, θ and H are the nondimensional temperature and concentration, respectively. The boundary conditions (2.2)-(2.3) are reduced to

$$
\begin{gathered}
f(0) = f_0, \quad f''(0) = -2(1+r), \quad \theta(0) = 1, \quad H(0) = 1, \\
f'(\infty) = 0, \quad \theta(\infty) = 0, \quad H(\infty) = 0,
\end{gathered}
\tag{2.11}
$$

where $r = \Delta h(d\sigma/dh)|_T/\Delta T(d\sigma/dT)|_h$ is the thermosolutal surface tension ratio.

The local Nusselt and Sherwood numbers are given by (see Al-Mudhaf and Chamkha [9])

$$
\begin{aligned}
Nu_x &= \frac{q''(x)x}{\lambda(T_w - T_\infty)} = -C_1 x\theta'(0), \\
Sh_x &= \frac{h''(x)x}{D(T_w - T_\infty)} = -C_1 x H'(0),
\end{aligned}
\tag{2.12}
$$

where D is the mass diffusivity, q'' is the heat flux, and h'' is the mass flux.

3. Results and Discussion

Numerical solutions of the ordinary differential equations (2.8)–(2.10) that subject to boundary conditions (2.11) have been solved using the Runge-Kutta-Fehlberg fourth-fifth order (RKF45) method using Maple 12 and the algorithm RKF45 in Maple has been well tested for its accuracy and robustness (Aziz [25]). In this method, it is most important to choose the appropriate finite value of the edge of boundary layer, $\eta \to \infty$ (say η_∞) that is between 4 to 10, which is in accordance with the standard practice in the boundary layer analysis. The influences of the magnetic field parameter (M), the suction/injection parameter (f_0), the thermosolutal surface tension ratio (r), the combined Dufour number D_f and Soret number S_r on the velocity, temperature and concentration, and the Nusselt and Sherwood numbers are presented in tables and some graphs. These findings are summarized and presented in the Tables 1–4 and Figures 1–9. We have compared the present results with the results attained by Al-Mudhaf and Chamkha [9] when the heat generation/absorption and first-order chemical reaction effects are neglected. It is seen that the results presented in Tables 1–3 are in very well agreement. Hence, this leads the confidence of the present results. It should be mentioned that$f'(0)$, $-\theta'(0)$ and $-H'(0)$ are related to the surface velocity, Nusselt number, and Sherwood numbers, respectively.

Figures 1, 2, and 3 display the velocity, temperature, and concentration profiles for different values of magnetic field parameter M when the other parameters are fixed. An application of a magnetic field within boundary layer has produced resistive-type force which known as Lorentz force. This force acts to retard the fluid motion along surface and

Table 1: Comparison values of $f'(0)$, $-\theta'(0)$ and $-H'(0)$ with different parameter M.

M	$f'(0)$		$-\theta'(0)$		$-H'(0)$	
	Al-Mudhaf and Chamkha [9]	Present	Al-Mudhaf and Chamkha [9]	Present	Al-Mudhaf and Chamkha [9]	Present
0	1.587671	1.587401	1.442203	1.442069	1.220880	1.220731
1	1.315181	1.314596	1.206468	1.205891	1.005541	1.005808
2	0.903945	0.9032119	0.7596045	0.7625145	0.6106418	0.6188354
3	0.6448883	0.6440222	0.4422402	0.4625877	0.3473967	0.37638077
4	0.4933589	0.4924782	0.2728471	0.3114736	0.2127706	0.25873328

Table 2: Comparison values of $f'(0)$, $-\theta'(0)$ and $-H'(0)$ with different parameter f_0.

f_0	$f'(0)$		$-\theta'(0)$		$-H'(0)$	
	Al-Mudhaf and Chamkha [9]	Present	Al-Mudhaf and Chamkha [9]	Present	Al-Mudhaf and Chamkha [9]	Present
−2	2.383451	2.382975	1.251341	1.250618	1.129218	1.128784
−1	2.000379	1.999999	1.336441	1.335853	1.173002	1.173006
0	1.587671	1.58740104	1.442203	1.442067	1.220880	1.220715
1	1.179708	1.17950902	1.634990	1.634360	1.328699	1.327979
2	0.8480268	0.8477075	2.020949	2.019468	1.593570	1.592596

Table 3: Comparison values of $f'(0)$, $-\theta'(0)$ and $-H'(0)$ with different parameter r.

r	$f'(0)$		$-\theta'(0)$		$-H'(0)$	
	Al-Mudhaf and Chamkha [9]	Present (2011)	Al-Mudhaf and Chamkha [9]	Present (2011)	Al-Mudhaf and Chamkha [9]	Present (2011)
0	1.587582	1.587297	1.442247	1.442412	1.220880	1.222427
1	2.520988	2.519819	1.817826	1.816999	1.538960	1.538688
5	5.244303	5.241482	2.621562	2.620417	2.219093	2.218261

Table 4: The values of $-\theta'(0)$ and $-H'(0)$ with different parameters D_f and S_r.

D_f	S_r	$-\theta'(0)$	$-H'(0)$
0.03	2.0	1.624748	−0.00587659
0.06	1.0	1.603973	0.6866464
0.15	0.4	1.541646	1.102160
0.3	0.2	1.437767	1.240664
0.6	0.1	1.230011	1.309917
2.0	0.03	0.260478	1.358394

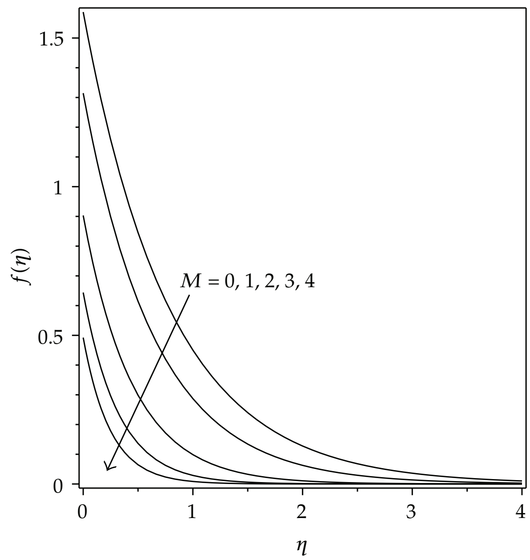

Figure 1: Velocity profiles for different values of M when $Pr = 0.78$, $Sc = 0.6$, $D_f = 0.03$, $S_r = 2$,$r = 0$, and $f_0 = 0$.

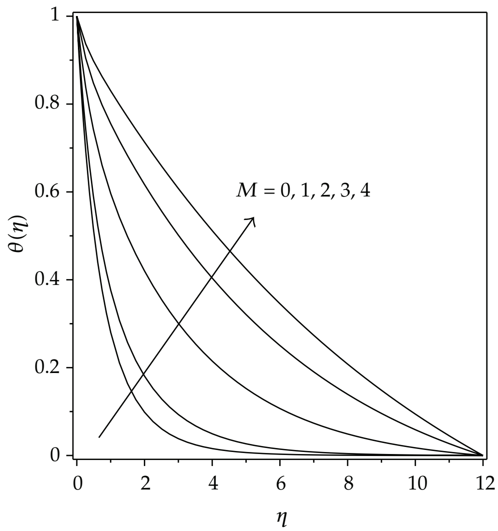

Figure 2: Temperature profiles for different values of M when $Pr = 0.78$, $Sc = 0.6$, $D_f = 0.03$, $S_r = 2$, $r = 0$, and $f_0 = 0$.

simultaneously increases its temperature and concentration values. In addition, the effect of the magnetic parameter of the viscous shearing force and the Lorentz force is given by

$$\nu \frac{u}{\delta_v^2} \approx \frac{\delta^* B_0^2}{\rho} u. \tag{3.1}$$

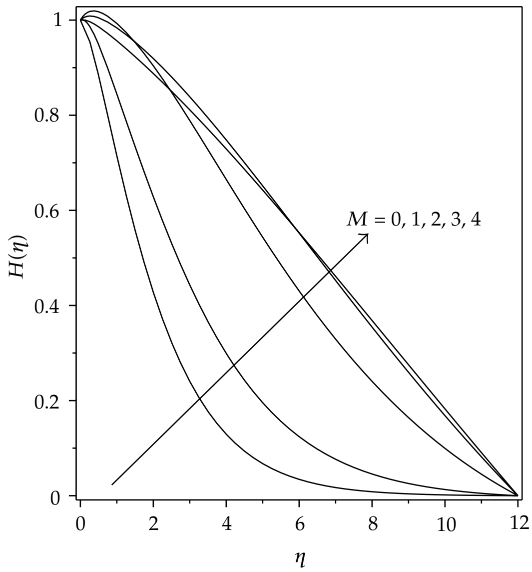

Figure 3: Concentration profiles for different values of M when $Pr = 0.78$, $Sc = 0.6$, $D_f = 0.03$, $S_r = 2$, $r = 0$, and $f_0 = 0$.

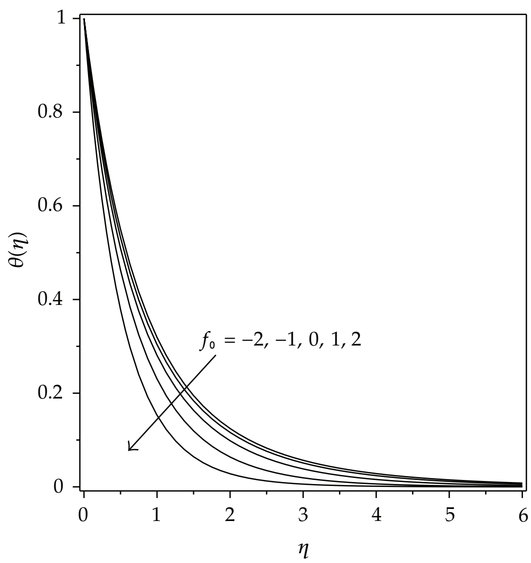

Figure 4: Temperature profiles for different values of f_0 when $Pr = 0.78$, $Sc = 0.6$, $D_f = 0.03$, $S_r = 2$, $r = 0$, and $M = 0$.

Thus, (3.1) gives

$$\eta_V \approx \frac{1}{M}. \tag{3.2}$$

However, the effect of surface tension can be obtained from (2.2) by the relation

$$\nu \frac{u}{\delta_v} \approx (\sigma_T 2Ax + \sigma_h A^* x). \tag{3.3}$$

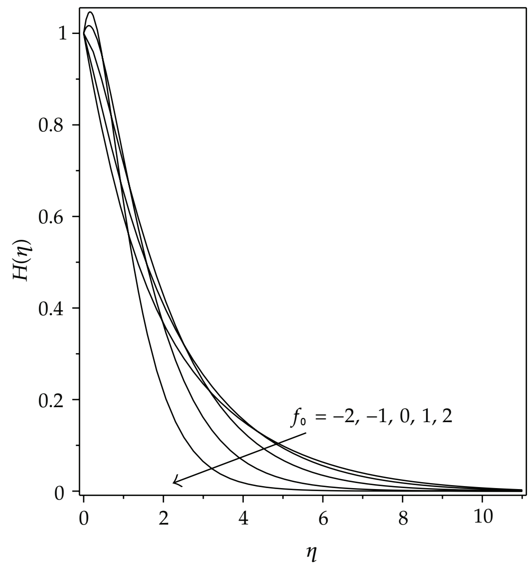

Figure 5: Concentration profiles for different values of f_0 when $Pr = 0.78$, $Sc = 0.6$, $D_f = 0.03$, $S_r = 2$, $r = 0$, and $M = 0$.

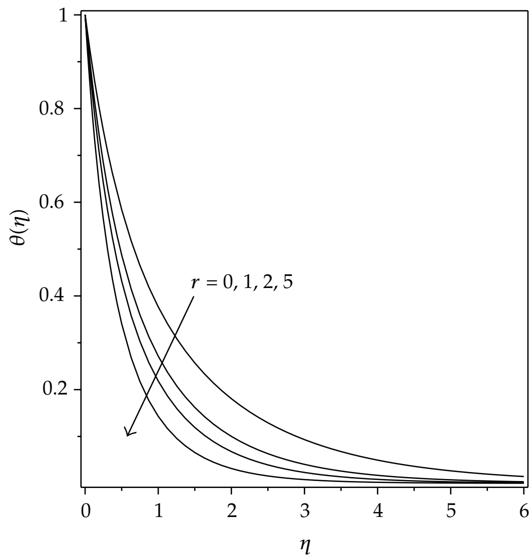

Figure 6: Temperature profiles for different values of r when $Pr = 0.78$, $Sc = 0.6$, $D_f = 0.03$, $S_r = 2$, $M = 1$, and $f_0 = 0$.

Then, (3.3) becomes

$$f'(0) \approx 2(1+r)\eta_V \approx \frac{2(1+r)}{M}. \tag{3.4}$$

Therefore, one can see that the velocity boundary layer thickness decreases with the increase of M as shown in Figure 1. However, the temperature and concentration increase

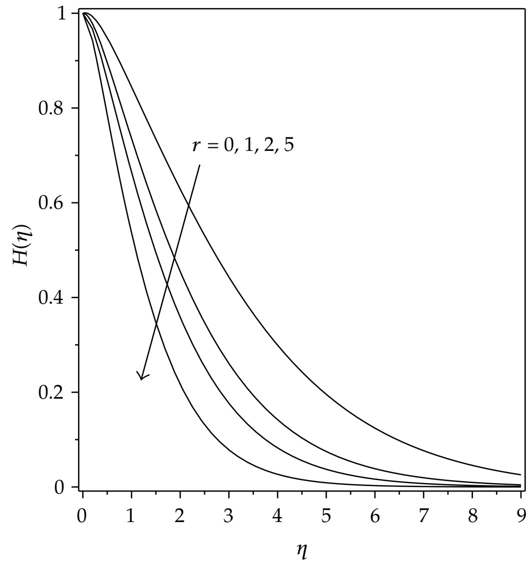

Figure 7: Concentration profiles for different values of r when $Pr = 0.78$, $Sc = 0.6$, $D_f = 0.03$, $S_r = 2$, $M = 1$, and $f_0 = 0$.

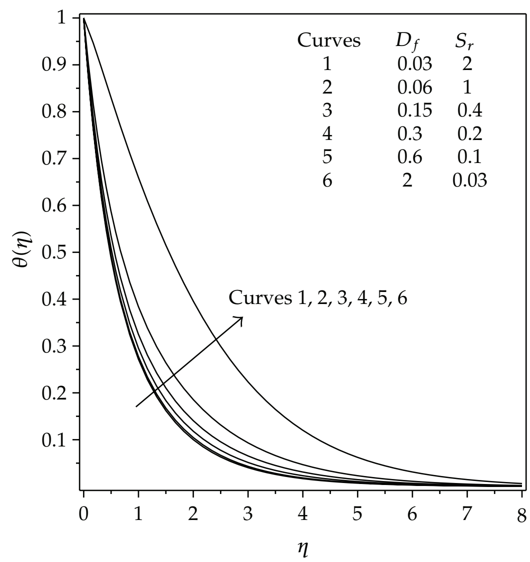

Figure 8: Temperature profiles for different values of D_f and S_r when $Pr = 0.78$, $Sc = 0.6$, $r = 1$, $M = 1$, and $f_0 = 0$.

with the increasing of the magnetic field parameter M. The temperature and concentration profiles are also affected by Pr, Sc, D_f, and S_r. Figures 4 and 5 show the influences of the suction and injection parameter f_0 on the temperature and concentration profiles. The results point out that increasing values in suction parameter ($f_0 > 0$) at the wall tend to decrease the temperature of the flow as shown in the Figure 4. Concurrently, the concentration profiles decrease as well with the inclusion of the suction parameter. This phenomenon is caused by

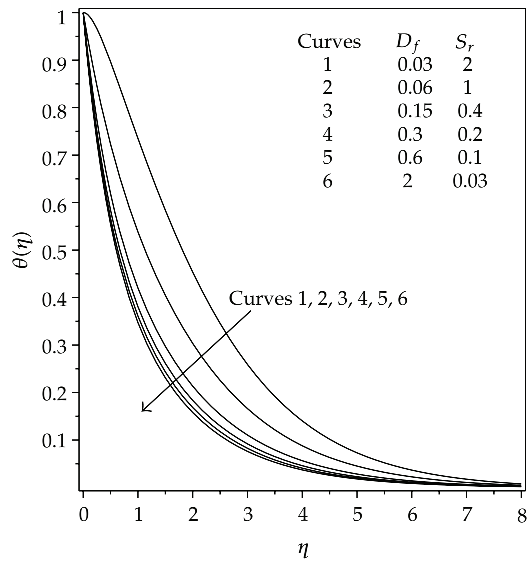

Figure 9: Concentration profiles for different values of D_f and S_r when $Pr = 0.78$, $Sc = 0.6$, $r = 1$, $M = 1$, and $f_0 = 0$.

the fluid moves nearer to the surface and decreases the thermal and concentration boundary layer thickness. Conversely, these observations are found to be opposite in the case of injection ($f_0 < 0$). It is seen that the imposition of the injection parameter will increase the fluid temperature and concentration.

The effect of the inclusion of the thermosolutal surface tension ratio r on the temperature and concentration profiles is illustrated in Figures 6 and 7, respectively. We observed that the parameter r significantly decreases the fluid temperature and concentration. This finding is obtained due to the increase of the Marangoni convection effect as r increases. From physical point of view, by increasing the Marangoni convection effect, more induced flows are produced. As consequences, the resulting flows will propagate within the boundary layers impling the maximum velocity obtained at the wall.

Figures 8 and 9 show the combination effects of the Dufour and Soret numbers on the fluid temperature and concentration. The Dufour D_f and Soret S_r numbers represent the thermal diffusion and diffusion thermal effects in this problem. Moreover, we have to be discriminating in selection of Dufour and Soret numbers in order to guarantee that the product of $S_r D_f$ is kept constant as well as assuming the mean temperature T_m is constant. To be practical, the Dufour and Soret values that are used in the present study are referred to the paper reported by Kafoussias and Williams [17]. Figure 8 specifically shows the influences of the Dufour and Soret number on the variations of the fluid temperature. For the case of increasing Dufour number and decreasing Soret number, it is seen that the temperature profiles show dissimilar increasing on its values. The Dufour term that describes the effect of concentration gradients as underlined in (2.9) plays a vital role in assisting the flow and is able to increase thermal energy in the boundary layer. This is the evident that as the parameter D_f increases and S_r decreases, the fluid temperature will increase.

In Figure 9, increasing Dufour number and simultaneously decreasing Soret number have implied significant effects on the concentration profiles. The Soret term exemplifies the temperature gradient effects on the variation of concentration as noted in (2.10). It is observed as the Dufour number increases and Soret number is decreased, the concentration values are found to decrease. For a small Soret number $S_r < 0.4$, it is seen that the concentration values decrease steadily and closely to each other with similar pattern. On the other hand, these observations are found to be contrary in the case of $S_r > 1$ when the graph shows large differences in concentration values compared to curves (3–5) with low values of Soret number. The physical reason of this phenomena that occurs is due to a strong concentration overshoot that happens nearly to the surface.

Furthermore, the results in the Figures 8 and 9 agree well with the data in Table 4. We can see that combination effects of the thermal diffussion and diffusion thermo can reduce the surface temperature gradient while increase the surface concentration gradient. Hence, the local Nusselt number decreases and the local Sherwood number increases by increasing the Dufour number and reducing the Soret number.

4. Conclusions

The problem of thermal diffusion and diffusion thermo effects on thermosolutal Marangoni convection boundary layer flow over a flat surface considering the fluid suction and injection in the presence of the magnetic field is studied. The governing partial differential equations associated with the boundary conditions were transformed into nonlinear ordinary differential equations before being solved using the Runge-Kutta-Fehlberg method. The effects of thermal diffusion (Soret number S_r) and diffusion thermo (Dufour number D_f), magnetic field parameter M, thermosolutal surface tension ratio r and suction or injection parameter f_0 on the velocity, temperature and concentration field, and the physical quantities interest in engineering problem such as surface velocity, the local Nusselt number and Sherwood number were plotted, tabulated, and analyzed. It is found that the inclusion of the magnetic field parameter on the flow increased the temperature, and concentration profiles while it decreased the velocity field as well as Nusselt an Sherwood numbers. The analysis also revealed that the same behavior was drawn as thermosolutal surface tension ratio r was decreased. We also observed that increasing the suction parameter f_0 has decreased the fluid velocity, temperature and concentration profiles as it increased the Nusselt and Sherwood numbers. In contrast, the opposite observation was attained for the imposition of the injection parameter. The current analysis also signifies that the temperature profile and Sherwood number increase with the increasing in Dufour number and decreasing in Soret number. Opposite behavior is identified on Nusselt number and concentration profile. We also noticed that the velocity field is insensitive by changing in Dufour and Soret numbers.

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