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Asymptotic non-null distributions of the LR criteria in a parallel profile model with random effects

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ABSTRACT. We consider a parallel profile model which is a mixture of the MANOVA and GMANOVA models. The covariance structure based on a random-effects model is assumed. Asymptotic non-null distributions of the likelihood ratio tests for two hypotheses are derived under the parallel profile model. A numerical example is also presented.

1. Introduction

Suppose that a response variable x has been measured at p different occasions on each of N individuals, and each individual belongs to one of k groups or treatments. Let $\boldsymbol{x}_j^{(g)} = (x_{1j}^{(g)}, \ldots, x_{pj}^{(g)})'$ be a p-vector of measurements on the j-th individual in the g-th group, and assume that $\boldsymbol{x}_j^{(g)}$ are independently distributed as $N_p(\boldsymbol{\mu}^{(g)}, \Sigma)$, where $j = 1, \ldots, N_g$, $g = 1, \ldots, k$. Further, we assume that profiles of k groups are parallel, i.e.,

$$\boldsymbol{\mu}^{(g)} = \delta^{(g)}\mathbf{1}_p + \boldsymbol{\mu}, \qquad g = 1, \ldots, k, \tag{1.1}$$

where $\mathbf{1}_p$ is a p-vector of ones. Without loss of generality we may assume that $\delta^{(k)} = 0$. In the following we shall do this. Let

$$X = [\boldsymbol{x}_1^{(1)}, \ldots, \boldsymbol{x}_{N_1}^{(1)}, \ldots, \boldsymbol{x}_1^{(k)}, \ldots, \boldsymbol{x}_{N_k}^{(k)}]'.$$

Then the model of X can be written as

$$X \sim N_{N\times p}(A_1\boldsymbol{\delta}\mathbf{1}_p' + \mathbf{1}_N\boldsymbol{\mu}', \Sigma \otimes I_N), \tag{1.2}$$

where $N = N_1 + \cdots + N_k$,

$$A_1 = \begin{bmatrix} \mathbf{1}_{N_1} & & O \\ & \ddots & \\ O & & \mathbf{1}_{N_{k-1}} \\ & \cdots\cdots & \\ & O & \end{bmatrix}$$

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is an $N \times (k-1)$ between-individual design matrix of rank $k-1$ $(\leq N-p-1)$, $\boldsymbol{\delta} = (\delta^{(1)}, \dots, \delta^{(k-1)})'$ and $\boldsymbol{\mu} = (\mu_1, \dots, \mu_p)'$ are vectors of unknown parameters, Σ is an unknown $p \times p$ positive definite matrix. The model (1.2) is called a parallel profile model, which is a special case of mixed MANOVA-GMANOVA models (see, Chinchilli and Elswick [1], Verbyla and Venables [6]).

This paper is concerned with profile analysis under a random-effects covariance structure, which is based on random-coefficients models (see, e.g., Rao [2], [3], Ware [7]). In our model the structure can be expressed as

$$\Sigma = \lambda^2 \mathbf{1}_p \mathbf{1}_p' + \sigma^2 I_p, \tag{1.3}$$

where $\lambda^2 \geq 0$ and $\sigma^2 > 0$. The parallel profile model with random effects can be written as

$$X \sim N_{N \times p}(A_1 \boldsymbol{\delta} \mathbf{1}_p' + \mathbf{1}_N \boldsymbol{\mu}', (\lambda^2 \mathbf{1}_p \mathbf{1}_p' + \sigma^2 I_p) \otimes I_N). \tag{1.4}$$

For an extension of the model (1.4) to the multiple-response case, see Yokoyama [9]. In order to examine whether or not the model (1.4) can be assumed, Yokoyama [8] obtained the likelihood ratio (=LR) criterion for the hypothesis

$$H_0\colon \Sigma = \lambda^2 \mathbf{1}_p \mathbf{1}_p' + \sigma^2 I_p \quad \text{vs.} \quad H_1\colon \text{not } H_0 \tag{1.5}$$

under the parallel profile model (1.2). Srivastava [4] obtained the LR tests for "no condition variation" hypothesis

$$H_{01}\colon \boldsymbol{\mu} = \nu \mathbf{1}_p \quad \text{vs.} \quad H_{11}\colon \boldsymbol{\mu} \neq \nu \mathbf{1}_p \tag{1.6}$$

and "level" hypothesis

$$H_{02}\colon \boldsymbol{\delta} = \mathbf{0} \quad \text{vs.} \quad H_{12}\colon \boldsymbol{\delta} \neq \mathbf{0} \tag{1.7}$$

under the assumptions that Σ is an unknown positive definite matrix and the response means $\boldsymbol{\mu}^{(g)}$'s satisfy (1.1), where $-\infty < \nu < \infty$. In this paper we consider to test these two hypotheses under the random-effects covariance structure (1.3). The LR criterion for the hypothesis (1.7), along with its asymptotic null distribution, has been obtained by Yokoyama and Fujikoshi [10]. In Section 2 we give a canonical reduction. In Section 3 we derive asymptotic non-null distributions of the LR criteria for the hypotheses (1.6) and (1.7) under the parallel profile model (1.4). In Section 4 we apply the results of the asymptotic null distributions of these tests to a data set of repeated measurements.

2. A canonical reduction

The random-effects covariance structure (1.3) is based on the following model:

$$x_j^{(g)} = (\delta^{(g)} + u_j^{(g)})\mathbf{1}_p + \boldsymbol{\mu} + e_j^{(g)}, \tag{2.1}$$

where $u_j^{(g)}$ and $e_j^{(g)}$ are independently distributed as $N(0, \lambda^2)$ and $N_p(\mathbf{0}, \sigma^2 I_p)$, respectively. Here, the latent variables $u_j^{(g)}$'s can be regarded as variables denoting variation between individuals for each group. From (2.1), we have

$$V(x_j^{(g)}) = \Sigma = \lambda^2 \mathbf{1}_p \mathbf{1}_p' + \sigma^2 I_p,$$

which implies (1.4).

We now derive a canonical reduction. Let $Q = [p^{-1/2}\mathbf{1}_p Q_2]$ and $H = [N^{-1/2}\mathbf{1}_N H_2]$ be the orthogonal matrices. Then a canonical form of the model (1.4) can be written as

$$H'XQ = \begin{bmatrix} z_1 & z_2' \\ y_1 & Y_2 \end{bmatrix} \sim N_{N\times p}\left(\begin{bmatrix} \theta_1 & \theta_2' \\ \tilde{A}_1\gamma & O \end{bmatrix}, \Psi \otimes I_N\right), \tag{2.2}$$

where $\theta_1 = N^{-1/2}\mathbf{1}_N' A_1 \gamma + N^{1/2}p^{-1/2}\boldsymbol{\mu}'\mathbf{1}_p$, $\theta_2' = N^{1/2}\boldsymbol{\mu}'Q_2$, $\tilde{A}_1 = H_2' A_1$, $\gamma = p^{1/2}\boldsymbol{\delta}$ and

$$\Psi = Q'\Sigma Q = \begin{pmatrix} p\lambda^2 + \sigma^2 & \mathbf{0} \\ \mathbf{0} & \sigma^2 I_{p-1} \end{pmatrix}.$$

We can express the hypotheses (1.6) and (1.7) as

$$H_{01}: \theta_2 = \mathbf{0} \quad \text{vs.} \quad H_{11}: \theta_2 \neq \mathbf{0} \tag{2.3}$$

and

$$H_{02}: \gamma = \mathbf{0} \quad \text{vs.} \quad H_{12}: \gamma \neq \mathbf{0}, \tag{2.4}$$

respectively.

3. LR tests for two hypotheses

We consider the LR test for the hypothesis (1.6) under the parallel profile model (1.4). This is equivalent to considering the LR test for the hypothesis (2.3) under the model (2.2). It is easily seen that the MLE's of θ_1, θ_2 and γ under H_{01} are given by

$$\hat{\theta}_1 = z_1, \qquad \hat{\theta}_2 = \mathbf{0}, \qquad \hat{\gamma} = (\tilde{A}_1'\tilde{A}_1)^{-1}\tilde{A}_1' y_1. \tag{3.1}$$

Let $L(\theta_1, \theta_2, \gamma, \sigma^2, \lambda^2)$ be the likelihood function of (z_1, z_2, y_1, Y_2). Then we have

$$g(\sigma^2, \lambda^2) = -2 \log L(\hat{\theta}_1, \hat{\theta}_2, \hat{\gamma}, \sigma^2, \lambda^2)$$
$$= N \log (p\lambda^2 + \sigma^2) + \frac{s_{11}}{p\lambda^2 + \sigma^2}$$
$$+ N(p-1) \log \sigma^2 + \frac{1}{\sigma^2}(z_2' z_2 + \mathrm{tr}\ Y_2' Y_2),$$

where $s_{11} = y_1'(I_{N-1} - P_{\tilde{A}_1})y_1$, $P_{\tilde{A}_1} = \tilde{A}_1(\tilde{A}_1'\tilde{A}_1)^{-1}\tilde{A}_1'$. The minimum of $g(\sigma^2, \lambda^2)$ is achieved at

$$\text{(3.2)} \quad \begin{aligned} \hat{\lambda}^2 &= \max\left\{\frac{1}{p}\left[\frac{1}{N}s_{11} - \frac{1}{N(p-1)}(z_2'z_2 + \mathrm{tr}\ Y_2'Y_2)\right], 0\right\}, \\ \hat{\sigma}^2 &= \min\left\{\frac{1}{N(p-1)}(z_2'z_2 + \mathrm{tr}\ Y_2'Y_2), \frac{1}{Np}(s_{11} + z_2'z_2 + \mathrm{tr}\ Y_2'Y_2)\right\}. \end{aligned}$$

Therefore, from (2.7) in Yokoyama and Fujikoshi [10] and $z_2'z_2 \geq 0$ we can write the LR criterion as

$$\text{(3.3)} \quad \Lambda_1^{N/2} = \begin{cases} R_1, & \text{if } z_2'z_2/\{N(p-1)\} \leq s_{11}/N - \mathrm{tr}\ Y_2'Y_2/\{N(p-1)\}, \\ R_2, & \text{if } 0 \leq s_{11}/N - \mathrm{tr}\ Y_2'Y_2/\{N(p-1)\} \leq z_2'z_2/\{N(p-1)\}, \\ R_3, & \text{if } s_{11}/N - \mathrm{tr}\ Y_2'Y_2/\{N(p-1)\} \leq 0, \end{cases}$$

where

$$R_1 = \left(\frac{\mathrm{tr}\ Y_2'Y_2}{z_2'z_2 + \mathrm{tr}\ Y_2'Y_2}\right)^{N(p-1)/2}, \qquad R_3 = \left(\frac{s_{11} + \mathrm{tr}\ Y_2'Y_2}{s_{11} + z_2'z_2 + \mathrm{tr}\ Y_2'Y_2}\right)^{Np/2},$$

$$R_2 = \frac{\left(\frac{1}{N}s_{11}\right)^{N/2}\left\{\frac{1}{N(p-1)}\mathrm{tr}\ Y_2'Y_2\right\}^{N(p-1)/2}}{\left\{\frac{1}{Np}(s_{11} + z_2'z_2 + \mathrm{tr}\ Y_2'Y_2)\right\}^{Np/2}}.$$

The LR criterion (3.3) can be expressed in terms of the original observations, using

$$s_{11} = \frac{1}{p}\mathbf{1}_p'S_w\mathbf{1}_p, \qquad \mathrm{tr}\ Y_2'Y_2 = \mathrm{tr}\ S_t - \frac{1}{p}\mathbf{1}_p'S_t\mathbf{1}_p, \qquad z_2'z_2 = N\left\{\bar{\boldsymbol{x}}'\bar{\boldsymbol{x}} - \frac{1}{p}(\bar{\boldsymbol{x}}'\mathbf{1}_p)^2\right\},$$

where S_t and S_w are the matrices of the sums of squares and products due to the total variation and within variation, i.e.,

$$S_t = \sum_{g=1}^{k}\sum_{j=1}^{N_g}(\boldsymbol{x}_j^{(g)} - \bar{\boldsymbol{x}})(\boldsymbol{x}_j^{(g)} - \bar{\boldsymbol{x}})', \qquad S_w = \sum_{g=1}^{k}\sum_{j=1}^{N_g}(\boldsymbol{x}_j^{(g)} - \bar{\boldsymbol{x}}^{(g)})(\boldsymbol{x}_j^{(g)} - \bar{\boldsymbol{x}}^{(g)})',$$

$\bar{x}$ and $\bar{x}^{(g)}$ are the sample mean vectors of observations of all the groups and the g-th group, respectively.

THEOREM 3.1. *Let $\Lambda_1^{N/2}$ be the LR criterion for testing $H_{01}: \boldsymbol{\mu} = \nu \mathbf{1}_p$ vs. $H_{11}: \boldsymbol{\mu} \neq \nu \mathbf{1}_p$. Then, under the sequence of local alternatives*

$$H_{11}^{(N)}: \boldsymbol{\mu} = \nu \mathbf{1}_p + \frac{1}{\sqrt{N}} \boldsymbol{\rho}\,,$$

it holds that

$$\lim_{N\to\infty} P(-N \log \Lambda_1 \le c) = P(\chi_{p-1}^2(\delta_1^*) \le c)\,,$$

where $\boldsymbol{\rho}$ is a constant vector, $\chi_{p-1}^2(\delta_1^*)$ denotes a χ^2 variate with $p-1$ degrees of freedom and noncentrality parameter $\delta_1^* = \{\boldsymbol{\rho}'\boldsymbol{\rho} - (\boldsymbol{\rho}'\mathbf{1}_p)^2/p\}/\sigma^2$.

PROOF. From the definition of $\Lambda_1^{N/2}$ we have

$$\begin{aligned} &P(-N \log \Lambda_1 \le c) \\ &\quad = P(-2 \log R_1 \le c,\, z_2' z_2/\{N(p-1)\} \le s_{11}/N - \operatorname{tr} Y_2' Y_2/\{N(p-1)\}) \\ &\qquad + P(-2 \log R_2 \le c,\, 0 \le s_{11}/N - \operatorname{tr} Y_2' Y_2/\{N(p-1)\} \le z_2' z_2/\{N(p-1)\}) \\ &\qquad + P(-2 \log R_3 \le c,\, s_{11}/N - \operatorname{tr} Y_2' Y_2/\{N(p-1)\} \le 0). \end{aligned}$$

Let

$$\frac{1}{\sqrt{2N}}\left(\frac{1}{\tau^2} s_{11} - N\right) = U_1\,, \qquad \frac{1}{\sqrt{2N(p-1)}}\left(\frac{1}{\sigma^2} \operatorname{tr} Y_2' Y_2 - N(p-1)\right) = U_2\,,$$

where $\tau = (p\lambda^2 + \sigma^2)^{1/2}$. Then U_1 and U_2 are independent, and the limiting distribution of U_i is $N(0, 1)$, $i = 1, 2$. Note that under $H_{11}^{(N)}$, $z_2' z_2$ is distributed as $\sigma^2 \chi_{p-1}^2\ (\boldsymbol{\rho}' Q_2 Q_2' \boldsymbol{\rho}/\sigma^2)$. Since $Q_2 Q_2' = I_p - \mathbf{1}_p \mathbf{1}_p'/p$, the noncentrality parameter is $\boldsymbol{\rho}' Q_2 Q_2' \boldsymbol{\rho}/\sigma^2 = \delta_1^*$. Using $\log(1+x) = x - x^2/2 + O(x^3)$, we can expand $-2 \log R_1$ as

$$-2 \log R_1 = \frac{1}{\sigma^2} z_2' z_2 + O_p(N^{-1/2})\,.$$

When $\lambda^2 > 0$, we have

$$\lim_{N\to\infty} P(z_2' z_2/\{N(p-1)\} \le s_{11}/N - \operatorname{tr} Y_2' Y_2/\{N(p-1)\}) = 1$$

and hence

$$\begin{aligned} \lim_{N\to\infty} P(-N \log \Lambda_1 \le c) &= \lim_{N\to\infty} P(-2 \log R_1 \le c) \\ &= P(\chi_{p-1}^2(\delta_1^*) \le c)\,. \end{aligned}$$

Let

$$Z_1^* = \sqrt{\frac{p-1}{p}}\,U_1 - \sqrt{\frac{1}{p}}\,U_2 .$$

Then the limiting distribution of Z_1^* is $N(0, 1)$. When $\lambda^2 = 0$, we have

$$\lim_{N\to\infty} P(0 \le s_{11}/N - \mathrm{tr}\, Y_2'Y_2/\{N(p-1)\} \le z_2'z_2/\{N(p-1)\}) = 0$$

and

$$-2\log R_3 = \frac{1}{\sigma^2} z_2'z_2 + O_p(N^{-1/2}) .$$

Here we note that $z_2'z_2/\{N(p-1)\} \le s_{11}/N - \mathrm{tr}\, Y_2'Y_2/\{N(p-1)\}$ is asymptotically equivalent to $Z_1^* \ge 0$, and $s_{11}/N - \mathrm{tr}\, Y_2'Y_2/\{N(p-1)\} \le 0$ is equivalent to $Z_1^* \le 0$. Therefore

$$\begin{aligned}
&\lim_{N\to\infty} P(-N\log \Lambda_1 \le c)\\
&= \lim_{N\to\infty} \left\{P\left(\frac{1}{\sigma^2} z_2'z_2 \le c, Z_1^* \ge 0\right) + P\left(\frac{1}{\sigma^2} z_2'z_2 \le c, Z_1^* \le 0\right)\right\}\\
&= P(\chi^2_{p-1}(\delta_1^*) \le c) ,
\end{aligned}$$

which proves the desired result.

Now we derive an asymptotic non-null distribution of the LR criterion for the hypothesis (1.7) under the parallel profile model (1.4). Noting that $y_1'y_1 \ge y_1'(I_{N-1} - P_{\tilde{A}_1})y_1 = s_{11}$, we can write the LR criterion as

$$(3.4) \qquad \Lambda_2^{N/2} = \begin{cases} R_4 , & \text{if } s_{11}/N \ge \mathrm{tr}\, Y_2'Y_2/\{N(p-1)\} ,\\ R_5 , & \text{if } y_1'y_1/N \ge \mathrm{tr}\, Y_2'Y_2/\{N(p-1)\} \ge s_{11}/N ,\\ R_6 , & \text{if } \mathrm{tr}\, Y_2'Y_2/\{N(p-1)\} \ge y_1'y_1/N ,\end{cases}$$

where

$$R_4 = \left(\frac{s_{11}}{y_1'y_1}\right)^{N/2} , \qquad R_6 = \left(\frac{s_{11} + \mathrm{tr}\, Y_2'Y_2}{y_1'y_1 + \mathrm{tr}\, Y_2'Y_2}\right)^{Np/2} ,$$

$$R_5 = \frac{\left\{\dfrac{1}{Np}(s_{11} + \mathrm{tr}\, Y_2'Y_2)\right\}^{Np/2}}{\left(\dfrac{1}{N} y_1'y_1\right)^{N/2}\left\{\dfrac{1}{N(p-1)}\,\mathrm{tr}\, Y_2'Y_2\right\}^{N(p-1)/2}}$$

(see Yokoyama and Fujikoshi [10]), where

$$y_1'y_1 = \frac{1}{p}\mathbf{1}_p'S_t\mathbf{1}_p .$$

THEOREM 3.2. *Let $\Lambda_2^{N/2}$ be the LR criterion for testing H_{02}: $\boldsymbol{\delta} = \mathbf{0}$ vs. H_{12}: $\boldsymbol{\delta} \neq \mathbf{0}$, and assume that $\tilde{A}_1'\tilde{A}_1 = O(N)$. Then, under the sequence of local alternatives*

$$H_{12}^{(N)}: \boldsymbol{\delta} = \frac{1}{\sqrt{N}}\boldsymbol{\beta},$$

it holds that

$$\lim_{N\to\infty} P(-N \log \Lambda_2 \leq c) = P(\chi_{k-1}^2(\delta_2^*) \leq c),$$

where $\boldsymbol{\beta}$ is a constant vector, $\delta_2^* = \lim_{N\to\infty} p\boldsymbol{\beta}'\tilde{A}_1'\tilde{A}_1\boldsymbol{\beta}/(\tau^2 N)$.

PROOF. From the definition of $\Lambda_2^{N/2}$ we have

$$\begin{aligned} P(-N \log \Lambda_2 \leq c) & \\ &= P(-2\log R_4 \leq c,\, s_{11}/N \geq \operatorname{tr} Y_2'Y_2/\{N(p-1)\}) \\ &\quad + P(-2\log R_5 \leq c,\, \boldsymbol{y}_1'\boldsymbol{y}_1/N \geq \operatorname{tr} Y_2'Y_2/\{N(p-1)\} \geq s_{11}/N) \\ &\quad + P(-2\log R_6 \leq c,\, \operatorname{tr} Y_2'Y_2/\{N(p-1)\} \geq \boldsymbol{y}_1'\boldsymbol{y}_1/N). \end{aligned}$$

Let U_i and Z_1^* be the same ones in the proof of Theorem 3.1. Note that under $H_{12}^{(N)}$, $\boldsymbol{y}_1'P_{\tilde{A}_1}\boldsymbol{y}_1$ is distributed as $\tau^2\chi_{k-1}^2(p\boldsymbol{\beta}'\tilde{A}_1'\tilde{A}_1\boldsymbol{\beta}/(\tau^2 N))$, and is independent of s_{11}. Then, by the same way as in Yokoyama and Fujikoshi [10], we can expand $-2\log R_4$ as

$$-2\log R_4 = \frac{1}{\tau^2}\boldsymbol{y}_1'P_{\tilde{A}_1}\boldsymbol{y}_1 + O_p(N^{-1/2}).$$

When $\lambda^2 > 0$, we have

$$\lim_{N\to\infty} P(s_{11}/N \geq \operatorname{tr} Y_2'Y_2/\{N(p-1)\}) = 1$$

and hence

$$\begin{aligned} \lim_{N\to\infty} P(-N\log \Lambda_2 \leq c) &= \lim_{N\to\infty} P(-2\log R_4 \leq c) \\ &= P(\chi_{k-1}^2(\delta_2^*) \leq c). \end{aligned}$$

When $\lambda^2 = 0$, we have

$$\lim_{N\to\infty} P(\boldsymbol{y}_1'\boldsymbol{y}_1/N \geq \operatorname{tr} Y_2'Y_2/\{N(p-1)\} \geq s_{11}/N) = 0$$

and

$$-2\log R_6 = \frac{1}{\tau^2} y_1' P_{\tilde{A}_1} y_1 + O_p(N^{-1/2})\ .$$

Here we note that $s_{11}/N \geq \operatorname{tr} Y_2'Y_2/\{N(p-1)\}$ is equivalent to $Z_1^* \geq 0$, and $\operatorname{tr} Y_2'Y_2/\{N(p-1)\} \geq y_1'y_1/N$ is asymptotically equivalent to $Z_1^* \leq 0$. Therefore

$$\begin{aligned}
&\lim_{N\to\infty} P(-N\log \Lambda_2 \leq c)\\
&\quad = \lim_{N\to\infty}\left\{P\left(\frac{1}{\tau^2} y_1' P_{\tilde{A}_1} y_1 \leq c,\, Z_1^* \geq 0\right) + P\left(\frac{1}{\tau^2} y_1' P_{\tilde{A}_1} y_1 \leq c,\, Z_1^* \leq 0\right)\right\},
\end{aligned}$$

which implies the desired result.

We note that under the null hypotheses H_{01} and H_{02}, the limiting distributions of the LR criteria in Theorems 3.1 and 3.2 are central χ^2-distributions with $p-1$ and $k-1$ degrees of freedom, respectively. The latter agrees with the results in Yokoyama and Fujikoshi [10].

4. An example

We apply the results of Section 3 to the data (see, e.g., Srivastava and Carter [5, Table 7.14]) of the price indices of hand soaps packaged in 4 ways, estimated by 12 consumers. Each consumer belongs to one of 2 groups. For this repeated measures data, we may assume the parallel profile model (1.4) in the case $p = 4$, $k = 2$ and $N = 12$, i.e.,

$$X \sim N_{12\times 4}\left(\begin{bmatrix}\mathbf{1}_6\\ \mathbf{0}\end{bmatrix}\delta\mathbf{1}_4' + \mathbf{1}_{12}\boldsymbol{\mu}',\ (\lambda^2\mathbf{1}_4\mathbf{1}_4' + \sigma^2 I_4)\otimes I_{12}\right). \tag{4.1}$$

The adequacy of the model (4.1) to the data has been examined in Yokoyama [8]. Now we consider testing the hypotheses (1.6) and (1.7) under the model (4.1). Since

$$s_{11} = \frac{1}{p}\mathbf{1}_p' S_w \mathbf{1}_p = .76635\,, \qquad y_1'y_1 = \frac{1}{p}\mathbf{1}_p' S_t \mathbf{1}_p = 1.8131\,,$$

$$\operatorname{tr} Y_2'Y_2 = \operatorname{tr} S_t - \frac{1}{p}\mathbf{1}_p' S_t \mathbf{1}_p = .35130\,, \qquad z_2'z_2 = N\left\{\bar{\boldsymbol{x}}'\bar{\boldsymbol{x}} - \frac{1}{p}(\bar{\boldsymbol{x}}'\mathbf{1}_p)^2\right\} = .78204$$

and $z_2'z_2/\{N(p-1)\} < s_{11}/N - \operatorname{tr} Y_2'Y_2/\{N(p-1)\}$, it follows from Theorems 3.1 and 3.2 that

$$\Lambda_1 = \left(\frac{\operatorname{tr} Y_2'Y_2}{z_2'z_2 + \operatorname{tr} Y_2'Y_2}\right)^{p-1} = .029782\,, \qquad \Lambda_2 = \frac{s_{11}}{y_1'y_1} = .42266\,,$$

and

$$-N\log\Lambda_1 = 42.166 > \chi^2_{p-1}(.01) = 11.345\,,$$

$$-N\log\Lambda_2 = 10.334 > \chi^2_{k-1}(.01) = 6.635\,.$$

Therefore, both hypotheses H_{01} and H_{02} should be rejected in this example.

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