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Acceptance sampling plans from truncated life tests based on the Marshall–Olkin extended exponential distribution for percentiles

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Abstract. In this article, acceptance sampling plans are developed for the Marshall–Olkin extended exponential distribution percentiles when the life test is truncated at a pre-specified time. The minimum sample size necessary to ensure the specified life percentile is obtained under a given customer's risk. The operating characteristic values (and curves) of the sampling plans as well as the producer's risk are presented. Two examples with real data sets are also given as illustration.

1 Introduction

A typical application of acceptance sampling is as follows: a company receives a shipment of product from a vendor. This product is often a component or raw material used in the company's manufacturing process. A sample is taken from the lot and the relevant quality characteristic of the units in the sample is inspected. On the basis of the information in this sample, a decision is made regarding lot disposition. Traditionally, when the life test indicates that the mean life of products exceeds the specified one, the lot of products is accepted, otherwise it is rejected. Accepted lots are put into production, while rejected lots may be returned to the vendor or may be subjected to some other lot disposition action. While it is customary to think of acceptance sampling as a receiving inspection activity, there are also other uses. For example, frequently a manufacturer samples and inspects its own product at various stages of production. Lots that are accepted are sent forward for further processing, while rejected lots may be reworked or scrapped. For the purpose of reducing the test time and cost, a truncated life test may be conducted to determine the smallest sample size to ensure a certain mean life of products when the life test is terminated at a preassigned time t_0, and the number of failures observed does not exceed a given acceptance number c.

A sampling inspection plans in the case that the sample observations are lifetimes of products put to test aims at verifying that the actual population mean exceeds a required minimum. The population mean stands for the mean lifetime

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of the product, say μ. If μ_0 is a specified minimum value, then one would like to verify that $\mu \geq \mu_0$, this means that the true unknown population mean lifetime of the product exceeds the specified value. On the basis of a random sample of size n, the lot is accepted, if by means of a suitable decision criterion, the acceptance sampling plan decides in favor of $\mu \geq \mu_0$. Otherwise, the lot is rejected. The decision criterion is naturally based on the number of observed failures in the sample of, n products during a specified time t_0 from which a lower bound for the unknown mean lifetime is derived. If the observed number of failures is large, say larger than a number c, the derived lower bound is smaller than μ_0 and the hypothesis $\mu \geq \mu_0$ is not verified. Hence, the lot cannot be accepted. Such a sampling plan is named Reliability test plan or Acceptance sampling plans on life tests.

A common practice in life testing is to terminate the life test by a pre-determined time t_0 and note the number of failures (assuming that a failure is well defined). One of the objectives of these experiments is to set a lower confidence limit on the mean life. It is then to establish a specified mean life with a given probability of at least p^* which provides protection to consumers. The decision to accept the specified mean life occurs if and only if the number of observed failures at the end of the fixed time t_0 does not exceed a given number 'c'—called the acceptance number. The test may get terminated before the time t_0 is reached when the number of failures exceeds 'c' in which case the decision is to reject the lot. For such a truncated life test and the associated decision rule; we are interested in obtaining the smallest sample size to arrive at a decision.

In this paper, we assume that the lifetime of product follows a Marshall–Olkin extended exponential distribution proposed and studied by Marshall and Olkin (1997). Adamidis et al. (2005) studied the reliability applications of Marshall–Olkin extended exponential distribution (they called as extension of the exponential-geometric distribution) with two sets of real data exhibiting increasing hazards. The first set of data involves results from an experiment concerning the tensile fatigue characteristics of a polyester/viscose yarn; 100 observations were obtained on the cycles to failure of a 100 cm yarn sample put to test under 2.3% strain level. The second set of data consists of 107 failure times for right rear brakes on D9G-66A caterpillar tractors. In addition to the exponential-geometric distribution, the gamma and Weibull distributions with respective were fitted to each set of data and they showed that exponential-geometric distribution better fitted than the gamma and Weibull distributions. They have given this model used under the concepts of population heterogeneity (through the process of compounding), the "series system" with identical components (or the "initial defects situation" with perfect repair) and the "parallel system" with identical components, appearing in many biological organisms or industrial components and units. Various statistical properties of the distribution along with its reliability features are explored and characterizations are given. The reliability applications of Marshall–Olkin extended exponential distribution was studied by Peña and Gupta (1990), Adamidis and Loukas (1998), Meintanis (2007) and Nadarajah (2008). Rao et al. (2009b) developed an acceptance sampling procedure for the Marshall–Olkin extended exponential distribution mean under a truncated life test. Some other studies regarding truncated life tests can be found in Epstein (1954), Sobel and Tischendrof (1959), Goode and Kao (1961), Gupta and Groll (1961), Gupta (1962), Fertig and Mann (1980), Kantam and Rosaiah (1998), Kantam et al. (2001), Baklizi (2003), Wu and Tsai (2005), Rosaiah and Kantam (2005), Rosaiah et al. (2006), Tsai and Wu (2006), Balakrishnan et al. (2007), Rao et al. (2008) and Rao et al. (2009a).

All these authors considered the design of acceptance sampling plans based on the population mean or median under a truncated life test. Whereas Lio et al. (2010) considered acceptance sampling plans from truncated life tests based on the Birnbaum–Saunders distribution for Percentiles and Rao and Kantam (2010) developed acceptance sampling plans from truncated life tests based on the log-logistic distribution for Percentiles, they proposed that the acceptance sampling plans based on mean may not satisfy the requirement of engineering on the specific percentile of strength or breaking stress. When the quality of a specified low percentile is concerned, the acceptance sampling plans based on the population mean could pass a lot which has the low percentile below the required standard of consumers. Furthermore, a small decrease in the mean with a simultaneous small increase in the variance can result in a significant downward shift in small percentiles of interest. This means that a lot of products could be accepted due to a small decrease in the mean life after inspection. But the material strengths of products are deteriorated significantly and may not meet the consumer's expectation. Therefore, engineers pay more attention to the percentiles of lifetimes than the mean life in life testing applications. Moreover, most of the employed life distributions are not symmetric. In viewing Marshall and Olkin (2007), the mean life may not be adequate to describe the central tendency of the distribution. This reduces the feasibility of acceptance sampling plans if they are developed based on the mean life of products. Actually, percentiles provide more information regarding a life distribution than the mean life does. When the life distribution is symmetric, the 50th percentile or the median is equivalent to the mean life. Hence, developing acceptance sampling plans based on percentiles of a life distribution can be treated as a generalization of developing acceptance sampling plans based on the mean life of items. Rao et al. (2009b) developed acceptance sampling plans based on the mean of the Marshall–Olkin extended exponential distribution. Balakrishnan et al. (2007) proposed the acceptance sampling plans could be used for the quantiles and derived the formulae whereas Lio et al. (2010) developed for the acceptance sampling plans for any other percentiles of the Birnbaum–Saunders (BS) model. They have developed the acceptance sampling plans for percentile by replace the scale parameter β by the $100q$th percentile in the BS distribution function. Rao and Kantam (2010) developed acceptance sampling plans from truncated life tests based on the log-logistic distribution for Percentiles. These reasons motivate to develop acceptance sampling plans based on the percentiles of the Marshall–Olkin extended exponential distribution under a truncated life test.

The rest of the article is organized as follows. The proposed sampling plans are established for the Marshall–Olkin extended exponential percentiles under a truncated life test, along with the operating characteristic (OC) and some relevant tables, is given in Section 2. Two examples based on real fatigue life data sets are provided for the illustration in Section 3 and discussion and some conclusions are made in Section 4.

2 Acceptance sampling plans

Assume that the lifetime of a product follows the Marshall–Olkin extended exponential distribution which has the following probability density function (p.d.f.) and cumulative distribution function (c.d.f.), respectively;

$$f(t;\alpha,\sigma)=\frac{(\alpha/\sigma)\exp(-t/\sigma)}{[1-\bar{\alpha}\exp(-t/\sigma)]^2}; \qquad t>0,\alpha,\sigma>0,\bar{\alpha}=1-\alpha, \tag{2.1}$$

and

$$F(t;\alpha,\sigma)=\frac{1-\exp(-t/\sigma)}{1-\bar{\alpha}\exp(-t/\sigma)}; \qquad t>0,\alpha,\sigma>0, \tag{2.2}$$

where σ is the scale parameter and α is the shape parameter. Given $0<q<1$ the $100q$th percentile (or the qth quantile) is given by

$$t_q=\sigma\ln\big[(1-\bar{\alpha}q)/(1-q)\big]. \tag{2.3}$$

The t_q is increasing with respect to α for $q>0.5$ and decreasing with respect to α for $q<0.5$. Therefore, the $100q$th percentile, t_q, is depend upon α. When $q=0.5$, then $t_{0.5}=\sigma\ln(1+\alpha)$ and $t_{0.5}$ is also the median of Marshall–Olkin extended exponential distribution. Let $\eta=\ln[(1-\bar{\alpha}q)/(1-q)]$. Then, equation (2.3) implies that

$$\sigma=t_q/\eta. \tag{2.4}$$

To develop acceptance sampling plans for the Marshall–Olkin extended exponential percentiles, the scale parameter σ in the Marshall–Olkin extended exponential c.d.f. is replaced by equation (2.4) and the Marshall–Olkin extended exponential c.d.f. is rewritten as

$$F(t)=\frac{1-\exp[-t/(t_q/\eta)]}{1-\bar{\alpha}\exp[-t/(t_q/\eta)]}; \qquad t>0.$$

Letting $\delta=t/t_q$, $F(t)$ can be rewritten emphasizing its dependence on δ as

$$F(t;\delta)=\frac{1-\exp(-\delta\eta)}{1-\bar{\alpha}\exp(-\delta\eta)}; \qquad t>0.$$

Taking partial derivative with respect to δ, we have

$$\frac{\partial F(t;\delta)}{\partial \delta} = \frac{\alpha\eta \exp(-\delta\eta)}{[1-\bar{\alpha}\exp(-\delta\eta)]^2}; \qquad t > 0.$$

A common practice in life testing is to terminate the life test by a pre-determined time t, the probability of rejecting a bad lot be at least p^*, and the maximum number of allowable bad items to accept the lot be c. The acceptance sampling plan for percentiles under a truncated life test is to set up the minimum sample size n for this given acceptance number c such that the consumer's risk, the probability of accepting a bad lot, does not exceed $1 - p^*$. A bad lot means that the true $100q$th percentile, t_q, is below the specified percentile, t_q^0. Thus, the probability p^* is a confidence level in the sense that the chance of rejecting a bad lot with $t_q < t_q^0$ is at least equal to p^*. Therefore, for a given p^*, the proposed acceptance sampling plan can be characterized by the triplet $(n, c, t/t_q^0)$.

2.1 Minimum sample size

For a fixed p^* our sampling plan is characterized by $(n, c, t/t_q^0)$. Here we consider sufficiently large sized lots so that the binomial distribution can be applied. The problem is to determine for given values of p^* $(0 < p^* < 1)$, t_q^0 and c, the smallest positive integer, n required to assert that $t_q > t_q^0$ must satisfy

$$\sum_{i=0}^{c} \binom{n}{i} p_0^i (1-p_0)^{n-i} \le 1 - p^*, \tag{2.5}$$

where $p = F(t;\delta_0)$ is the probability of a failure during the time t given a specified $100q$th percentile of lifetime t_q^0 and depends only on $\delta_0 = t/t_q^0$, since $\partial F(t;\delta)/\partial\delta > 0$, $F(t;\delta)$ is a nondecreasing function of δ. Accordingly, we have

$$\begin{aligned} F(t,\delta) < F(t,\delta_0) \quad &\Leftrightarrow \quad \delta \le \delta_0, \quad \text{or equivalently,} \\ F(t,\delta) \le F(t,\delta_0) \quad &\Leftrightarrow \quad t_q \ge t_q^0. \end{aligned}$$

The smallest sample size n satisfying the inequality (2.5) can be obtained for any given q, t/t_q^0, p^* and α. Whereas, the smallest sample size n calculation in Rao et al. (2009b) only needs input values for t/σ_0, p^* and α. Hence, the proposed process to find the smallest sample size in this case is the same as the procedure provided by Rao et al. (2009b) for the Marshall–Olkin extended exponential model except in place of t/σ_0 replace by t/t_q^0 at q. To save space, only the results of small sample sizes for $q = 0.1$, $t/t_q^0 = 0.7$, 0.9, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5; $p^* = 0.75$, 0.90, 0.95, 0.99; $c = 0$, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and $\alpha = 2$ are reported in Table 1.

If $p = F(t;\delta_0)$ is small and n is large, the binomial probability may be approximated by Poisson probability with parameter $\lambda = np$ so that the left side of (2.5)

Table 1 *Minimum sample sizes necessary to assert the* 10*th percentile to exceed a given values,* $t_{0.1}^{0}$, *with probability* p^* *and the corresponding acceptance number,* c, *for the Marshall–Olkin extended exponential distribution with* $\alpha = 2$ *using the binomial approximation*

		$t/t_{0.1}^{0}$							
p^*	c	0.7	0.9	1.0	1.5	2.0	2.5	3.0	3.5
0.75	0	20	15	14	9	7	5	5	4
0.75	1	38	30	27	18	13	11	9	8
0.75	2	55	43	39	26	19	15	13	11
0.75	3	72	56	51	34	25	20	17	15
0.75	4	89	69	62	41	31	25	21	18
0.75	5	105	82	73	49	37	29	25	21
0.75	6	121	94	85	56	42	34	28	24
0.75	7	137	107	96	64	48	39	32	28
0.75	8	153	119	107	71	54	43	36	31
0.75	9	169	131	118	79	59	47	40	34
0.75	10	185	144	129	86	65	52	43	37
0.90	0	32	25	22	15	11	9	7	6
0.90	1	55	42	38	25	19	15	12	10
0.90	2	75	58	52	34	26	20	17	14
0.90	3	94	73	65	43	32	26	21	18
0.90	4	112	87	78	52	39	31	26	22
0.90	5	131	101	91	60	45	36	30	26
0.90	6	148	115	104	69	51	41	34	29
0.90	7	166	129	116	77	57	46	38	33
0.90	8	183	142	128	85	64	51	42	36
0.90	9	200	156	140	93	70	56	46	40
0.90	10	217	169	152	101	75	60	50	43
0.95	0	42	32	29	19	14	11	9	8
0.95	1	66	51	46	30	23	18	15	13
0.95	2	88	68	61	40	30	24	20	17
0.95	3	109	84	76	50	37	30	25	21
0.95	4	128	100	89	59	44	35	29	25
0.95	5	148	114	103	68	51	40	34	29
0.95	6	166	129	116	77	57	46	38	32
0.95	7	185	143	129	85	64	51	42	36
0.95	8	203	158	142	94	70	56	46	40
0.95	9	221	171	154	102	76	61	51	43
0.95	10	239	185	167	110	83	66	55	47

can be written as

$$\sum_{i=0}^{c} \frac{\lambda^i}{i!} e^{-\lambda} \leq 1 - p^*, \tag{2.6}$$

Table 1 (*Continued*)

p^*	c	$t/t^0_{0.1}$ 0.7	0.9	1.0	1.5	2.0	2.5	3.0	3.5
0.99	0	64	49	44	29	21	17	14	12
0.99	1	92	71	64	42	31	25	20	17
0.99	2	117	91	81	53	40	31	26	22
0.99	3	140	108	97	64	48	38	31	26
0.99	4	162	125	113	74	55	44	36	31
0.99	5	183	142	127	84	62	50	41	35
0.99	6	204	158	142	94	70	55	46	39
0.99	7	224	174	156	103	77	61	50	43
0.99	8	244	189	170	112	83	66	55	47
0.99	9	263	204	183	121	90	72	60	51
0.99	10	283	219	197	130	97	77	64	55

where $\lambda = nF(t;\delta_0)$. The minimum values of n satisfying (2.6) are obtained for the same combination of q, t/t_q^0, p^* and α values as those used for (2.5). The results are reported in Table 2.

2.2 Operating characteristic of the sampling plan $(n, c, t/t_q^0)$

The operating characteristic (OC) function of the sampling plan $(n, c, t/t_q^0)$ is the probability of accepting a lot. It is given as

$$L(p) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i}, \tag{2.7}$$

where $p = F(t;\delta)$. It should be noticed that $F(t;\delta)$ can be represented as a function of $\delta = t/t_q$. Therefore, $p = F(\frac{t}{t_q^0}\frac{1}{d_q})$ where $d_q = t_q/t_q^0$. Using equation (2.7), the OC values and OC curves can be obtained for any sampling plan, $(n, c, t/t_q^0)$, and any α. To save space, we present Table 3 to show the OC values for the sampling plan $(n, c = 5, t/t^0_{0.1})$ with $\alpha = 2$. Figure 1 shows the OC curves for the sampling plan $(n, c, t/t^0_{0.1})$ with $p^* = 0.90$ for $\delta_0 = 1$, $\alpha = 2$, where $c = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$.

2.3 Producer's risk

The producer's risk is defined as the probability of rejecting the lot when $t_q > t_q^0$. For a given value of the producer's risk, say γ, we are interested in knowing the value of d_q to ensure the producer's risk is less than or equal to γ if a sampling plan $(n, c, t/t_q^0)$ is developed at a specified confidence level p^*. Thus, one needs

Table 2 *Minimum sample sizes necessary to assert the* 10*th percentile to exceed a given values,* $t^0_{0.1}$*, with probability* p^* *and the corresponding acceptance number,* c*, for the Marshall–Olkin extended exponential distribution with* $\alpha = 2$ *using the Poisson approximation*

		$t/t^0_{0.1}$							
p^*	c	0.7	0.9	1.0	1.5	2.0	2.5	3.0	3.5
0.75	0	20	16	14	10	8	6	5	5
0.75	1	32	25	23	15	12	10	8	7
0.75	2	54	42	38	26	20	16	13	12
0.75	3	73	57	51	34	26	21	18	16
0.75	4	90	70	63	42	32	26	22	19
0.75	5	106	83	75	50	38	31	26	22
0.75	6	123	96	86	58	44	35	30	26
0.75	7	139	108	97	65	49	40	34	29
0.75	8	155	120	109	73	55	44	37	33
0.75	9	170	133	120	80	61	49	41	36
0.75	10	186	145	131	88	66	53	45	39
0.90	0	33	26	24	16	12	10	8	7
0.90	1	52	40	36	25	19	15	13	11
0.90	2	75	59	53	36	27	22	18	16
0.90	3	95	74	67	45	34	28	23	20
0.90	4	114	89	80	54	41	33	28	24
0.90	5	133	103	93	63	47	38	32	28
0.90	6	151	117	106	71	54	43	37	32
0.90	7	168	131	118	79	60	48	41	35
0.90	8	186	145	130	87	66	53	45	39
0.90	9	203	158	143	96	72	58	49	43
0.90	10	220	172	155	104	78	63	53	46
0.95	0	43	34	30	21	16	13	11	9
0.95	1	65	51	46	31	23	19	16	14
0.95	2	90	70	63	42	32	26	22	19
0.95	3	111	86	78	52	40	32	27	23
0.95	4	131	102	92	62	47	38	32	28
0.95	5	150	117	106	71	54	43	36	32
0.95	6	169	132	119	80	60	49	41	36
0.95	7	188	146	132	89	67	54	45	39
0.95	8	206	161	145	97	73	59	50	43
0.95	9	224	175	158	106	80	64	54	47
0.95	10	242	189	170	114	86	70	59	51

to find the smallest value d_q according to equation (2.7) as

$$\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \geq 1-\gamma, \tag{2.8}$$

Table 2 (*Continued*)

		$t/t^0_{0.1}$							
p^*	c	0.7	0.9	1.0	1.5	2.0	2.5	3.0	3.5
0.99	0	66	52	47	31	24	19	16	14
0.99	1	93	72	65	44	33	27	23	20
0.99	2	120	94	84	57	43	35	29	25
0.99	3	144	112	101	68	51	41	35	30
0.99	4	166	129	117	78	59	48	40	35
0.99	5	187	146	132	88	67	54	45	39
0.99	6	208	162	146	98	74	60	50	44
0.99	7	229	178	160	108	81	66	55	48
0.99	8	249	194	175	117	88	71	60	52
0.99	9	268	209	188	126	95	77	65	56
0.99	10	288	224	202	135	102	82	69	60

where $p = F(\frac{t}{t_q^0}\frac{1}{d_q})$, $d_q = t_q/t_q^0$. To save space, based on sampling plans $(n, c, t/t_q^0)$ established in Table 1 the minimum ratios of $d_{0.1}$ for the acceptability of a lot under $\alpha = 2$, at the producer's risk of $\gamma = 0.05$ are presented in Table 4.

3 Illustrative examples

In this section, we consider two examples with real data sets are given to illustrate the proposed acceptance sampling plans. The first data set is of the data given arisen in tests on endurance of deep groove ball bearings (Lawless, 1982, p. 28). The data are the number of million revolutions before failure for each of the 23 ball bearings in life test and they are: 51.84, 51.96, 54.12, 55.56, 67.80, 68.44, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04 and 173.40. The second data set regarding the software reliability was presented by Wood (1996), analyzed via the acceptance sampling viewpoint by Rosaiah and Kantam (2005), Balakrishnan et al. (2007), Rao et al. (2008), Rao et al. (2009a, 2009b), Lio et al. (2010) and Rao and Kantam (2010). The software reliability data set was reported in hours as 519, 968, 1430, 1893, 2490, 3058, 3625, 4422, and 5218. As the confidence level is assured by this acceptance sampling plan only if the lifetimes are from the Marshall–Olkin extended exponential distribution. Then, we should check if it is reasonable to admit that the given sample comes from the Marshall–Olkin extended exponential distribution by the goodness of fit test and model selection criteria. The first data set was used by Sultan (2007) to demonstrate the goodness of fit for generalized exponential distribution and Parikh et al. (2008) showed that Marshall–Olkin extended exponential distribution is a suitable model for this data set. Balakrishnan et al. (2007) compared the goodness of fits among the Rayleigh, generalized BS, and BS distributions for the software reliability data set presented

Table 3 *Operating characteristic values of the sampling plan* $(n, c = 5, t/t_{0.1}^0)$ *for a given* p^* *under Marshall–Olkin extended exponential distribution with* $\alpha = 2$

			$t_{0.1}/t_{0.1}^0$							
p^*	n	$t/t_{0.1}^0$	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75
0.75	105	0.7	0.2470	0.4587	0.6317	0.7539	0.8355	0.8889	0.9240	0.9471
0.75	82	0.9	0.2415	0.4541	0.6286	0.7521	0.8345	0.8884	0.9237	0.9470
0.75	73	1.0	0.2499	0.4647	0.6386	0.7603	0.8408	0.8932	0.9272	0.9496
0.75	49	1.5	0.2401	0.4574	0.6346	0.7586	0.8404	0.8933	0.9276	0.9501
0.75	37	2.0	0.2313	0.4509	0.6312	0.7573	0.8402	0.8937	0.9281	0.9506
0.75	29	2.5	0.2480	0.4741	0.6539	0.7763	0.8549	0.9047	0.9363	0.9566
0.75	25	3.0	0.2961	0.5305	0.7033	0.8145	0.8829	0.9248	0.9507	0.9669
0.75	21	3.5	0.2363	0.4666	0.6512	0.7765	0.8564	0.9065	0.9380	0.9581
0.90	131	0.7	0.0966	0.2501	0.4209	0.5705	0.6873	0.7736	0.8357	0.8801
0.90	101	0.9	0.0990	0.2559	0.4287	0.5787	0.6949	0.7802	0.8412	0.8845
0.90	91	1.0	0.0976	0.2542	0.4273	0.5778	0.6944	0.7799	0.8411	0.8844
0.90	60	1.5	0.0990	0.2603	0.4372	0.5890	0.7053	0.7895	0.8491	0.8910
0.90	45	2.0	0.0955	0.2576	0.4368	0.5905	0.7077	0.7922	0.8518	0.8934
0.90	36	2.5	0.0926	0.2557	0.4371	0.5926	0.7106	0.7953	0.8546	0.8959
0.90	30	3.0	0.0904	0.2545	0.4382	0.5953	0.7140	0.7987	0.8576	0.8985
0.90	26	3.5	0.0833	0.2440	0.4283	0.5876	0.7085	0.7950	0.8552	0.8969
0.95	148	0.7	0.0482	0.1570	0.3044	0.4528	0.5811	0.6837	0.7621	0.8210
0.95	114	0.9	0.0498	0.1618	0.3120	0.4618	0.5902	0.6920	0.7694	0.8271
0.95	103	1.0	0.0479	0.1582	0.3078	0.4577	0.5867	0.6891	0.7672	0.8254
0.95	68	1.5	0.0479	0.1609	0.3141	0.4666	0.5964	0.6986	0.7757	0.8328
0.95	51	2.0	0.0452	0.1575	0.3120	0.4663	0.5976	0.7006	0.7780	0.8351
0.95	40	2.5	0.0490	0.1689	0.3298	0.4869	0.6180	0.7189	0.7937	0.8481
0.95	34	3.0	0.0411	0.1525	0.3096	0.4675	0.6013	0.7056	0.7834	0.8402
0.95	29	3.5	0.0410	0.1543	0.3143	0.4741	0.6087	0.7127	0.7898	0.8457
0.99	183	0.7	0.0099	0.0526	0.1388	0.2538	0.3764	0.4913	0.5911	0.6739
0.99	142	0.9	0.0097	0.0522	0.1387	0.2545	0.3779	0.4933	0.5933	0.6762
0.99	127	1.0	0.0100	0.0536	0.1419	0.2593	0.3835	0.4992	0.5991	0.6815
0.99	84	1.5	0.0096	0.0535	0.1435	0.2632	0.3896	0.5065	0.6068	0.6891
0.99	62	2.0	0.0100	0.0564	0.1509	0.2750	0.4043	0.5223	0.6223	0.7034
0.99	50	2.5	0.0084	0.0514	0.1428	0.2658	0.3954	0.5147	0.6161	0.6986
0.99	41	3.0	0.0090	0.0549	0.1513	0.2790	0.4115	0.5316	0.6325	0.7136
0.99	35	3.5	0.0087	0.0546	0.1522	0.2817	0.4158	0.5369	0.6382	0.7191

here using probability plots and showed that the generalized BS model (R-square (RS) = 0.97) was slightly better than the BS model (RS = 0.96) and both models were much better than the Rayleigh model (RS = 0.87). However, the acceptance sampling plans under the truncated life test based on the Marshall–Olkin extended exponential distribution for percentiles has not yet been developed. We have applied QQ plot and RS method to test the goodness of fit for both data sets for

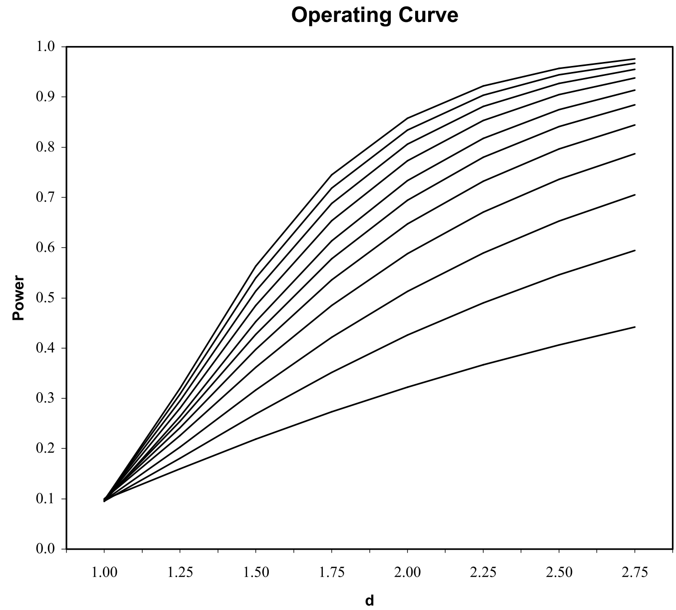

Figure 1 *OC curves for* $c = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$, *respectively under* $p^* = 0.90$, $\delta_0 = 1$ *and* $\alpha = 2$, *based on the* 10*th percentile*, $d = d_{0.1}$, *of Marshall–Olkin extended exponential distribution.*

Marshall–Olkin extended exponential distribution and we got RS = 0.9819 for first data set and RS = 0.9834 for second data set. Hence, the Marshall–Olkin extended exponential distribution could also provide reasonable goodness of fits for both data sets.

3.1 Example 1

Assume that the lifetime distribution is Marshall–Olkin extended exponential distribution with $\alpha = 2$ and that the experimenter is interested to establish the true unknown 10th percentile lifetime for the ball bearings to be at least 20 million revolutions with confidence $p^* = 0.75$ and the life test would be ended at 60 million revolutions, which should have led to the ratio $t/t^0_{0.1} = 3.0$. Thus, for an acceptance number $c = 5$ and the confidence level $p^* = 0.75$, the required sample size n found from Table 1 should be at least 25. Therefore, in this case, the acceptance sampling plan from truncated life tests for the Marshall–Olkin extended exponential distribution 10th percentile should be $(n, c, t/t^0_q) = (25, 5, 3.0)$. Based on the ball bearings data, the experimenter must have decided whether to accept or reject the lot. The lot should be accepted only if the number of items of which lifetimes were less than or equal to the scheduled test lifetime, 60 million revolutions, was at most 5 among the first 25 observations. Since there were 4 items with a failure time less than or equal to 60 million revolutions in the given sample of $n = 25$ observations, the experimenter would accept the lot, assuming the 10th percentile lifetime

Table 4 *Minimum ratio of true $d_{0.1}$ for the acceptability of a lot for the Marshall–Olkin extended exponential distribution with $\alpha = 2$ and producer's risk of* 0.05

		$t/t_{0.1}^0$							
p^*	c	0.7	0.9	1.0	1.5	2.0	2.5	3.0	3.5
0.75	0	26.5182	27.4801	26.5182	27.5634	24.5821	29.5334	27.5634	31.5856
0.75	1	7.5873	7.5873	7.5245	7.1685	7.5873	7.3421	7.5873	7.5245
0.75	2	4.6904	4.7148	4.6904	4.5269	4.4170	4.5725	4.4603	5.0839
0.75	3	3.6536	3.6982	3.6536	3.5537	3.5125	3.5537	3.6245	4.1545
0.75	4	3.1153	3.1046	3.0525	3.0525	3.0423	3.0423	3.0120	3.4329
0.75	5	2.7988	2.7563	2.7563	2.7480	2.6596	2.7315	2.6364	3.0221
0.75	6	2.5478	2.5549	2.4988	2.4783	2.4851	2.4254	2.3935	2.7397
0.75	7	2.3935	2.3810	2.3624	2.3381	2.3502	2.2910	2.3143	2.6441
0.75	8	2.2568	2.2512	2.2183	2.2292	2.1968	2.1863	2.1706	2.4783
0.75	9	2.1501	2.1501	2.1400	2.1101	2.0764	2.1004	2.0576	2.3563
0.75	10	2.0812	2.0670	2.0483	2.0437	2.0255	1.9857	1.9685	2.2512
0.90	0	44.1696	43.1220	44.1696	43.3276	44.1696	41.3565	41.3565	47.1476
0.90	1	10.5932	10.7181	10.4712	10.5932	10.4712	9.9010	9.5877	10.9769
0.90	2	6.3654	6.3211	6.1501	6.2344	5.9488	6.0277	5.7604	6.5963
0.90	3	4.7893	4.7148	4.6664	4.5956	4.6189	4.4385	4.4170	5.0277
0.90	4	3.9557	3.9216	3.9047	3.8715	3.8226	3.8066	3.7286	4.2717
0.90	5	3.4590	3.4590	3.3944	3.3693	3.3322	3.3080	3.3201	3.7908
0.90	6	3.1260	3.1368	3.1046	3.0321	3.0221	2.9824	2.9343	3.3568
0.90	7	2.8969	2.8877	2.8514	2.7902	2.7988	2.7480	2.7563	3.1586
0.90	8	2.6991	2.6991	2.6752	2.6596	2.6288	2.5767	2.5478	2.9155
0.90	9	2.5694	2.5549	2.5265	2.5195	2.4988	2.4384	2.4516	2.8074
0.90	10	2.4450	2.4450	2.4190	2.3747	2.3502	2.3321	2.3202	2.6518
0.95	0	56.5291	56.8828	55.8347	55.1572	54.1712	53.2198	55.1572	63.2111
0.95	1	13.0208	13.0208	12.6582	12.8370	12.4844	12.4844	12.6582	14.4718
0.95	2	7.4627	7.4627	7.2833	7.2254	7.2254	7.1685	7.0572	8.0580
0.95	3	5.5157	5.5494	5.4171	5.3220	5.3533	5.3220	5.2002	5.9102
0.95	4	4.5496	4.4823	4.4385	4.3745	4.3328	4.2717	4.2717	4.8662
0.95	5	3.9047	3.9216	3.8551	3.8226	3.7286	3.7750	3.7286	4.2517
0.95	6	3.5125	3.5125	3.4722	3.3944	3.4072	3.3445	3.2605	3.7286
0.95	7	3.2144	3.2144	3.1586	3.1476	3.1153	3.0525	3.0321	3.4590
0.95	8	3.0120	3.0021	2.9630	2.9155	2.8969	2.8337	2.8514	3.2605
0.95	9	2.8161	2.8161	2.7816	2.7397	2.7315	2.7152	2.6518	3.0321
0.95	10	2.6831	2.6911	2.6364	2.6364	2.5988	2.5767	2.5478	2.9155

$t_{0.1}$ of at least 20 million revolutions with a confidence level of $p^* = 0.75$. The OC values for the acceptance sampling plan $(n, c, t/t_q^0) = (25, 5, 3.0)$ and confidence level $p^* = 0.75$ under Marshall–Olkin extended exponential distribution with $\alpha = 2$ from Table 3 is as follows in Table 5.

This shows that if the true 10th percentile is equal to the required 10th percentile $(t_{0.1}/t_{0.1}^0 = 1.00)$ the producer's risk is approximately 0.7039 $(= 1 - 0.2961)$. The

Table 4 (*Continued*)

		$t/t^0_{0.1}$							
p^*	c	0.7	0.9	1.0	1.5	2.0	2.5	3.0	3.5
0.99	0	86.7303	86.7303	85.9107	82.7815	83.5422	82.7815	82.7815	94.8767
0.99	1	18.2482	18.2482	17.8891	17.5439	17.5439	16.8919	16.5837	19.0114
0.99	2	10.0100	9.9010	9.6899	9.6899	9.3897	9.3897	9.1996	10.5932
0.99	3	7.1124	7.1124	7.0028	6.9493	6.8446	6.6445	6.5020	7.4019
0.99	4	5.6883	5.7241	5.5835	5.5157	5.4825	5.3533	5.3533	6.1087
0.99	5	4.8662	4.8403	4.7893	4.6664	4.6904	4.5956	4.5496	5.2002
0.99	6	4.3122	4.3122	4.2517	4.1929	4.0984	4.0800	4.0080	4.5956
0.99	7	3.9216	3.9047	3.8388	3.8066	3.7439	3.6684	3.6536	4.1736
0.99	8	3.6101	3.6101	3.5398	3.4722	3.4329	3.4200	3.3818	3.8715
0.99	9	3.3693	3.3568	3.3080	3.2605	3.2373	3.2258	3.1696	3.6245
0.99	10	3.1807	3.1807	3.1260	3.0941	3.0525	3.0221	3.0120	3.4329

Table 5 *The OC values for the acceptance sampling plan with plan* $(n, c, t/t^0_q) = (25, 5, 3.0)$

	$t_{0.1}/t^0_{0.1}$							
	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75
OC	0.2961	0.5305	0.7033	0.8145	0.8829	0.9248	0.9507	0.9669

producer's risk is almost equal to 0.0493 when the true 10th percentile is greater than or equal to 2.5 times the specified 10th percentile.

From Table 4, the experimenter could get the values of $d_{0.1}$ for different choices of c and $t/t^0_{0.1}$ in order to assert that the producer's risk was less than 0.05. In this example, the value of $d_{0.1}$ should be 2.7563 for $c = 5$, $t/t^0_{0.1} = 1.0$ and $p^* = 0.75$. This means the product can have a 10th percentile life of 2.7563 times the required 10th percentile lifetime in order that under the above acceptance sampling plan the product is accepted with probability of at least 0.95.

Alternatively, assume that products have a Marshall–Olkin extended exponential distribution with $\alpha = 2$, and consumers wish to reject a bad lot with probability of $p^* = 0.75$. What should the true 10th percentile life of products be so that the producer's risk is 0.05 if the acceptance sampling plan is based on an acceptance number $c = 3$ and $t/t^0_{0.1} = 0.7$? From Table 4, we can find that the entry for $\alpha = 2$, $p^* = 0.75$, $c = 3$, and $t/t^0_{0.1} = 0.7$ is $d_{0.1} = 3.6536$. Thus, the manufacturer's product should have a 10th percentile life at least 3.6536 times the specified 10th percentile life in order for the products to be accepted with probability 0.75 under the above acceptance sampling plan. Table 1 indicates that the number of products required to be tested is $n = 72$ so that the sampling plan is $(n, c, t/t^0_{0.1}) = (72, 3, 0.7)$.

3.2 Example 2

Suppose an experimenter would like to establish the true unknown 10th percentile lifetime for the software mentioned above to be at least 150 h and the life test would be ended at 450 h, which should have led to the ratio $t/t_{0.1}^0 = 3.0$. The goodness of fit test for these nine observations were verified and showed that Marshall–Olkin extended exponential model as a reasonable goodness of fit for these nine observations. Thus, with $c = 0$ and $p^* = 0.95$, the experimenter should find from Table 1 the sample size n must be at least 9 and the sampling plan to be $(n, c, t/t_{0.1}^0) = (9, 0, 3.0)$. Since there were no items with a failure time less than or equal to 450 h in the given sample of $n = 9$ observations, the experimenter would accept the lot, assuming the 10th percentile lifetime $t_{0.1}$ of at least 150 h with a confidence level of $p^* = 0.95$.

4 Discussion and conclusions

The sampling plans based on the Marshall–Olkin extended exponential population mean developed by Rao et al. (2009b) to the Marshall–Olkin extended exponential models with $\alpha = 2$. It shows that the minimum sample sizes are smaller than those reported in Tables 1 and 2 of this article for the 10th percentile for both binomial and poisson approximation. Here, $\delta_0 = t/t_{0.1}^0$ for the sampling plans based on 10th percentile is replaced by $\delta_0 = t/\mu_0$ with μ_0 as a specific population mean for the acceptance plans based on the Marshall–Olkin extended exponential population mean. Therefore, the acceptance sampling plans based on the Marshall–Olkin extended exponential population mean could have less chance to report a failure than the acceptance sampling plans based on 10th percentile. The acceptance sampling plans based on population mean could accept the lot of bad quality of the 10th percentiles. The minimum sample sizes are reported in Table 1 of this article for the 10th percentiles are compared with the minimum sample sizes are reported in Table 1 of Lio et al. (2010) and Rao and Kantam (2010). It shows that the minimum sample sizes using Marshall–Olkin extended exponential population are smaller than those reported in Table 1 of Lio et al. (2010) and Rao and Kantam (2010) population for the 10th percentile when $\delta_0 \leq 1.0$ whereas, the minimum sample sizes using Marshall–Olkin extended exponential population are larger than those reported in Table 1 of Lio et al. (2010) and Rao and Kantam (2010) for Birnbaum–Saunders and log-logistic populations respectively for the 10th percentile when $\delta_0 > 1.0$.

This article has derived the acceptance sampling plans based on the Marshall–Olkin extended exponential percentiles when the life test is truncated at a pre-fixed time. The procedure is provided to construct the proposed sampling plans for the percentiles of the Marshall–Olkin extended exponential distribution with known parameter $\alpha = 2$. To ensure that the life quality of products exceeds a specified one in terms of the life percentile, the acceptance sampling plans based on percentiles should be used. Some useful tables are provided and applied to establish acceptance sampling plans for two examples. The developed sampling plans are the extension work for the acceptance sampling plan based on the Marshall–Olkin extended exponential mean by Rao et al. (2009b).

References

Adamidis, K. and Loukas, S. (1998). A lifetime distribution with decreasing failure rate. *Statistics & Probability Letters* **39**, 35–42. MR1649319

Adamidis, K., Dimitrakopoulou, T. and Loukas, S. (2005). On an extension of the exponential-geometric distribution. *Statistics & Probability Letters* **73**, 259–269. MR2179285

Baklizi, A. (2003). Acceptance sampling based on truncated life tests in the Pareto distribution of the second kind. *Advances and Applications in Statistics* **3**, 33–48. MR1985754

Balakrishnan, N., Leiva, V. and Lopez, J. (2007). Acceptance sampling plans from truncated life tests based on the generalized Birnbaum–Saunders distribution. *Communications in Statistics. Simulation and Computation* **36**, 643–656. MR2370927

Epstein, B. (1954). Truncated life tests in the exponential case. *Annals of Mathematical Statistics* **25**, 555–564. MR0064372

Fertig, F. W. and Mann, N. R. (1980). Life-test sampling plans for two-parameter Weibull populations. *Technometrics* **22**, 165–177.

Goode, H. P. and Kao, J. H. K. (1961). Sampling plans based on the Weibull distribution. In *Proceedings of Seventh National Symposium on Reliability and Quality Control* 24–40. Philadelphia.

Gupta, S. S. (1962). Life test sampling plans for normal and lognormal distribution. *Technometrics* **4**, 151–175. MR0138182

Gupta, S. S. and Groll, P. A. (1961). Gamma distribution in acceptance sampling based on life tests. *Journal of the American Statistical Association* **56**, 942–970. MR0144437

Kantam, R. R. L. and Rosaiah, K. (1998). Half logistic distribution in acceptance sampling based on life tests. *IAPQR Transactions* **23**, 117–125.

Kantam, R. R. L., Rosaiah, K. and Rao, G. S. (2001). Acceptance sampling based on life tests: Log-logistic model. *Journal of Applied Statistics* **28**, 121–128. MR1834427

Lawless, J. F. (1982). *Statistical Models and Methods for Lifetime Data*. New York: Wiley. MR0640866

Lio, Y. L., Tsai, T.-R. and Wu, S.-J. (2010). Acceptance sampling plans from truncated life tests based on the Birnbaum–Saunders distribution for percentiles. *Communications in Statistics. Simulation and Computation* **39**, 119–136. MR2784557

Marshall, A. W. and Olkin, I. (1997). A new method for adding a parameter to a family of distributions with application to the exponential and Weibull families. *Biometrika* **84**, 641–652. MR1603936

Marshall, A. W. and Olkin, I. (2007). *Life Distributions-Structure of Nonparametric, Semiparametric, and Parametric Families*. New York: Springer. MR2344835

Meintanis, S. G. (2007). Test of fit for Marshall–Olkin distributions with applications. *Journal of Statistical Planning and Inference* **137**, 3954–3963. MR2368540

Nadarajah, S. (2008). Marshall and Olkin's distributions. *Acta Applicandae Mathematicae* **103**, 87–100. MR2415173

Parikh, R, V., Ghosh, D. K. and Muralidharan, K. (2008). On Marshall–Olkin generalized exponential distribution. *IAPQR Transactions* **33**, 71–81.

Peña, E. A. and Gupta, A. K. (1990). Bayes estimation for the Marshall–Olkin exponential distribution. *Journal of the Royal Statistical Society, Ser. B* **52**, 379–389. MR1064424

Rao, G. S. and Kantam, R. R. L. (2010). Acceptance sampling plans from truncated life tests based on the log-logistic distribution for percentiles. *Economic Quality Control* **25**, 153–167. MR2799747

Rao, G. S., Ghitany, M. E. and Kantam, R. R. L. (2008). Acceptance sampling plans for Marshall–Olkin extended Lomax distribution. *International Journal of Applied Mathematics* **21**, 315–325.

Rao, G. S., Ghitany, M. E. and Kantam, R. R. L. (2009a). Marshall–Olkin extended Lomax distribution: An economic reliability test plan. *International Journal of Applied Mathematics* **22**, 139–148. MR2493679

Rao, G. S., Ghitany, M. E. and Kantam, R. R. L. (2009b). Reliability test plans for Marshall–Olkin extended exponential distribution. *Applied Mathematical Sciences* **3**, 2745–2755. MR2563116

Rosaiah, K. and Kantam, R. R. L. (2005). Acceptance sampling based on the inverse Rayleigh distribution. *Economic Quality Control* **20**, 277–286. MR2197464

Rosaiah, K., Kantam, R. R. L. and Santosh Kumar, C. (2006). Reliability test plans for exponentiated log-logistic distribution, *Economic Quality Control* **21**, 165–175. MR2364118

Sobel, M. and Tischendrof, J. A. (1959). Acceptance sampling with sew life test objective. In *Proceedings of Fifth National Symposium on Reliability and Quality Control* 108–118. Philadelphia.

Sultan, K. S. (2007). Order statistics from the generalized exponential distribution and applications. *Communications in Statistics. Theory and Methods* **36**, 1409–1418. MR2405266

Tsai, T.-R. and Wu, S.-J. (2006). Acceptance sampling based on truncated life tests forgeneralized Rayleigh distribution. *Journal of Applied Statistics* **33**, 595–600. MR2240929

Wood, A. (1996). Predicting software reliability. *IEEE Transactions on Software Engineering* **22**, 69–77.

Wu, C.-J. and Tsai, T.-R. (2005). Acceptance sampling plans for Birnbaum–Saunders distribution under truncated life tests. *International Journal of Reliability, Quality and Safety Engineering* **12**, 507–519.

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