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APPROXIMATING FIXED POINTS OF CONTRACTIVE SET-VALUED MAPPINGS

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Abstract

We establish two uniform convergence theorems for an iterative scheme which approximates fixed points of strictly contractive set-valued mappings in complete metric spaces.

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1 Introduction and preliminaries

In the last forty-five years there has been considerable interest in the fixed point theory of single- and set-valued contractive and nonexpansive mappings. See, for, example, de Blasi and Myjak [BMa, BMb], Goebel and Kirk [GK], Goebel and Reich [GR], Kirk [Ki], Nadler [N], Ricceri [R], and the references mentioned therein. More recently, set-valued dynamical systems induced by set-valued nonexpansive mappings have been investigated and some new iterative methods for approximating the corresponding fixed points have been obtained (de Blasi, Myjak, Reich and Zaslavski [BMRZ], Reich and Zaslavski [RZa, RZb, RZc]).

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It is well known that there is a strong connection between functional analysis, approximation theory and numerical analysis (see, for instance, Kantorovich [Ka], Kantorovich and Akilov [KA], and Lax [L]). Indeed, convergence of many iterative schemes in numerical analysis follows from the properties of certain operators acting on functional spaces. Moreover, the Banach fixed point theorem plays an important role in some of these schemes. This connection also works in the opposite direction. For example, Banach's theorem is usually established by proving the convergence of iterates. Also, Nadler [N] establishes the existence of a fixed point of a set-valued contractive mapping by using a certain iterative scheme. A more general form of this scheme is employed in [BMRZ].

In the present paper we consider the question of approximating the fixed points of strictly contractive set-valued mappings. More precisely, we provide sufficient conditions for the iterative scheme of [BMRZ] to converges either to a fixed point or to the fixed point set, uniformly for all initial points taken from any bounded set. See Theorems 5 and 6 in Sections 2 and 3, respectively. Although our results are stated for complete metric spaces, we emphasize that they are new even in Banach spaces.

We begin with some notation and terminology which are used throughout the paper.

Let (X,ρ) be a complete metric space. For $x \in X$ and a nonempty subset A of X, set

$$\rho(x,A) = \inf\{\rho(x,y) :\ y \in A\}.$$

For each pair of nonempty closed sets $A,B \subset X$, put

$$H(A,B) = \max\{\sup_{x\in A} \rho(x,B),\ \sup_{y\in B} \rho(y,A)\}.$$

Let $T : X \to 2^X \setminus \{\emptyset\}$ be such that $T(x)$ is a closed subset of X for each $x \in X$ and assume that

$$H(T(x),T(y)) \leq c\rho(x,y) \text{ for all } x,y \in X, \tag{1}$$

where $c \in [0,1)$ is a constant .

We now quote Theorems 4.1-4.4 of [BMRZ]. These theorems provide information on the asymptotic behavior of certain trajectories of a dynamical system induced by T.

Theorem 1. *Let $T : X \to 2^X \setminus \{\emptyset\}$ be a strict contraction such that $T(x)$ is a closed set for each $x \in X$ and T satisfies (1). Assume that $x_0 \in X$, $\{\varepsilon_i\}_{i=0}^{\infty} \subset (0,\infty)$, $\sum_{i=0}^{\infty} \varepsilon_i < \infty$, and that for each integer $i \geq 0$,*

$$x_{i+1} \in T(x_i),\ \rho(x_i,x_{i+1}) \leq \rho(x_i,T(x_i)) + \varepsilon_i.$$

Then $\{x_i\}_{i=0}^{\infty}$ converges to a fixed point of T.

Theorem 2. *Let $T : X \to 2^X \setminus \{\emptyset\}$ be a strict contraction such that $T(x)$ is a closed set for all $x \in X$ and T satisfies (1). Let $\varepsilon > 0$ be given. Then there exists $\delta > 0$ such that if $x \in X$ and $\rho(x,T(x)) < \delta$, then there is $\bar{x} \in X$ such that $\bar{x} \in T(\bar{x})$ and $\rho(x,\bar{x}) \leq \varepsilon$.*

Theorem 3. *Let $T : X \to 2^X \setminus \{\emptyset\}$ be a strict contraction such that $T(x)$ is a closed set for all $x \in X$ and T satisfies (1). Fix $\theta \in X$. Let $\varepsilon > 0$ and $M > 0$ be given. Then there exist $\delta \in (0,\varepsilon)$ and an integer $n_0 \geq 1$ with the following property:*

for each sequence $\{x_i\}_{i=0}^{\infty} \subset X$ *such that* $\rho(x_0,\theta) \le M$ *and such that for each integer* $n \ge 0$*,*

$$x_{n+1} \in T(x_n) \text{ and } \rho(x_{n+1},x_n) \le \delta + \rho(x_n,T(x_n)),$$

we have

$$\rho(x_{n+1},x_n) < \varepsilon \text{ for all integers } n \ge n_0.$$

Theorem 4. *Let* $T : X \to 2^X \setminus \{\emptyset\}$ *be a strict contraction such that* $T(x)$ *is a closed set for all* $x \in X$ *and* T *satisfies (1). Fix* $\theta \in X$*. Let positive numbers* ε *and* M *be given. Then there exist* $\delta > 0$ *and an integer* $n_0 \ge 1$ *such that if a sequence* $\{x_i\}_{i=0}^{\infty} \subset X$ *satisfies*

$$\rho(x_0,\theta) \le M,\ x_{n+1} \in T(x_n) \text{ and } \rho(x_n,x_{n+1}) \le \rho(x_n,T(x_n)) + \delta$$

for all integers $n \ge 0$*, then for each integer* $n \ge n_0$*, there is* $y \in X$ *such that* $y \in T(y)$ *and* $\rho(y,x_n) < \varepsilon$*.*

The following example [BMRZ] shows that Theorem 4 cannot be improved in the sense that the fixed point y, the existence of which is guaranteed by this theorem, is not, in general, the same for all integers $n \ge n_0$.

Example 1.

Let $X = [0,1]$, $\rho(x,y) = |x-y|$, $x,y \in X$ and $T(x) = [0,1]$ for all $x \in [0,1]$. Let $\delta > 0$. Choose a natural number k such that $1/k < \delta$. Put

$$x_0 = 0,\ x_i = i/k,\ i = 0,\dots,k,$$

$$x_{i+k} = 1 - i/k,\ i = 0,\dots,k,$$

and for all integers $p \ge 0$ and any $i \in \{0,\dots,2k\}$, put

$$x_{2pk+i} = x_i.$$

Then $\{x_i\}_{i=0}^{\infty} \subset X$ and for any integer $i \ge 0$, we have

$$x_{i+1} \in T(x_i) \text{ and } |x_i - x_{i+1}| \le k^{-1} < \delta.$$

On the other hand, for all $x \in X$ and any integer $p \ge 0$,

$$\max\{|x - x_i| :\ i = 2kp,\dots,2pk+2k\} \ge 1/2.$$

2 Uniform convergence to a fixed point

In this section we state and prove Theorem 5, which shows that the iterative scheme of Theorem 1 with a summable sequence $\{\varepsilon_i\}_{i=0}^{\infty}$ converges to a fixed point, uniformly for all initial points taken from any bounded set.

Theorem 5. *Let* $T : X \to 2^X \setminus \{\emptyset\}$ *be a strict contraction such that* $T(x)$ *is a closed set for each* $x \in X$ *and* T *satisfies (1). Fix* $\theta \in X$*. Assume that* $\{\varepsilon_i\}_{i=0}^{\infty} \subset (0,\infty)$*,* $\sum_{i=0}^{\infty} \varepsilon_i < \infty$*,* $M > 0$ *and* $\delta > 0$*. Then there exists a natural number* n_0 *such that each sequence* $\{x_i\}_{i=0}^{\infty} \subset X$ *which satisfies*

$$\rho(x_0,\theta) \le M, \tag{2}$$

$$x_{i+1} \in T(x_i),\ \rho(x_i, x_{i+1}) \le \rho(x_i, T(x_i)) + \varepsilon_i,\ i = 0, 1, \dots, \tag{3}$$

converges in (X, ρ), $\lim_{i\to\infty} x_i$ *is a fixed point of* T, *and*

$$\rho(x_j, \lim_{i\to\infty} x_i) \le \delta \textit{ for all integers } j \ge n_0.$$

Proof. : Set

$$M_0 = M + (\sum_{n=0}^{\infty} c^n)(\sum_{n=1}^{\infty} \varepsilon_n) + (\sum_{n=0}^{\infty} c^n + 1)(2M + \varepsilon_0 + \rho(\theta, T(\theta)). \tag{4}$$

Choose a natural number $p_0 > 2$ such that

$$(\sum_{p=p_0+1}^{\infty} \varepsilon_p)(\sum_{j=0}^{\infty} c^j) < \delta/8, \tag{5}$$

and a natural number $n_0 > 2p_0 + 2$ such that

$$c^{n_0}(\sum_{i=0}^{\infty} c^i)(2M + \varepsilon_0 + \rho(\theta, T(\theta))) < \delta/8 \tag{6}$$

and

$$(\sum_{i=1}^{\infty} \varepsilon_i) \sum_{j=n_0-p_0}^{\infty} c^j < \delta/8. \tag{7}$$

Let $\{x_i\}_{i=0}^{\infty} \subset X$ satisfy (2) and (3). By Theorem 1, $\{x_i\}_{i=0}^{\infty}$ converges in (X, ρ) and $\lim_{i\to\infty} x_i$ is a fixed point of T. In the proof of Theorem 1 (see Theorem 4.1 of [BMRZ]) it was shown by induction that for each integer $n \ge 1$,

$$\rho(x_n, x_{n+1}) \le c^n \rho(x_0, x_1) + \sum_{i=0}^{n-1} c^i \varepsilon_{n-i}. \tag{8}$$

This implies that

$$\sum_{n=0}^{\infty} \rho(x_n, x_{n+1}) \le \rho(x_0, x_1) + \sum_{n=1}^{\infty} (c^n \rho(x_0, x_1) + \sum_{i=1}^{n} c^{n-i} \varepsilon_i)$$

$$\le \rho(x_0, x_1) \sum_{n=0}^{\infty} c^n + (\sum_{i=1}^{\infty} \varepsilon_i)(\sum_{n=0}^{\infty} c^n). \tag{9}$$

It follows from (9) that for each integer $k \ge 1$,

$$\rho(x_0, x_k) \le \sum_{n=0}^{\infty} \rho(x_n, x_{n+1})$$

$$\le (\sum_{n=0}^{\infty} c^n)(\sum_{n=1}^{\infty} \varepsilon_n) + (\sum_{n=0}^{\infty} c^n) \rho(x_0, x_1). \tag{10}$$

By (3), (2) and (1),

$$\rho(x_0, x_1) \le \rho(x_0, T(x_0)) + \varepsilon_0 \le \rho(x_0, \theta) + \rho(\theta, T(x_0)) + \varepsilon_0$$

$$\leq \rho(x_0,\theta)+\rho(\theta,T(\theta))+H(T(\theta),T(x_0))+\varepsilon_0$$

$$\leq M+\rho(\theta,T(\theta))+M+\varepsilon_0. \tag{11}$$

By (2), (10), (11) and (4), for each integer $k \geq 1$, we have

$$\rho(\theta,x_k) \leq \rho(\theta,x_0)+\rho(x_0,x_k)$$

$$\leq M+(\sum_{n=0}^{\infty} c^n)(\sum_{n=1}^{\infty} \varepsilon_n)+\sum_{n=0}^{\infty} c^n(2M+\varepsilon_0+\rho(\theta,T(\theta))) \leq M_0. \tag{12}$$

By (12),

$$\rho(\theta,\lim_{i\to\infty} x_i) \leq M_0. \tag{13}$$

By (8), (11), (6), (5) and (7), for each integer $k \geq n_0$,

$$\rho(x_k,\lim_{i\to\infty} x_i) = \lim_{n\to\infty} \rho(x_k,x_n) \leq \lim_{n\to\infty}[\sum_{i=k}^{n-1} \rho(x_i,x_{i+1})]$$

$$\leq \lim_{n\to\infty}[\sum_{i=k}^{n-1}(c^i\rho(x_0,x_1)+\sum_{p=0}^{i-1} c^p\varepsilon_{i-p})]$$

$$\leq \rho(x_0,x_1)c^k\sum_{i=0}^{\infty} c^i+\lim_{n\to\infty}\sum_{i=k}^{n-1}[\sum_{p=1}^{i} c^{i-p}\varepsilon_p]$$

$$\leq \rho(x_0,x_1)c^k\sum_{i=0}^{\infty} c^i+\sum_{p=1}^{p_0}(\sum_{j=k-p}^{\infty} c^j)\varepsilon_p+\sum_{p=p_0+1}^{\infty}(\sum_{j=0}^{\infty} c^j)\varepsilon_p$$

$$\leq c^{n_0}[\sum_{i=0}^{\infty} c^i(2M+\varepsilon_0+\rho(\theta,T(\theta))]$$

$$+(\sum_{i=1}^{\infty}\varepsilon_i)(\sum_{j=n_0-p_0}^{\infty} c^j)+(\sum_{j=0}^{\infty} c^j)(\sum_{p=p_0+1}^{\infty}\varepsilon_p) < 3\delta/8.$$

Theorem 5 is proved.

□

3 Uniform convergence to the fixed point set

In this section we state and prove Theorem 6, which shows that the iterative scheme of Theorem 1 with a possibly nonsummable null sequence $\{\varepsilon_i\}_{i=0}^{\infty}$ still converges to the fixed point set of T, uniformly for all initial points taken from any bounded set. We also provide an example (see Example 2 below) which shows that Theorem 6 cannot be improved in the sense that under its conditions the constructed sequence $\{x_i\}_{i=0}^{\infty}$ need not converge to a fixed point of T.

Theorem 6. *Let $T : X \to 2^X \setminus \{\emptyset\}$ be a strict contraction such that $T(x)$ is a closed set for each $x \in X$ and T satisfies (1). Fix $\theta \in X$.*

Let F be the set of all fixed points of T, $M > 0$, and let the sequence $\{\varepsilon_i\}_{i=0}^{\infty} \subset (0,\infty)$ satisfy $\lim_{i\to\infty} \varepsilon_i = 0$. Then for each $\delta > 0$, there exists a natural number n_0 such that for each sequence $\{x_i\}_{i=0}^{\infty} \subset X$ which satisfies

$$\rho(x_0,\theta) \le M, \tag{13}$$

$$x_{i+1} \in T(x_i),\ \rho(x_{i+1},x_i) \le \rho(x_i,T(x_i)) + \varepsilon_i,\ i = 0,1,\dots, \tag{14}$$

and each integer $j \ge n_0$, we have $\rho(x_j,F) \le \delta$.

Proof. :

Set

$$\bar{\varepsilon} = \max\{\varepsilon_i :\ i = 0,1,\dots\}. \tag{15}$$

Fix $\theta_1 \in X$ such that

$$\theta_1 \in T(\theta). \tag{16}$$

For each $x \in X$,

$$\rho(x,T(x)) \le \rho(x,\theta) + \rho(\theta,T(x)) \le \rho(x,\theta) + \rho(\theta,\theta_1) + \rho(\theta_1,T(x))$$

$$\le \rho(x,\theta) + \rho(\theta,\theta_1) + H(T(\theta),T(x)) \le 2\rho(x,\theta) + \rho(\theta,\theta_1). \tag{17}$$

Put

$$M_0 = 2M + \rho(\theta,\theta_1) + \bar{\varepsilon}(1-c)^{-1}. \tag{18}$$

Let $\delta > 0$. By Theorem 2, there is $\gamma_0 > 0$ such that

$$\text{if } x \in X,\ \rho(x,T(x)) \le \gamma_0,\ \text{ then } \rho(x,F) \le \delta. \tag{19}$$

Choose a positive number

$$\gamma_1 < \gamma_0(1-c)4^{-1}. \tag{20}$$

There is a natural number n_1 such that

$$\varepsilon_j \le \gamma_1 \text{ for all integers } j \ge n_1. \tag{21}$$

Next, choose a natural number k such that

$$k(1-c)\gamma_0 16^{-1} > M_0 + 1. \tag{22}$$

Put

$$n_0 = n_1 + k. \tag{23}$$

Assume that a sequence $\{x_i\}_{i=0}^{\infty} \subset X$ satisfies (13) and (14). Let $j \ge 0$ be an integer. By (14),

$$\rho(x_{j+1},x_{j+2}) \le \rho(x_{j+1},T(x_{j+1})) + \varepsilon_{j+1}$$

$$\le H(T(x_j),T(x_{j+1})) + \varepsilon_{j+1} \tag{24}$$

and, in view of (1) and (15),

$$\rho(x_{j+1},x_{j+2}) \le c\rho(x_j,x_{j+1}) + \bar{\varepsilon}. \tag{25}$$

By (14), (17), (15), (13) and (18),

$$\rho(x_0,x_1) \le \rho(x_0,T(x_0)) + \varepsilon_0 \le 2\rho(x_0,\theta) + \rho(\theta,\theta_1) + \bar{\varepsilon} < M_0. \tag{26}$$

We now show by induction that for all integers $j \ge 0$,

$$\rho(x_j,x_{j+1}) \le M_0. \tag{27}$$

By (26), inequality (27) indeed holds for $j = 0$.

Assume that (27) holds for an integer $j \ge 0$.

Then we have by (25), (27) and (18),

$$\begin{aligned}\rho(x_{j+1},x_{j+2}) &\le c\rho(x_j,x_{j+1}) + \bar{\varepsilon} \le cM_0 + \bar{\varepsilon}\\ &c(2M + \rho(\theta,\theta_1)) + c\bar{\varepsilon}(1-c)^{-1} + \bar{\varepsilon}\\ &= c(2M + \rho(\theta,\theta_1)) + \bar{\varepsilon}(1-c)^{-1} \le M_0.\end{aligned}$$

Thus (27) is true for all integers $j \ge 0$.

Let an integer $j \ge n_1$ satisfy

$$\rho(x_j,x_{j+1}) \ge \gamma_0/2. \tag{28}$$

By (24), (1), (21), (20) and, (28),

$$\begin{aligned}\rho(x_{j+1},x_{j+2}) &\le c\rho(x_j,x_{j+1}) + \varepsilon_{j+1} \le c\rho(x_j,x_{j+1}) + \gamma_1\\ &\le \rho(x_j,x_{j+1}) - (1-c)\rho(x_j,x_{j+1}) + (1-c)\gamma_0/4\\ &\le \rho(x_j,x_{j+1}) - (1-c)\gamma_0/4.\end{aligned}$$

Thus we have shown that the following property holds:

(P1) If an integer $j \ge n_1$ satisfies $\rho(x_j,x_{j+1}) \ge \gamma_0/2$, then

$$\rho(x_{j+1},x_{j+2}) \le \rho(x_j,x_{j+1}) - (1-c)\gamma_0/4.$$

Assume now that an integer $j \ge n_1$ satisfies

$$\rho(x_j,x_{j+1}) \le \gamma_0/2. \tag{29}$$

By (24), (1), (29), (21) and (20),

$$\rho(x_{j+1},x_{j+2}) \le c\rho(x_j,x_{j+1}) + \varepsilon_{j+1} \le c\gamma_0/2 + \gamma_1 \le c\gamma_0/2 + (1-c)\gamma_0/4 \le \gamma_0/2.$$

This means that the following property also holds:

(P2) If an integer $j \ge n_1$ satisfies $\rho(x_j,x_{j+1}) \le \gamma_0/2$, then $\rho(x_{j+1},x_{j+2}) \le \gamma_0/2$.

By (P2), (19) and (14), in order to complete the proof it is sufficient to show that $\rho(x_j,x_{j+1}) \le \gamma_0/2$ for some integer $j \in [n_1,n_0]$.

Assume the contrary. Then for each integer $j \in [n_1, n_0]$,

$$\rho(x_j, x_{j+1}) > \gamma_0/2$$

and, in view of (P1),

$$\rho(x_j, x_{j+1}) - \rho(x_{j+1}, x_{j+2}) \geq (1-c)\gamma_0/4.$$

When combined with (27), this inequality implies that

$$M_0 \geq \rho(x_{n_1}, x_{n_1+1}) - \rho(x_{n_0}, x_{n_0+1})$$

$$= \sum_{j=n_1}^{n_0} [\rho(x_j, x_{j+1}) - \rho(x_{j+1}, x_{j+2})] \geq (1-c)\gamma_0 4^{-1}(k+1).$$

This contradicts (22). The contradiction we have reached proves Theorem 6. □

Example 2.
Let $X = [0,1]$, $\rho(x,y) = |x-y|$, $x, y \in X$ and $T(x) = [0,1]$ for all $x \in [0,1]$. Put

$$x_0 = 0,\ x_1 = 2^{-1}, x_2 = 1,\ \ x_3 = 2^{-1},\ x_4 = 0.$$

Let n be a natural number and assume we have already defined $x_i \in X$, $i = 0, \ldots, \sum_{j=1}^{n} 2^{j+1}$, and that $x_{\sum_{j=1}^{n} 2^{j+1}} = 0$. Set

$$S_n = \sum_{i=1}^{n} 2^{i+1}.$$

For $i = 1, \ldots, 2^{n+1}$, put

$$x_{S_n+i} = 2^{-n-1} i$$

and

$$x_{S_n+2^{n+1}+i} = 1 - 2^{-n-1} i.$$

Using induction, we have thus constructed a sequence $\{x_j\}_{j=0}^{\infty}$ such that

$$\lim_{j \to \infty} [|x_{j+1} - x_j| - \rho(x_j, T(x_j))] = 0$$

and any $z \in X$ is one of its limit points.

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