of Frobenius are proven. Here the exposition differs necessarily from the standard treatment of the finite-dimensional case, and is very pretty. Except for the theory of the Lie derivative, the main features of the standard theory are all present. The integration of forms is excluded of course.

In addition to these chapters, the book contains secondary material on differential calculus, including an elegant proof of the inverse function theorem (Chapter I), on Riemannian metrics, including their relation to sprays (Chapter VII), on the spectral theorem for Hermitian operators (Appendix I gives a complete exposition of the subject in 7 pages), and on the classical language of the finite-dimensional calculus (Appendix II).

The extra care required by the infinite-dimensional extension is more than compensated by the clarification of the standard theory it provides, and the intrinsic geometric intuition it teaches. Thus the author's claim is justified, that the generalization is achieved "at no extra cost." Nevertheless, many a reader will feel frustrated by the lack of a significant example of a differentiable manifold which is not finite-dimensional. An example (credited to J. Eells, Jr.) is mentioned in the foreword, however, which is of central importance in current applications of differential calculus, and strongly reinforces the author's choices of definitions.

Finally, the author should be rebuked for allowing several careless errors to appear in the book. Although these are mathematically insignificant, they obscure the most important virtue of the book: the really subtle pitfalls have been expertly skirted. In spite of this minor negligence, the *Introduction* at once provides the expert with a fundamentally reliable handbook in an area of current research, and the novice with an elegant exposition of a basic category.

RALPH ABRAHAM

Differential geometry and symmetric spaces. By S. Helgason. Pure and Applied Mathematics Series, Vol. 12. Academic Press, New York, 1962. 14+486 pp. $12.50.

The mathematical community has long been in need of a book on symmetric spaces. S. Helgason has admirably satisfied this need with his book *Differential geometry and symmetric spaces.* It is a remarkably well written book that takes the "and" in its title seriously in both a material and spiritual sense. Indeed, about the first third of the book is devoted to a concise exposition of the differential geometry of abstract manifolds and Lie groups. But in addition to this obvious physical fact, the author has, whenever possible, chosen to emphasize the geometric point of view rather than the algebraic. The end result of this is that the geometric point of view is always before the reader. Unfortunately, the algebraic structure of the subject sometimes has to suffer on this account. This is apparent, for instance, in the piecemeal way that the structure theory of the semi-simple groups was treated. It is cut up and spread throughout the first two-thirds of the book. Indeed in the second third, devoted to the elementary and detailed treatment of the symmetric spaces, the author has to stop at a certain point and insert the algebraic material he needs. The third and final part of the book contains an extremely enlightening account of the function theory of symmetric spaces. This material is extremely current and the author is to be highly complimented on his treatment of this subject. With these general comments made, we will try to briefly outline the material in this book and point out places where the exposition is unusual.

The first chapter is a masterpiece of concise, lucid mathematical exposition with the author beginning with the basic definitions of manifolds, vector fields, connections, etc. and culminating seventy odd pages later with the concepts of complete riemannian manifolds, sectional curvature, and the structure of riemann manifolds of negative curvature. The treatment of differential geometry is essentially that first given by Koszul and publicly exposed in print in the early papers of Nomizu. The most unusual feature is his careful treatment of the exponential mapping in an affinely connected manifold which culminates, perhaps, with the formula for the differential of the exponential map in terms of the torsion and curvature of the connection. He has also unified the treatment of the exponential map for affinely connected manifolds and Lie groups. This is certainly an extremely pretty point in the author's exposition.

In the second chapter the author treats the basic ideas in the theory of Lie groups and Lie algebras that center about the correspondence between Lie groups, Lie algebras, subalgebras and subgroups, etc. One of the most elegant features of the author's treatment of this subject is his presentation of the proof that a closed subgroup of a Lie group is a Lie group. The author begins his study in §3 by introducing a left invariant affine connection for Lie groups and, as remarked before, this enables him to obtain the exponential map for Lie groups from the previously treated exponential mapping for affine connections. It is not clear, however, why the author introduces the universal enveloping algebra so early and with no motivation. It is introduced in §2 immediately following the basic definitions and without any motivation at all. This seems rather unfortunate in an otherwise beautifully written chapter. Again in the definition of a semi-simple Lie group (non-degenerate Killing form) the author has been forced to choose a definition that is hard to motivate. Of course given the amount of material he covers, the author must occasionally make a convenient rather than an intuitive definition. Indeed, the only objection one can seriously raise to the first three chapters, Chapter III treats in a standard fashion some of the structure theory for semi-simple Lie algebras, is that the limitations of space have forced the author sometimes to adopt an expository point of view that is satisfactory for a person with some experience in the field, but which might be difficult for a novice in the field due to the lack of any clear motivation or treatment of examples.

The second third of the book, Chapters IV, V, VI, VII and IX are beautifully written and whereas in the first third there is, of necessity, much very concise writing, in this part the author's exposition is brilliant. Indeed, it might be used as a textbook for "how to write mathematics." All the little difficulties inherent in the subject are neatly treated, while the reader always has before him the global picture of where he is going and why. The author has succeeded in presenting us with both the forest and the trees. Chapter IV treats affine and riemannian symmetric spaces, local and global, and leads naturally to the decomposition of symmetric spaces given in Chapter V. One of the high points of Chapter V is the discussion of duality in §2. Chapters VI and VII treat the symmetric spaces of non-compact and compact type respectively. In Chapter VII the important and deep Weyl group is admirably and completely (for the first time to the reviewer's knowledge) treated. Chapter IX deals with the listing of various classifications of spaces that have played such an important role in the theory of symmetric spaces, enabling many theorems to be proven initially by case checking.

The last part of the book consists of Chapters VIII and X. Chapter VIII treats Hermitian symmetric spaces and bounded symmetric domains. In Chapter X, we have a treatment of analysis on symmetric spaces in the spirit of Harish-Chandra. This centers about the spherical functions on such spaces and again the exposition is extremely clear.

In conclusion one should make two remarks—one that the author has chosen not to assume or use the language of fiber bundles or any algebraic topology except the concept of covering spaces; second, this book should make this important topic available to mathematicians who are not specialists in differential geometry.

Louis Auslander