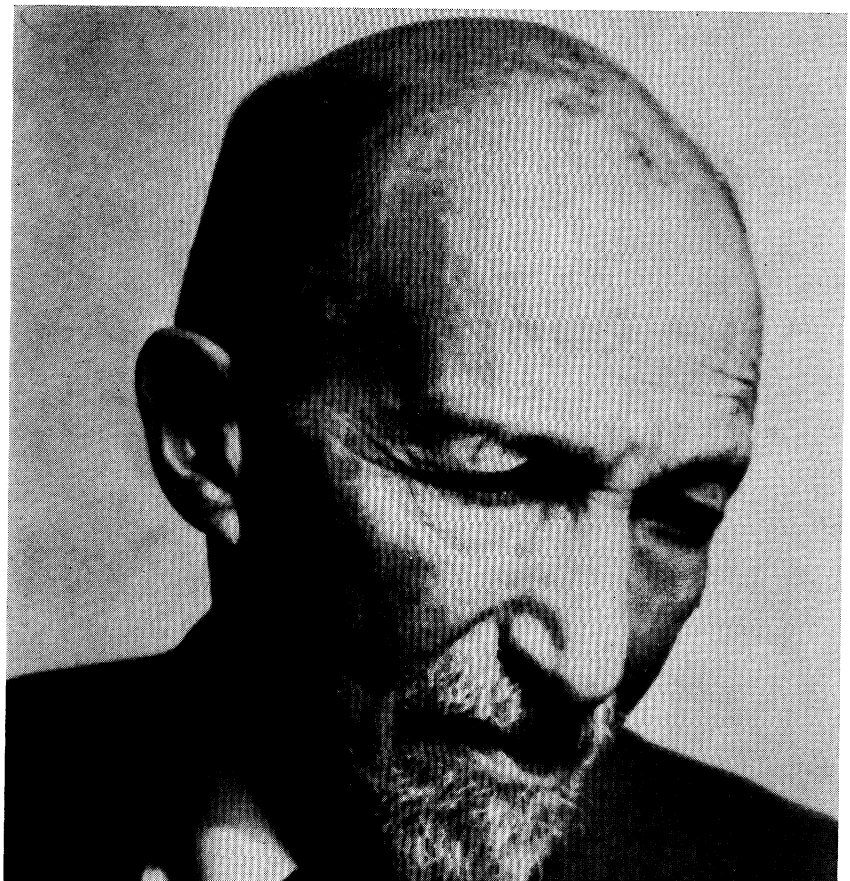

JACQUES HADAMARD (1865–1963)

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Jacques Hadamard, whose mathematical work extends over almost three quarters of a century, passed away on October 17, 1963 at the age of nearly 98 years: he was born on December 8, 1865.

Only a very few mathematicians of the last hundred years left so rich a heritage in so many fields as Hadamard did. As a matter of fact, few of the fundamental fields of mathematics were not deeply influenced by his genius. Many of the most important ones owe to Hadamard their intimate nature and ways of research, some of them their existence.

In introducing Hadamard to the London Mathematical Society in 1944, where he was asked to speak about his most striking discoveries, Hardy called him, I remember, the "living legend" in mathematics.

I will have to limit myself to only the most important ideas and results spread through this legendary work.

The first important papers of Hadamard dealt with analytic functions, or, more precisely, with the study of the nature of an analytic function defined by its Taylor series. Weierstrass, and Méray in France, on defining rigorously the meaning of analytic continuation of such a series, took that series as the starting point of the definition of an analytic function. But this is merely an existence and uniqueness theorem, and the problem of *actually* indicating the properties of the function so defined, starting from the properties of the coefficients, remained almost untouched.

A few results on the inverse problem were known. For instance, the arithmetical nature of the coefficients of an algebraic function were indicated by Eisenstein and Tchebycheff; another theorem, false as stated, was given by Lecornu. The inverse problem was also treated, in some instances, by Darboux. He studied the rate of growth of the coefficients in relation to the "growth" of the function near the circle of convergence.[1]

But Hadamard is the real creator of the theory of detection (and nature) of singularities of the analytic continuation of a Taylor series (see *Thèse* 1892 and a few notes in the C. R. Acad. Sci. Paris which preceded it).

[1] Some interesting papers of Worpitzky written around 1860, published in a very obscure journal and treating related subjects, were discovered at the beginning of this century.

He first gave the value of the radius of convergence in introducing the lim sup of $|A_n|^{1/n}$ $(f(z)=\sum A_n z^n)$. Until then, only when the limit of this expression exists could one find the radius (Cauchy).

He then gave a necessary and sufficient condition bearing on the A_n in order that a given point on the circle of convergence be a singularity. This condition, given by the limit superior of expressions each containing a finite number of coefficients, has since yielded, under its original form, or correspondingly adapted, a wealth of important results. First of all, the famous "Hadamard gap theorem": for $\sum A_n z^{\lambda_n}$, with $\lambda_{n+1}/\lambda_n \geqq \lambda > 1$, the circle of convergence (if finite) is a cut. This gap theorem is only a particular case of a result, rich in content, which follows from the criterion on singularities.

The "gap idea" applied to Taylor series, or to Fourier series, has since become an important principle in trying to "homogenize" a property of a function over its entire range of arguments, or over an interval.

The use of symmetrical determinants $D_{n,m}$ formed with $2m+1$ (m fixed) successive coefficients starting with A_n gives, by analyzing the values of lim sup $|D_{n,m}|^{1/n}$ for different m's, a condition for the function to be meromorphic up to a given circle around the origin.

The corresponding theorems and methods of proof are of a rare elegance.

Hadamard classified singularities on the circle of convergence by their "order." The Riemann-Liouville fractional derivative (or, rather, Hadamard's version of it) of order $-\alpha$ of the series, considered as a function of the argument on the circle of convergence, being continuous and of "écart fini" in a neighborhood of the singularity, the order is the infimum of such α's. A function $\phi(\theta)$ is of "écart fini" on $[a, b]$ if the expressions $|n\int_\alpha^\beta \phi(\theta)e^{in\theta}\,d\theta|$ are bounded by a constant independent of α, β, for $a \leqq \alpha < \beta \leqq b$, and n. As an answer to a question set by Hadamard in his *Thèse* it has since been proved that there exist continuous functions of écart fini which are not of bounded variation. The "order on the circle of convergence," the greatest order of singularities on the circle, is given by a formula recalling, in a very suggestive way, that of the radius of convergence.

It appears to me that Hadamard's idea of order has not been fully exploited by mathematicians of younger generations. Much deeper results should certainly be obtained from this notion, so rich in content, than those obtained in the early twenties in a monograph devoted to this subject by one of Hadamard's followers. The study of determinants, also introduced in the *Thèse* and attached to the idea of order in a way similar to that in which the $D_{n,m}$ are attached to meromorphism, should furnish great possibilities for important research.

Hadamard's theorem on composition of singularities was proved in 1898. When stated without much rigour, it reads as follows. $\sum A_n B_n z^n$ has no other singularities than those which can be expressed as products of the form $\alpha\beta$, where α is a singularity of $\sum A_n z^n$ and β a singularity of $\sum B_n z^n$.

The theorem is proved by the use of Parseval's integral, which Hadamard adapted to Dirichlet series (1898 and 1928), not for the research of the singularities of the composite series, but for the study of interesting relationships between the values of Riemann's ζ function at different points, or between different types of ζ functions.

The two papers on analytic continuation of Taylor series just quoted, and the admirable little monograph *La série de Taylor et son prolongement analytique*, written by Hadamard in 1901, were to inspire a great number of very well-known mathematicians, and it is not exaggerating to say that almost all of the 350 publications of the 150 authors quoted in Bieberbach's recent monograph on analytic continuation are inspired directly, or indirectly, by Hadamard's work just analysed.

The year 1892 is one of the richest in the history of Function Theory, since then not only did Hadamard's thesis appear, but also his famous work on entire functions, which enabled him, a few years later (1896), to solve one of the oldest and most important problems in the Theory of Numbers.

The general results obtained, establishing a relationship between the rate of decrease of the moduli of the coefficients of an entire function and its genus (the converse of Poincaré's theorem), applied to the entire function $\xi(z)$, related to $\zeta(s)$, shows that its genus, considered as a function of z^2, is (as stated, but not proved correctly, by Riemann) zero. This relationship (for general entire functions) between the moduli of the zeros of an entire function and the rate of decrease of its coefficients is obtained by using the results of the *Thèse*, quoted above, and concerning the determinants $D_{n,m}$ of a suitable meromorphic function (the reciprocal of the considered entire function).

This paper on entire functions was written for the Grand Prix de l'Académie des Sciences in 1892. As a matter of fact, the mathematical world in Paris was sure that Stieltjes would get the prize, since Stieltjes thought that he had proved the famous "Riemannische Vermutung," and it is interesting, I believe, to quote a sentence from Hadamard's extremely famous paper of 1896 with the suggestive title, "Sur la distribution des zéros de la fonction $\zeta(s)$ et ses conséquences arithmétiques." Hadamard writes: "Stieltjes avait démontré, conformément aux prévisions de Riemann, que ces zéros sont tous de la forme $\frac{1}{2}+ti$ (le nombre t étant réel), mais sa démonstration n'a jamais été publiée, et il n'a même pas été établi que la fonction ζ n'ait pas de zéros sur la droite $R(s)=1$. C'est cette dernière conclusion que je me propose de démontrer."

The "modesty," and the grandeur, of the purpose: to prove that $\zeta(s)\neq 0$ for $\sigma=1$ $(s=\sigma+it)$, after the assertion that Stieltjes had "proved" the Riemannische Vermutung, are remarkably moving. The more so that, due to this proof, Hadamard could prove, in the same paper of 1896, the most important proposition on the distribution of primes: $\pi(x)$ being the number of primes smaller than x $(x>0)$, $\pi(x)\sim x/\log x$ $(x\to\infty)$.

The event had certainly a great historical bearing. The assumption was made, at the beginning of the last century, by Legendre (in the form $\pi(x)=x/(\log x-A(x))$, with $A(x)$ tending to a finite limit). Tchebycheff had shown that $.92129\leqq\pi(x)\log x/x\leqq 1.10555\cdots$, but did not prove that the expression tends to a limit, and there was no hope that his method could yield any such proof. Many mathematicians, Sylvester among them, were able, in using the same methods as Tchebycheff, to improve these inequalities. But there was nothing fundamentally new in these improvements. Let us quote Sylvester (1881) on this matter (quotation given by Landau). "But to pronounce with certainty upon the existence of such possibility ($\lim \pi(x)\log x/x=1$) we should probably have to wait until someone is born into the world as far surpassing Tchebycheff in insight and penetration as Tchebycheff proved himself superior in these qualities to the ordinary run of mankind."

And, as Landau says, when Sylvester wrote these words Hadamard was already born.

It should be pointed out that independently, and at the same time, de La Vallée-Poussin also proved the nonvanishing of ζ on $\sigma=1$ and, thus, the prime-number theorem; however, Hadamard's proof is much simpler.

Hadamard's study of the behavior of the set of zeros of $\zeta(s)$ is based on his result quoted above (proved in his paper of 1892, written for the Grand Prix), on the genus of $\xi(z)$.

It seems to me of importance to insist upon the "chain of events" in Hadamard's discoveries: relationship between the position of the poles of a meromorphic function and the coefficients of its Taylor series; this result yields later the genus of an entire function by the rate of decrease of its Taylor coefficients; and from there, four years later, the important properties of $\zeta(s)$, and finally, as a consequence, the prime-number theorem.

Clearly, one of the most beautiful theories on analytic continuation, so important by itself, and so rich by its own consequences, seems to have been directed in Hadamard's mind, consciously or not, towards one aim: the prime-number theorem.

He proved also the analogous theorems on the distribution of primes belonging to a given arithmetical progression, since by his methods he was able to study Dirichlet series which, with respect to these primes, play the same role as the ζ function plays with respect to all primes.

Since we were just speaking about "Hadamard's determinants," let us mention the estimates he gave for the values of general determinants (1893). And, again in the theory of functions of a complex variable, his famous "three-circle theorem": $M(r)$ being the maximum modulus of a function holomorphic in a circle $|z| < R$ ($|z| = r < R$), $\log M(r)$ is a convex function of $\log r$.

We must not leave the Theory of Functions without mentioning that the problem of quasi-analyticity for infinitely differentiable functions, as distinct from the problem of generalizing the notion of analytic functions in the complex domain, was clearly pointed out by Hadamard in a short paper in 1912. This problem was suggested to Hadamard by the properties of the data for Cauchy's problem for the heat equation (Holmgren's properties of the derivative of the solution on the frontier $x=0$).

It should be pointed out here that Hadamard was the first to indicate a relation between the norms of a function and its first two derivatives (1919), a relation which, later, improved and generalized, played an important role in the theory of infinitely differentiable functions.

In a quite different order of ideas, stepping into classical differential geometry, Hadamard, by using rather elementary considerations on maxima and minima, studied the behavior of the real geodesics on general surfaces. He found particularly interesting and simple results when the surfaces are everywhere of positive curvature (1897) (Prix Bordin de l'Académie des Sciences).

However, differential geometry being replaced by topological considerations or rather by what was called Analysis Situs, the study of geodesics on surfaces of negative curvature became the subject of one of the most beautiful papers of Hadamard (1898). (The importance of Analysis Situs in the study of differential equations was pointed out in Poincaré's work, for which Hadamard always professed the greatest admiration.)

In this work Hadamard abandoned completely hypotheses on the analytic nature of the surface, and his results are thus of an essentially more general character than those concerning surfaces of positive curvature.

On the surfaces of negative curvature, considered by Hadamard, surfaces with a finite number of expanding infinite nappes, the geodesics behave in a way which presents an interesting philosophical problem to physicists and astronomers.

There are: closed geodesics, or geodesics asymptotic to such closed ones; geodesics which tend to infinity on any of the nappes. But there is also a third category of geodesics on these surfaces: those of which entire segments approach successively corresponding segments on each of a sequence of closed geodesics, the length of these segments increasing constantly.

But the most remarkable feature is the following: the set E of tangents to the geodesics passing through a point and remaining at a finite distance is *perfect and nowhere dense* and, in the neighborhood of each geodesic of which the tangent belongs to E (neighborhood of directions), there is a geodesic which goes to infinity in an arbitrarily chosen nappe. In each such neighborhood there are also geodesics of the third category, that is to say, those which approach even more tightly a denumerable set of closed geodesics.

In other words, "The smallest change in the direction of a geodesic, which remains at a finite distance, suffices to introduce absolutely any final direction to the curve, the new geodesic might take any of the stated above forms."

And Hadamard wonders if such circumstances could be met in other problems of mechanics. Could they occur, in particular, in the study of the motion of celestial bodies? It is probable, points out Hadamard, that the results obtained in such difficult cases are analogous to the one studied in this paper. But in a problem in physics a slight modification in the data at a moment should have little influence on the future of a solution, since only approximate data are known anyway. Thus the *final* behavior of a trajectory could well depend on *arithmetical* properties of the constants of integration.

Hadamard was very interested by Volterra's functional calculus. As a matter of fact, it is Hadamard who suggested the term "fonctionnelle" to replace Volterra's older term "fonction de ligne." It should be pointed out that already at the very beginning of the century (1903) Hadamard gave a general expression for linear functionals defined on the class of continuous functions on an interval. The expression given as a limit of the form

$$U(f) = \lim_{\mu \to \infty} \int_a^b f(x)\Phi(x, \mu)\, dx$$

should be considered as a precursor of F. Riesz's famous theorem.

In introducing the functional derivative, in Volterra's sense, of a Green's function, considered as a functional of the boundary surface (and as a function of the two interior points), Hadamard was able, by integrating the corresponding functional equation, to determine the Green's function corresponding to $\Delta U = 0$ when it is known for a given surface.

Hadamard had always been interested in mechanics and differential equations and published various interesting papers on these subjects, some of which already contain his main ideas on problems suggested by physics. But his work on partial differential equations, which became one of his best contributions to the advancement of science, started relatively late. One of his papers devoted to this subject dates from 1900, two from 1901. In 1903 appeared the "Leçons sur la propagation des ondes et les équations de l'hydrodynamique." His chief ideas were developed in the following years. He insisted on them, persistently, in order to persuade mathematicians and physicists of their importance. He carefully examined the different kinds of problems; in Dirichlet problems, for a partial differential equation of the second order, the boundary datum is only one function while, for Cauchy problems, one gives two data on the initial subspace $t = 0$: for an elliptic operator, for instance, the Laplace operator Δ, the Dirichlet problem is well posed (to find a solution of $\Delta u = f$, a given function, in an open set Ω of $\boldsymbol{R}^m$, with the boundary value $u = u_0$, a given function, on the boundary S of Ω) while, for a hyperbolic operator, as for instance the wave operator, $\square = \partial^2/\partial t^2 - \Delta_x$, the Cauchy problem is well posed (to find a solution of $\square u = f$, with the initial data $u(0, t) = u_0(x)$, $\partial u(x, 0)/\partial t = u_1(x)$).

Of course, all these ideas are now very well known, but at that time, it was not the case, due to the impregnation by simple differential equations and to the result of Cauchy-Kowalewskaya on analytic partial differential equations; different types of equations and types of problems were not as well settled as they are now, and probably even we, his contemporaries and successors, owe very much to the insistence of Hadamard on this classification. He claimed, moreover, that a "well-posed" problem is not only one for which the solution exists and is unique for given data; the solution must depend continuously on the data. He explains that, if the solution varies considerably for a small variation of the data, it is not actually a solution, in the sense of physics; since in the physical reality, we never know the data completely, but only with a certain degree of accuracy, so that it would mean that we actually do not know the solution. He repeated this idea constantly so that, even now, a problem is called "well posed in the sense of Hadamard" if it has the property of continuity of the solution with respect to the data. This idea was even more fruitful than he himself imagined; for the analysts were then obliged to examine, as he says, the "different types of neighborhoods and continuity," which led unavoidably to functional spaces, general topology and functional analysis; it is surely one of the sources of functional analysis, and it is still now one of the best fields for applications of functional analysis. The modern ways for solving partial differential equations use "a priori estimates," which means that one actually proves the existence and the uniqueness of a solution by proving, first, its continuity with respect to the data; functional analysis (essentially Banach's and F. Riesz' theorems) yields then the result. On the other hand, the famous theorem of Banach, the so-called "closed-graph theorem," showed later that, in most cases, existence and uniqueness of the solution imply its continuous dependence on the data. These investigations on partial differential equations, together with the introduction by Fredholm of integral equations, and Hilbert's methods (Hilbert spaces) became the source of the modern theory of operators. One must also stress how much Hadamard tried to remain close to physics; he liked to work with the rigor of a mathematician and the practical sense of a physicist, and liked to repeat Poincaré's words, "La Physique ne nous donne pas seulement l'occasion de résoudre des problèmes . . . , elle nous fait pressentir la solution." But probably the most important contribution of Hadamard to partial differential equations is the complete solution of equations of the second order of normal hyperbolic type with the use of the elementary solution (he preferred this denomination to that of fundamental solution; today, one says fundamental solution in English, elementary solution in French). The first among the known fundamental solutions is that of the Laplace equation: the kernel $1/r$ of potential theory. In the elliptic case, more general equations have been successfully studied by Picard, E. E. Levi, Hilbert (with also the famous parametrix), Fredholm, Herglotz, Leroux, Zeilon, and even for higher order equations, with constant or variable coefficients. But the hyperbolic case remained mysterious. Riemann's function was only relative to the very special case of dimension two, and could not be extended directly. Poisson solved the wave equation in the four-dimensional case (three dimensions for space, one for time) by an explicit expression of the solution in terms of the inhomogeneous term and the initial data, but this solution was given as such, with no method whatsoever to find it and therefore to generalize it for more difficult cases, or even for dimensions different from four. The problem had been thoroughly investigated by Kirchhoff (*Zur Theorie des Lichtstrahlen*), Volterra, Tedone; they had been able to solve completely wave equations for higher dimensions, but by indirect processes; they obtained some repeated integrals of the solutions along arbitrary lines, which allowed them to find the solutions themselves by differentiation. It should be noted that this indirect procedure must not be rejected, as it has some internal justifications, but it did not furnish a direct and simple expression of the solution, and also it could be found only in a very complicated way so that it did not show how it could be generalized to the case of variable coefficients. The complete solution by the study of the elementary solution is explained in Hadamard's famous book, *Le problème de Cauchy et les équations aux dérivées partielles linéaires hyperboliques*, his lectures at Yale University in 1920. This book is a real masterpiece and, by its content, its clarity, and the abundance of its ideas, it has inspired all the investigators on partial differential equations of the following generation. First of all, he built the elementary solution, defined as a solution of the homogeneous equation, having a given type of singularity on the characteristic cone; $\Gamma = 0$ being the normal equation of this cone, the solution must be of the form $U/\Gamma^{(m-2)/2}$ (m the dimension of the space) for m odd, and of the form $U/\Gamma^{(m-2)/2} + V \log \Gamma$ for m even, U and V regular. This difference between odd and even dimensions of the space had already been noticed for elliptic equations, and still today plays an essential role. Now, how can one build the solution of a given Cauchy problem with this elementary solution? Green's formula furnishes the normal way; but it leads to divergent integrals. Hadamard introduced at this point a new process to compute divergent integrals, by subtracting the "infinite part" of nonintegral order; he called it the "finite part of the divergent integral." It proved to be a very powerful tool and, indeed, he was able to express completely the solution of the problem ("Synthèse de la solution") by a Green's formula with the elementary solution, involving finite parts of divergent integrals, at least in the case of m odd. For the case of m even, the procedure failed; he used then the "méthode de descente," solving the problem for the dimension $m+1$, and then going down to m. This descent is, of course, a very trivial idea, but it furnishes here a fantastic simplification and is, in spite of its simplicity, a beautiful and fruitful mathematical idea. The expression he found for the solution in the even case is completely different from the one in the odd case; this result also has many important implications, which were studied by a great number of authors after him. In fact, we know today that, especially in the even case, the concept of an elementary or fundamental solution has to be put in another way, using a Dirac distribution δ as the inhomogeneous term; but this could never have been understood without Hadamard's method of resolution, and the finite parts of divergent integrals.

The expression of the solution by integrals enables one to see how it depends on the data. First of all, the existence of the solution implies the existence of a large number of derivatives of the data, depending on the dimension m of the space. This is rather surprising since, for an equation of the second order, it is not natural to assume that the solution or the data have more than two derivatives. Here one must confess that physical intuition would give a wrong idea. This intervention of higher derivatives also played an important role later on. The introduction of Hilbert spaces of functions gives a method to avoid it ("energy integrals"). But there is another kind of dependence of the solution on the data: in order to know the solution in a given bounded region of the space, it is sufficient to know the data in a fixed bounded region ("rayon d'action" des données). It can be formulated in a very precise way which expresses, here, an important physical feature: the waves are propagating, there are rays of propagation, the bicharacteristic lines, playing the role of the light rays in the wave equation for light. One can also interpret wave surfaces and Huygens' principle. Hadamard restated this principle, making a careful distinction between two kinds of Huygens' principles: one is a universal law; for all kinds of evolution equations, it simply expresses the existence of a groupoid of transformations, expressing the passage from the time t to the time t'', as a composition of a passage from t to t' and a passage from t' to t'' (however, computed for various types of equations, it yields many interesting addition formulas for Bessel hypergeometric theta functions); another one, which expresses the existence of "lacunae" in the elementary solutions; he introduced here the notion of diffusion or nondiffusion of waves; there is always diffusion in the odd-dimensional case and in dimension two (vibrating strings); in even dimensions, he could not find the general result, which inspired very remarkable works of more recent mathematicians.[2]

He also studied the so-called "mixed problems" for a hyperbolic equation; one gives both Cauchy initial data for $t=0$, and Dirichlet data on the space boundary. This part of his work contains some of the most difficult investigations; he follows the rays after the reflection on the boundary, and studies the "caustics." If the waves propagate, if the movement can be followed along the time with rays, why should not one act the same way for boundary-value problems, when there are reflections? It seems, however, that this approach is too difficult. Most of the modern methods for boundary-value problems are global ones involving, for instance, Laplace transform, or semi-group theory, or eigenfunctions and stationary waves, or operational methods, but one loses the idea of propagation and reflection.

Until the very end of his life, Hadamard was interested in partial differential equations. One could meet him in colloquia and seminars, following everything carefully, asking questions at the end. The language had become so different that it was difficult sometimes to recognize even familiar ideas, but he always was aware of the similarity between the new concepts and the old ideas. One must add that he never revolted against this renovation of language and concepts, and tried to understand the new ones; he considered mathematics, not as a static science, but as being in a continuous state of progression, and he knew that this change, not only in the methods, but also in the shape itself of mathematics, is the price for this progression. New ideas could always find in him a fervent supporter.

Those of us who have had the privilege of attending Hadamard's Seminars at the Collège de France, where he taught from 1909 to 1937[3] would probably be unable to recall more inspiring hours of mathematical thought. Well-known mathematicians all over the world considered it as an honor, and sometimes as a redoubtable task, to be asked to state and to prove their recent results, or the results in their fields just discovered by others. But, without trying to diminish the contribution of the talent of the lecturers, we must say that

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[2] He hardly studied equations of order higher than two, which are today the object of most of the publications.

[3] Hadamard also taught at the Ecole Polytechnique and Ecole Centrale. He began his teaching career (as it was always done in France, even for the greatest, until the late twenties) as a high school teacher; then, Maître de Conférences at the University de Bordeaux, from where he went to the Sorbonne, before being elected to the Collège de France.

the bulk of our feelings, of the richness of our inspiration, and our desire to continue to work, or at least to think, on the subject just treated in the Seminar, came from Hadamard's analysis of the lecture, from his critical views, from his interruptions, simple remarks and some prophecies on the future of the subject.

One of the characteristics of Hadamard's Seminar was its variety. It was not a Seminar on one branch of mathematics—it was one on Mathematics, pure and applied, on the philosophy of mathematics and numerical analysis as well. Often the lecture was an important mathematical event, where for the first time a very significant result was expounded. Sometimes new results were born at the Seminar, and published a few weeks later in the C. R. Acad. Sci. Paris.

Mathematical life in Paris in the twenties and early thirties was for the large part described by two words: "Séminaire d'Hadamard."

Scientific Papers and Books of Jacques Hadamard

1884

Sur le limaçon de Pascal, J. Math. Spéc. (2) **3**, 80–83.

Sur l'hypocycloïde à trois rebroussements, J. Math. Spéc. (2) **3**, 226–232.

1885

Sur l'hypocycloïde à trois rebroussements, J. Math. Spéc. (2) **4**, 41–42.

1888

Sur le rayon de convergence des séries ordonnées suivant les puissances d'une variable, C. R. Acad. Sci. Paris **106**, 259.

Recherche des surfaces anallagmatiques par rapport à une infinité de pôles d'inversion, Bull. Sci. Math. (2) **12**, 118–121.

1889

Sur la recherche des discontinuités polaires, C. R. Acad. Sci. Paris **108**, 722.

1892

Essai sur l'étude des fonctions données par leur développement de Taylor. Thèse de Doctorat de la Faculté des Sciences, J. Math. (4) **8**, 101–186.

Sur les fonctions entières de la forme $e^{G(x)}$, C. R. Acad. Sci. Paris **114**, 1053.

Etude sur les propriétés des fonctions entières et en particulier d'une fonction considérée par Riemann (Grand Prix des Sciences mathématiques), J. Math. (4) **9**, 171–215.

1893

Sur le module maximum que puisse atteindre un déterminant, C. R. Acad. Sci. Paris **116**, 1500.

Résolution d'une question relative aux déterminants, Bull. Sci. Math. (2) **17**, 240–246.

Sur les caractères de convergence des séries à termes positifs, C. R. Acad. Sci. Paris **117**, 844.

1894

Sur les caractères de convergence des séries à termes positifs et sur les fonctions indéfiniment croissantes (avec note complémentaire), Acta Math. **18**, 319–336 et 421.

Remarque sur les rayons de courbure des roulettes, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux, 19 avril.

Sur les mouvements de roulement, C. R. Acad. Sci. Paris **118**, 911–912.

Sur le théorème de Jacobi relatif au mouvement d'un corps pesant de révolution fixé par un point de son axe, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux, 19 juillet.

Sur l'élimination, C. R. Acad. Sci. Paris **119**, 995–997.

1895

Sur le tautochronisme, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux, 7 février.

Sur l'expression du produit $1 \cdot 2 \cdot 3 \cdots (n-1)$ *par une fonction entière*, Bull. Sci. Math. (2) **19**, 69–71.

Sur une congruence remarquable et sur un problème fonctionnel qui s'y rattache, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux, 14 février.

Sur les éléments infinitésimaux du second ordre dans les transformations perpétuelles, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux, 19 décembre.

Sur la précession dans le mouvement d'un corps pesant de révolution fixé par un point de son axe, Bull. Sci. Math. (2) **19**, 228–230.

Sur la stabilité des rotations d'un corps solide pesant, Ass. Franç., Congr. Bordeaux.

Sur les mouvements de roulement, Mém. Soc. Sci. Phys. Natur., Bordeaux.

Sur certains systèmes d'équations aux différentielles totales, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux.

Ces deux travaux ont été réimprimés dans le volume de M. Appell: "*Les roulements en dynamique*," Coll. Scientia, Carré et Naud, Paris, 1899, 47–68 et 69–70.

1896

Mémoire sur l'élimination, Acta Math. **20**, 201–238.

Une propriété des mouvements sur une surface, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux, 30 avril.

Une propriété des mouvements sur une surface, C. R. Acad. Sci. Paris **122**, 983.

Sur l'instabilité de l'équilibre, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux, 21 mai.

Sur les fonctions entières, C. R. Acad. Sci. Paris **122**, 1257–1258.

Sur certaines propriétés des trajectoires en dynamique, Mémoire couronné par l'Académie des Sciences (Prix Bordin), J. Math. (5) **3**, 331–387.

Sur les lignes géodésiques des surfaces spirales et les équations différentielles qui s'y rapportent, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux, 4 juin.

Sur les zéros de la fonction $\zeta(s)$ *de Riemann*, C. R. Acad. Sci. Paris **122**, 1470–1473.

Sur la fonction $\zeta(s)$, C. R. Acad. Sci. Paris **123**, 93.

Sur les fonctions entières, Bull. Soc. Math. France **24**, 186–187.

Sur la distribution des zéros de la fonction $\zeta(s)$ *et ses conséquences arithmétiques*, Bull. Soc. Math. France **14**, 199–220.

Sur une forme de l'intégrale de l'équation d'Euler, Bull. Sci. Math. (2) **20**, 263–266.

Sur la décomposition de deux figures géométriques équivalentes en un nombre fini d'éléments superposables chacun à chacun, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux, Communication de Brunel, 24 décembre.

1897

Sur les notions d'aire et de volume, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux, 21 janvier.

Sur les séries de Dirichlet, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux, 18 février.

Sur les lignes géodésiques des surfaces à courbures opposées, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux, 4 mars.

Théorème sur les séries entières, C. R. Acad. Sci. Paris **124**, 135.

Sur les principes fondamentaux de la mécanique, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux, 18 mars.

Sur la démonstration d'un théorème d'algèbre, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux, 1 avril.

Sur les conditions de décomposition d'une forme ternaire, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux, 13 mai.

Sur les séries entières, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux, 3 juin.

Sur les lignes géodésiques, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux, 17 juin.

Sur les lignes géodésiques des surfaces à courbures opposées, C. R. Acad. Sci. Paris **124**, 149.

Sur les lignes géodésiques, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux, 1 juillet.

Sur certaines applications possibles de la théorie des ensembles, Ier Congr. Internat. Math., Zurich.

Sur une surface à courbures opposées, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux, 22 juillet.

1898

Théorèmes sur les séries entières, Acta Math. **22**, 55–64.

Sur la généralisation du théorème de Guldin, Bull. Soc. Math. France **26**, 264–265.

Les surfaces à courbures opposées et leurs lignes géodésiques, J. Math. (5) **4**, 27–73.

Sur la courbure dans les espaces à plus de deux dimensions, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux, 3 février.

Sur la forme de l'espace, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux, 3 février.

Les invariants intégraux et l'optique, C. R. Acad. Sci. Paris **126**, 82.

Sur la forme des géodésiques à l'infini et sur les géodésiques des surfaces réglées du second ordre, Bull. Soc. Math. France **26**, 195–216.

Sur le billard non euclidien, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux, 5 mai.

Leçons de géométrie élémentaire (*Géométrie plane*), A. Colin, Paris.

1899

Sur les conditions de décomposition des formes, Bull. Soc. Math. France **27**, 34–47.

1900

Sur les points doubles des contours fermés, Proc.-Verb. Soc. Sci. Phys. Natur. Bordeaux, 12 janvier.

Sur les intégrales d'un système d'équations différentielles ordinaires, considérées comme fonctions des données initiales, Bull. Soc. Math. France **28**, 64–66.

Sur l'intégrale résiduelle, Bull. Soc. Math. France **28**, 69–90.

Sur les équations aux dérivées partielles à caractéristiques réelles, IIième Congr. Internat. Math., Paris.

Note sur l'induction et la généralisation en mathématiques, Congr. Internat. Philos., Paris.

Sur les singularités de certaines séries, Intermédiaire des Mathématiciens.

1901

La série de Taylor et son prolongement analytique, Coll. Scientia, Carré et Naud, Paris.

Sur la propagation des ondes, Bull. Soc. Math. France **29**, 50–60.

Sur les réseaux de coniques, Bull. Soc. Math. **25**, 27–30.

Sur les éléments linéaires à plus de deux dimensions, Bull. Sci. Math. **25**, 37–60.

Leçons de géométrie élémentaire (*Géométrie dans l'espace*), A. Colin, Paris.

Sur l'équilibre des plaques circulaires libres ou appuyées et sur celui de la sphère isotrope, Ann. Sci. École Norm. Sup. (3) **18**, 313–342.

Sur l'itération et les solutions asymptotiques des équations différentielles, Bull. Soc. Math. France **29**, 224–228.

La bosse des mathématiques, Rev. Gén. Sci. **11**.

1902

Sur les problèmes aux dérivées partielles et leur signification physique, Bull. Univ. Princeton **13**, 49–52.

La théorie des plaques élastiques planes, Trans. Amer. Math. Soc. **3**, 401–422.

Deux théorèmes d'Abel sur la convergence des séries, Acta Math. **26**, 177–183.

Sur certaines surfaces minima, Bull. Sci. Math. (2) **26**, 357–361.

Sur les dérivées des fonctions de lignes, Bull. Soc. Math. France **30**, 40–43.

Sur une classe d'équations différentielles, Bull. Soc. Math. France **30**, 208–220.

Sur une question de calcul des variations, Bull. Soc. Math. France **30**, 253–256.

Sur une condition qu'on peut imposer à une surface, Bull. Soc. Math. France **30**, 111.

Compte rendu de Larmor, Aether and Matter, Bull. Sci. Math. (2) **26**, 319–328.

Compte rendu de Bouvier, la méthode mathématique en economie politique, Rev. Gén. Sci. **13**.

Sur les fonctions entières, C. R. Acad. Sci. Paris **135**, 1309.

1903

Sur les glissements dans les fluides, C. R. Acad. Sci. Paris **136**, 299.

Sur les glissements dans les fluides (Note complémentaire), C. R. Acad. Sci. Paris **136**, 545.

Sur les opérations fonctionnelles, C. R. Acad. Sci. Paris **136**, 351.

Sur un problème mixte aux dérivées partielles, Bull. Soc. Math. France **31**, 208–224.

Sur les surfaces à courbure positive, Bull. Soc. Math. France **31**, 300–301.

Leçons sur la propagation des ondes et les équations de l'hydrodynamique, Hermann, Paris.

Sur les équations aux dérivées partielles linéaires du deuxième ordre, C. R. Acad. Sci. Paris **137**, 1028.

Les sciences dans l'enseignement secondaire (Conférence faite à l'Ecole des Hautes Etudes sociales), Alcan, Paris.

1904

Résolution d'un problème aux limites pour les équations linéaires du type hyperbolique, Bull. Soc. Math. France **32**, 242–268.

Recherches sur les solutions fondamentales et l'intégration des équations linéaires aux dérivées partielles, Ann. École Norm. Sup. (3) **21**, 535–556.

Sur un point de la théorie des percussions, Nouv. Ann. Math. (4) **4**, 533–535.

Sur les séries de la forme $\sum a_n e^{-\lambda_n z}$, Nouv. Ann. Math. (4) **4**, 529–533.

Sur les solutions fondamentales des équations linéaires aux dérivées partielles, IIIe Congrès Internat. Math., Heidelberg.

Sur les données aux limites dans les équations aux dérivées partielles de la physique mathématique, IIIe Congr. Internat. Math., Heidelberg.

1905

Recherches sur les solutions fondamentales et l'intégration des équations linéaires aux dérivées partielles (2ième Mémoire), Ann. École Norm. Sup. (3) **22**, 101–141.

Sur les équations aux dérivées partielles (2ième Note), C. R. Acad. Sci. Paris **140**, 425.

Sur quelques questions de calcul des variations, Bull. Soc. Math. France **33**, 73–80.

Sur la théorie des coniques, Nouv. Ann. Math. (4) **5**, 145–152.

Lettres sur la théorie des ensembles (*Correspondance avec Mm. Borel, Baire et Lebesgue*), Bull. Soc. Math. France **33**, 261–273.

Remarque au sujet d'une note de M. Gyözö-Zemplen, C. R. Acad. Sci. Paris **141**, 713.

A propos d'enseignement, Rev. Gén. Sci. **16.**
La théorie des ensembles, Rev. Gén. Sci. **16.**
Réflexions sur la méthode heuristique, Rev. Gén. Sci. **16.**

1906

Sur un théorème de M. Osgood, relatif au calcul des variations, Bull. Soc. Math. France **34,** 61.
Sur la mise en équation des problèmes de mécanique, Nouv. Ann. Math. (4) **6,** 97–100.
Sur les transformations planes, C. R. Acad. Sci. Paris **142,** 74.
Sur les transformations ponctuelles, Bull. Soc. Math. France **34,** 71–84.
Compte rendu de Elementary Principles in Statistical Mechanics, de Gibbs, Bull. Amer. Math. Soc. **12,** 194–210; Réimprimé dans Bull. Sci. Math. (2) **30,** 161–179.
Les problèmes aux limites dans la théorie des équations aux dérivées partielles (Conférences faites à la Société de Mathématique de France et à la Société française de Physique), Bull. Soc. Franç. Phys., J. Phys.
Sur une méthode de calcul des variations, C. R. Acad. Sci. Paris **143,** 1127.
La logistique et l'induction complète, Rev. Gén. Sci. **17.**
Les principes de la théorie des ensembles, Rev. Gén. Sci. **17.**
Sur les caractéristiques des systèmes aux dérivées partielles, Bull. Soc. Math. France **24,** 48–52.
Sur le principe de Dirichlet, Bull. Soc. Math. France **24,** 135–138.
La logistique et la notion de nombre entier, Rev. Gén. Sci. **17.**

1907

Sur quelques questions de calcul des variations, Ann. École Norm. Sup. (3) **24,** 203–231.
Sur l'interprétation théorique des raies spectrales, Bull. Soc. Franç. Phys.
Sur la variation des intégrales doubles, C. R. Acad. Sci. Paris **144,** 1092.
Sur le problème d'analyse relatif à l'équilibre des plaques élastique encastrées, Mémoire couronné par l'Académie des Sciences (Prix Vaillant), Mém. Sav. Étrang. **33.**

1908

Sur les séries de Dirichlet, Rend. Circ. Mat. **25,** 326–330.
Rectification à la note "Sur les séries de Dirichlet," Rend. Circ. Mat. **25,** 395–396.
Théorie des équations aux dérivées partielles linéaires hyperboliques et du problème de Cauchy, Acta Math. **31,** 333–380.
Sur l'expression asymptotique de la fonction de Bessel, Bull. Soc. Math. France **36,** 77–85.
Sur certaines particularités du calcul des variations, IVe Congr. Math., Rome.
Sur certains cas inéressants du problème biharmonique, IVe Congr. Math., Rome.
Les paradoxes de la théorie des ensembles, Rev. Gén. Sci. **19.**

1909

Sur les lignes géodésiques (à propos d'une note de M. Drach), C. R. Acad. Sci. Paris **148,** 272.
Sur une propriété fonctionnelle de la fonction $\zeta(s)$ de Riemann, Bull. Soc. Math. France **37,** 59–60.
Détermination d'un champ électrique, Ann. Chim. Phys. (8) **16.**
Notions élémentaires sur la géométrie de situation, Nouv. Ann. Math. (4) **9,** 193–235.
La géométrie de situation et son rôle en mathématique, Leçon d'ouverture professée au Collège de France, Rev. du mois **8.**

1910

Leçons sur le calcul des variations, Hermann, Paris.

Sur les ondes liquides, C. R. Acad. Sci. Paris **150**, 609.

Sur les ondes liquides. 2,[4] C. R. Acad. Sci. Paris **150**, 772.

Quelques propriétés des fonctions de Green, C. R. Acad. Sci. Paris **150**, 764.

Sur quelques applications de l'indice de Kronecker (Note additionnelle à la deuxième édition de l'"Introduction à la Théorie des fonctions d'une variable" de J. Tannery), Hermann, Paris.

Sur un problème de cinématique navale, Nouv. Ann. Math. (4) **10**, 337–361. Réimprimé dans Rev. mar., avril 1911.

1911

Sur les trajectoires de Liouville, Bull. Sci. Math. (2) **35**, 106–113.

Relations entre les solutions des équations aux dérivées partielles des types parabolique et hyperbolique, Bull. Soc. Math. France **39**, C.-R. des séances.

Solution fondamentale des équations linéaires aux dérivées partielles du type parabolique, C. R. Acad. Sci. Paris **152**, 1148.

Mouvement permanent lent d'une sphère liquide et visqueuse dans un liquide visqueux, C. R. Acad. Sci. Paris **152**, 1735.

Sur l'inégalité

$$[\delta g_A^A \delta g_B^B - (\delta g_B^A)^2][\delta g_C^C \delta g_D^D - (\delta g_D^C)^2] > (\delta g_C^A \delta g_D^B - \delta g_D^A \delta g_C^B)^2,$$

à laquelle satisfont les variations de la fonction de Green quand on passe d'un contour à un contour voisin, Bull. Soc. Math. France, **39**, C. R. des séances.

Sur les propriétés des fonctions de Green dans le plan, Bull. Soc. Math. France **39**, C. R. des séances.

Le calcul fonctionnel (Recueil de travaux dédiés à la mémoire de Louis Olivier), Reproduit dans Enseignement Math., 1912, 1–18.

Propriétés générales des corps et domaines algébriques (en collaboration avec M. Kürschak), Encycl. Sci. Math., édition française, **1**, Vol. 2, 233–385.

Maurice Lévy, Rev. Gén. Sci. **22**.

Four lectures on mathematics (données à Columbia University en octobre 1911), Columbia Univ. Press, New York, 1915.

1912

Sur une question relative aux liquides visqueux (Note rectificative), C. R. Acad. Sci. Paris **154**, 109.

Sur les variations unilatérales et les principes du calcul des variations, Bull. Soc. Math., C. R. des séances, 20.

Sur les extrémales du problème isopérimétrique dans le cas des intégrales doubles, Bull. Soc. Math., C. R. des séances, 20.

Sur la généralisation de la notion de fonction analytique, Bull. Soc. Math., C. R. des séances, 28.

Sur la loi d'inertie des formes quadratiques, Bull. Soc. Math., C. R. des séances, 29.

Propositions transcendantes de la théorie des nombres (en collaboration avec M. Maillet), Encycl. Sci. Math., édition française, **1**, Vol. 3, 215–387.

Itération des noyaux infinis dans le cas des intégrales doubles (Note additionnelle à Fréchet et Heywood, "L'équation de Fredholm et ses applications à la Physique mathématique"), Hermann, Paris.

Observation à propos de la communication de M. Borel "Remarque sur la théorie des résonateurs," Bull. Soc. Franç. Phys.

[4] Ces deux notes sont développées dans un travail ultérieur de M. Bouligand [Bull. Soc. Math. France **40**.]

Sur la série de Stirling, Ve Congr. Internat. Math., Cambridge.
Henri Poincaré, Rev. Métaph. Mor. **21**, 617–658; Revue du mois **16**, 385–418.
L'oeuvre mathématique de H. Poincaré, Acta Math. **38** (1921), 203–287.

1913

La construction de Weierstrass et l'existence de l'extremum dans le problème isopérimétrique, Ann. Mat. (3) **21** (volume du Centenaire de Lagrange), 251–287.
Observations à propos d'une note de M. Bouligand, C. R. Acad. Sci. Paris **156**, 1364.

1914

Points pinces, arêtes de rebroussement et représentation paramétrique des surfaces, Enseignement Math. **16**, 356–359.
L'infini mathématique et la réalité, Revue du mois.
Sur la limitation du module des dérivées, Bull. Soc. Math. France **42**, C. R. des séances, 68–72.
A propos d'une note de M. Paul Lévy sur la fonction de Green, C. R. Acad. Sci. Paris **158**, 1010–1011.

1915

Sur un mémoire de M. Sundman, Bull. Sci. Math. (2) **39**, 249–264.

1916

Sur les ondes liquides, Rend. Acad. Lincei (5) **25**, 716–719.
Sur l'élimination entre équations différentielles, Nouv. Ann. Math. (4) **17**, 81–84.

1919

Remarques sur l'intégrale résiduelle, C. R. Acad. Sci. Paris **168**, 533–534.
Sur les correspondances ponctuelles, Bull. Soc. Math. France **47**, C. R. des séances, 28–29 et Rev. Mat. Hisp.-Amer.
Sur les singularités des séries entières, Bull. Soc. Math. France **47**, C. R. des séances, 40.
Sur un théorème fondamental de la théorie des fonctions analytiques de plusieurs variables, Bull. Soc. Math. France **47**, C. R. des séances, 44–46.
Démonstration directe d'un théorème de Poincaré sur les périodes des intégrales abéliennes attachées à une courbe algébrique qui satisfait à une équation différentielle linéaire, Bull. Soc. Math. France **47**, C. R. des séances, 46.
Recherche du balourd dynamique des obus, Travaux du Laboratoire d'Essais des Arts et Métiers.

1920

Sur certaines solutions d'une équation aux dérivées fonctionnelles linéaires hyperboliques non analytiques, C. R. Acad. Sci. Paris **170**, 355–359.
La solution élémentaire des équations aux dérivées partielles, C. R. Acad. Sci. Paris **170**, 149–154.
Rapport sur les travaux examinés et retenus par la Commission de Balistique de l'Académie des Sciences, C. R. Acad. Sci. Paris **170**, 436–445.
Sur la solution élémentaire des équations aux dérivées partielles et sur les propriétés des géodésiques, Congr. Internat. Math., Strasbourg, pp. 179–184.
Sur le problème mixte pour les équations linéaires aux dérivées partielles, Congr. Internat. Math., Strasbourg, pp. 499–503.

1921

On some topics connected with linear partial differential equations, Proc. Bénarès Math. Soc. **3**, 39–48.
A propos d'enseignement secondaire, Rev. Internat. Enseignement Bull. Union Natur.
Sur la comparaison des problèmes aux limites pour les deux principaux types d'équations aux dérivées partielles, Bull. Soc. Math. France **49**, C. R. des séances, 28.

1922

Lectures on Cauchy's problem in linear partial differential equations (Cambridge-New Haven).

L'enseignement secondaire et l'esprit scientifique, Rev. France.

Einstein en France, Rev. Internat. Enseignmement.

Les principes du calcul des probabilités, Rev. Métaph. Mor. **39.**

A propos des notions d'homogénéité et de dimension, Soc. Franç. Phys. et J. Phys.

Sur un théorème de géométrie élémentaire, Bull. Off. Recherc. Inv., décembre.

Sur la fonction harmonique la plus voisine d'une fonction donnée, Assoc. Franç. Avanc. Sci.

Sur une question de calcul des probabilités, Assoc. Franç. Avanc. Sci.

The early scientific work of H. Poincaré, Rice Institute Pamphlet **9,** 111–183.

1923

La notion de différentielle dans l'enseignement, Scripta Univ. Jérusalem **1,** no. 4.

Poincaré i la teoria de la ecuacions differentials, Conf. Inst. d'Études Catalanes de Barcelone.

La réforme de l'enseignement secondaire, Conférence Assemblée Générale Étud., Bull. Scient. Etud. Paris.

Sur les points doubles des lieux géométriques et sur la construction par régions, Nouv. Ann. Math. (5) **1,** 364–379.

La pensée française dans l'évolution des sciences exactes, France et Monde.

Sur une formule déduite de la théorie des fonctions elliptiques, Bull. Soc. Math. France **51,** 295–296.

Sur les tourbillons et les surfaces de glissement dans les fluides, C. R. Acad. Sci. Paris **177,** 505–506.

1924

Principle de Huyghens et prolongement analytique, Bull. Soc. Math. France **52,** 241–278.

Quelques conséquences analytiques du principe de Huyghens (13e Réunion Soc. Ital. Avancement Sci. Naples), Atti Soc. Ital. Prog. Sci. **16,** 164–168.

Sobre la represencion grafica de l'espacio de quatro dimensiones, Rev. Mat. Hisp.-Amer.

Comment je n'ai pas découvert la relativité?, Congr. Philos., Naples.

Le développement de la notion de fonction, Conf. École Polytechnique Rio de Janeiro (Portuguese), redigées par J. Nicoleti, Rev. Acad. Brasil. Ciencias.

Le principe de Huyghens (Conférence pour le cinquantenaire de la Soc. Math. de France), Bull. Soc. Math. France **52,** 610–640.

1925

On quasi analytic functions, Proc. Nat. Acad. Sci. U.S.A. **11,** 447–448.

Sur le calcul approché des intégrales définies, Proc. Nat. Acad. Sci. U.S.A. **11,** 448–450; Bull. Soc. Math. France, C. R. des séances, 21–22.

Itération et fonctions quasi analytiques, Rev. Gén. Sci.

Sobre un tipo de ecuaciones integrales singulares, Rev. Acad. Madrid **22,** 187–191.

1926

Sur une série entière en relation avec le dernier théorème de Fermat, Bull. Soc. Math. France, C. R. des séances, 21–22.

Sur les équations intégrables par la méthode de Laplace, Bull. Soc. Math. France, C. R. des séances, 33–35.

Sur la géométrie anallagmatique, Bull. Soc. Math. France, C. R. des séances, 35–39.

Préface de Gonseth "Les fondements des mathématiques," reproduit dans Bull. Sci. Math. (2) **51** (1927), 66–73.

Quelques cas d'impossibilité du problème de Cauchy (In memoriam N. I. Lobatchewsky), édité par la Soc. Math. de Kazan, Centenaire de Lobatchewsky 2.
Sur la théorie des séries entières, Nouv. Ann. Math. (6) **1**, 161–164.
A propos du nouveau programme de mathématiques spéciales, Nouv. Ann. Math. (6) **1**, 257–276, 391–393.
Cours d'analyse de l'école polytechnique. 1, Hermann, Paris.
Le principe de Huyghens dans le cas de quatre variables indépendantes, Acta Math. **49**, 203–244.
La serie de Taylor et son prolongement analytique, 2 ième édition revisée et completée Gauthier-Villars, Paris. (en collaboration avec S. Mandelbrojt)

1927

Récents progrès de la géométrie anallagmatique, Rev. Mat. Hisp.-Amer. **2**, Nouv. Ann. Math. (6) **2**, 257–273, 289–320.
Sur la théorie des fonctions entières, Bull. Soc. Math. France **55**, 135–137.
Sur les éléments riemanniens et le déplacement parallèle, Bull. Soc. Math. France **55**, C. R. des séances, 30–31.
Sulle funzioni intere di genere finito. Lettre à M. Landau, Rend. Acad. Lincei **6**, 3–9.
L'oeuvre de Duhem sous son aspect mathématique, Mém. Soc. Phys. Natur. Bordeaux **17**.
Sur le battage des cartes, C. R. Acad. Sci. Paris **185**, 5–9.

1928

Observations sur une note de M. Hostinsky, C. R. Acad. Sci. Paris **186**, 62.
Sur les opérations itérées en calcul des probabilités, C. R. Acad. Sci. Paris **186**, 189–192.
Sur le principe ergodique, C. R. Acad. Sci. Paris **186**, 275–276.
Deux exercices de mécanique, Enseignement Sci. **1**.
A propos de géométrie anallagmatique, Enseignement Sci. **1**.
Les méthodes d'enseignement des sciences expérimentales, Rev. Internat. Enseignement, 47éme année.
Une propriété de la fonction $\zeta(s)$ *et des séries de Dirichlet*, Bull. Soc. Math. France, C. R. des séances, 43–44; Congr. Ass. Franç. Avanc. Sci., La Rochelle.
Sur l'enseignement de la mécanique, Congr. Ass. Franç. Avanc. Sci., La Rochelle.
Le développement et le rôle scientifique du calcul fonctionnel, Congr. Internat. Math., Bologne.
Sur le battage des cartes et ses relations avec la mécanique statistique, Congr. Internat. Math., Bologne.

1929

Huyghensuv princip (Conférence faite à l'Université Charles à Prague et à l'Université Mazaryk à Brno), Casopis Pest Mat. Fys.
Le principe de Huygens pour les équations à trois variables indépendantes, J. Math. (9) **8**, 197–228.
On ordinary restricted extrema in connection with point transformations, Bull. Amer. Math. Soc. **35**, 823–828.
Analyse du livre de E. Landau "Vorlesungen über Zahlentheorie," Bull. Sci. Math. (en collaboration avec S. Mandelbrojt)

1930

Sur les arêtes de rebroussement de certaines enveloppes, Comptes rendus du 1er Congrès des mathématiciens des pays slaves in 1929.
Remarques géométriques sur les enveloppes et la propagation des ondes, Acta Math. **54**, 247–261.

La physique et la culture générale, Oeuvre, 30 janvier.
La question de la physique, Oeuvre, 16 février.
Un nouveau pas à faire dans la voie de la paix: conventions scolaires, La paix par le Droit.
Equations aux dérivées partielles et fonctions de variables réelles, Congrès des mathématiciens russes à Kharkov; Ukrainian traduction dans Comm. Soc. Math., Kharkow (4) **5** (1932), 11–20.
Cours d'Analyse de l'Ecole Polytechnique, 2, Hermann, Paris.

1931

Parlons culture générale, Oeuvre, 13 janvier.
Formation ou déformation intellectuelle, Oeuvre, 19 janvier.
Une culture qu'il ne faudrait pas détruire, Oeuvre, 24 janvier.
La question de la physique, Oeuvre, 16 février.
Multiplication et division, Enseignement Sci.

1932

La propagation des ondes et les caustiques, Comment Math. Helv. **5**, 137–173.
Sur les équations aux dérivées partielles d'ordre supérieur, Congr. Internat. Math., Zurich.
Sur la théorie des équations aux dérivées partielles du premier ordre, Enseignement Sci.
Réponse à une enquête sur l'histoire des sciences dans l'enseignement, Enseignement Sci.
Coordination d'enseignements, Enseignement Sci.
Le problème de Cauchy et les équations aux dérivées partielles linéaires hyperboliques, (traduction des leçons professées à Yale), Hermann, Paris.

1933

Propriétés d'une équation linéaire aux dérivées partielles du quatrième ordre, Tôhoku Math. J. **37**, 133–150.
Painlevé, le savant, Vu.
Sur les probabilités discontinues des événements en chaîne (en collaboration avec M. Fréchet), Z. Angew. Math. Mech. **13**, 92–97.

1934

Sur un résultat relatif aux équations algébriques, Bull. Soc. Math. France **62**, C. R. des séances, 25.
Sur une question relative aux congruences de sphères, Bull. Soc. Math. France **62**, C. R. des séances, 25.
L'oeuvre scientifique de Paul Painlevé, Rev. Métaph. Mor.
Un terme à effacer de l'enseignement mathématique "Effectuer," Enseignement Sci.
Réponse à l'enquête sur les bases de l'enseignement mathématique, Enseignement Sci.
La non-résolubilité de l'équation du cinquième degré, Enseignement Sci.
Un cas simple de diffusion des ondes, Rec. Math. Moscou. **41**, 402–407.
Préface, Hasse au livre de, "Über Gewisse Ideale in einer Einfachen Algebra," Actualités Sci. Indust., 1934, no. 109, 5–6.
Observation au sujet de la note de M. Mursi, C. R. Acad. Sci. Paris **199**, 179–180.

1935

Polynomes linéaires adjoints, Enseignement Sci.
Réponse à l'enquête sur l'enseignement de la mécanique, Enseignement Sci.
Les développables circonscrites à la sphère, Enseignement Sci.
La théorie des équations du premier degré, Enseignement Sci.
Extrait d'une lettre à M. T. Kubota, Tôhoku Math. J. **40**, 198.

Les conditions définies dans les problèmes aux dérivées partielles, Généralités des cas hyperboliques, Conférences d'introduction à la réunion mathématique tenue à l'Université de Genève, 17 à 29 juin.

Les caustiques des enveloppes à deux paramètres, J. Math., volume publiéen hommage à M. Goursat.

Un problème topologique sur les équations différentielles, Prace Mat. Fiz. **44**, volume en hommage à la mémoire de Lichtenstein.

La notion de différentielle dans l'enseignement, Math. Gaz. **19**, 341–342.

1936

La caustique des enveloppes à deux paramètres, J. Math. Pures Appl. (9) **15**, 333–337.

Sur les caustiques des enveloppes à deux paramètres, C. R. Soc. Math. France **1935**, 29–30.

Selecta, Jubilé Scientifique, 1935, Gauthier-Villars, Paris.

1937

Observations sur la note de M. Krasner et M^{lle}. Ranulac, C. R. Acad. Sci. Paris **204**, 399.

Observations sur la note de M. Mandelbrojt, C. R. Acad. Sci. Paris **204**, 1458–1459.

Observations sur les notes précédentes de MM. Destouches et Appart, C. R. Acad. Sci. Paris **204**, 458.

Le problème de Dirichlet pour les équations hyperboliques, J. Chinese Math. Soc. **2**, 6–20.

Calcul des variations et différentiation des intégrales, Akad. Nauk S.S.S.R., Trudy Inst. Mat. Tbilissi **1**, 55–64.

1938

Sur certaines questions de calcul intégral, An. Soc. Ci. Argentina **125**, 1–18.

L'homogénéité en mécanique, Bull. Sci. Math. (2) **62**, 6–10.

Remarque sur l'intégration approachée des équations différentielles, Ann. Soc. Polon. Math. **10**, 126.

1940

Les mathématiques dans l'encyclopédie française, Mathematica (Cluj) **16**, 1–5.

Les diverses formes et les diverses étapes de l'esprit scientifique, Thalès **4**, 23–27.

1942

The problem of diffusion of waves, Ann. of Math. (2) **43**, 510–522.

On the Dirichlet problem for the hyperbolic case, Proc. Nat. Acad. Sci. U.S.A. **28**, 258–263.

1943

Obituary: Emile Picard, J. London Math. Soc. **18**, 114–128.

1944

Two works on iteration and related questions, Bull. Amer. Math. Soc. **50**, 67–75.

A known problem of geometry and its cases of indetermination, Bull. Amer. Math. Soc. **50**, 520–528.

1945

The psychology of invention in the mathematical field, Princeton Univ. Press, Princeton, N. J.

Obituary: George David Birkhoff, C. R. Acad. Sci. Paris **220**, 719–721.

On the three-cusped hypocycloid, Math. Gaz. **29**, 66–67.

Remarques sur le cas parabolique des equations aux dérivées partielles, Publ. Inst. Mat. Univ. Nac. Littoral **5**, 3–11.

Problèmes à apparence difficile, Mat. Sb. (N.S.) **17**(59), 3–8.

1947

Observation sur la note de M. Bureau, C. R. Acad. Sci. Paris **225**, 854.

1948

Sur le cas anormal du problème de Cauchy pour l'équation des ondes. Studies and essays presented to R. Courant on his 60th birthday, January 8, Interscience, New York; pp. 161–165.

1950

Célébration du deuxième centenaire de la naissance de P. S. Laplace, Arch. Internat. Hist. Sci.(N.S.) **3**, 287–290.

Les fonctions de classe supérieure dans l'équation de Volterra, J. Analyse Math. **1**, 1–10.

1951

Non-Euclidean geometry in the theory of automorphic functions, GITTL, Moscow-Leningrad. (Russian)

Partial differential equations and functions of real variables, Gaz. Mat. (Lisboa) **12**, no. 50, 3–6. (Portuguese)

La géométrie non-euclidienne dans la theorie des fonctions automorphes, GITTL, Moscow-Leningrad. (Russian)

1953

Lectures on Cauchy's problem in linear partial differential equations, Dover, New York.

Non-Euclidian geometry and axiomatic definitions, Magyar Tud. Akad. Mat. Fiz. Oszt. Közl. **3**, 199–208. (Hungarian)

1954

An essay on the psychology of invention in the mathematical field, Princeton Univ. Press, Princeton, N. J., 1945.

La géométrie non-euclidienne et les définitions axiomatiques, Acta Math. Acad. Sci. Hungar. **5**, suppl. 95–104.

Sur des questions d'histoire des sciences. La naissance du calcul infinitésimal, An. Acad. Brasil Ci. **26**, 83–89.

Equations du type parabolique dépourvues de solutions, J. Rational Mech. Anal. **3**, 3–12.

History of science and psychology of invention, Mathematica **1**, 1–3; traduction en hongrois, Mat. Lapok **9** (1958), 64–66.

Le centenaire de Henri Poincaré, Rev. Histoire Sci. Appl. **7**, 101–108.

1955

Extension à l'équation de la chaleur d'un théorème de A. Harnak, Rend. Circ. Mat. Palermo (2) **3**, 337–346.

1957

Sur le théorème de A. Harnak, Publ. Inst. Statist. Univ. Paris **6**, 177–181; voir aussi Bul. Inst. Politehn Iaşi (N.S.) **3**.

1959

Essai sur la psychologie de l'invention dans le domaine mathématique. Traduit de l'anglais par Jacqueline Hadamard. Première édition française revue et augmentée par l'auteur, Librairie Scientifique Albert Blanchard, Paris.

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