AXIOMATIC APPROACH TO THE HOMOTOPY GROUPS

SZE-TSEN HU

1. **Introduction.** The history of homotopy groups traces back to the fundamental groups of Poincaré [9; 10; 14].[1] For a given topological space X and a given point x_0 in X, the funnamental group $\pi_1(X, x_0)$ is defined by considering the loops in X with x_0 as basic point, i.e., the continuous maps $\lambda: S^1 \to X$ of the circle S^1 into X with a given point s_0 of S^1 mapped into x_0.

Replacing the circle S^1 by a higher dimensional sphere S^n, Hurewicz [7] introduced the homotopy groups $\pi_n(X, x_0)$ in 1935 which turned out to be very useful and prolific. In 1941, relative homotopy groups $\pi_n(X, A, x_0)$, $n \geqq 2$, of a topological space X modulo a subspace A at a given point x_0 were introduced by a joint paper of Hurewicz and Steenrod [4] and also independently by J. H. C. Whitehead [16]. These groups are defined by considering the continuous maps of an n-dimensional cell E^n into X with the boundary sphere S^{n-1} mapped into A and a given point s_0 of S^{n-1} mapped into x_0.

For each $n \geqq 2$, a boundary homomorphism

$$\partial: \pi_n(X, A, x_0) \to \pi_{n-1}(A, x_0)$$

is defined by taking restrictions of the maps on the boundary sphere S^{n-1} of the cell E^n.

For any continuous map $f: (X, A, x_0) \to (Y, B, y_0)$, an induced homomorphism

$$f_*: \pi_n(X, A, x_0) \to \pi_n(Y, B, y_0)$$

is defined by means of composition.

These are the entities of the so-called homotopy theory [5]. One observes that the homotopy theory looks quite like a homology theory.

Since Eilenberg and Steenrod [2] established their celebrated axiomatic approach to homology theory in 1945, it has been a natural problem to ask whether a similar approach is possible for homotopy theory.

An address delivered before the Knoxville meeting of the Society on November 18, 1955, by invitation of the Committee to Select Hour Speakers for Southeastern Sectional Meetings; received by the editors February 23, 1956.

[1] Numbers in brackets refer to the bibliography at the end of the paper.